Ptolemy’s
ALMAGEST

Translated and Annotated by
G. J. Toomer

.Duckworth
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Preface

A new English translation of the Almagest needs no apology. As one of the most influential scientific works in history, and a masterpiece of technical exposition in its own right, it deserves a much wider audience than can be found amongst those able to read it in the original. The existing English translation by R. Catesby Taliaferro, besides being difficult to acquire, is such that silence is the kindest comment one can make. The French translation by N. Halma, virtually unobtainable, suffers from excessive literalness, particularly where the text is difficult. The other modern version, Karl Manitius' German translation, is on an entirely different level from these. It was done by a man who had studied the text and made a strenuous and on the whole successful effort to understand Ptolemy's meaning and methods. I have used it constantly for twenty years, and those to whom it is familiar will recognise how much I owe to it. Nevertheless, it is not free from mistakes, and, to my taste, errs in the direction of paraphrasing where it should be translating. Most important, one can no longer assume that those with a serious interest in history are able to read German with ease. I have been able to improve on Manitius' translation, in part because of work published since he made it, in part because I had independent access to much of the textual evidence, notably the mediaeval Arabic translations. I have drawn attention to a few passages where I have noticed that he is in error, but I have made no systematic comparison between my translation and his or any other version.

Every translator, and especially one dealing with an ancient language, is confronted with the dilemma of being faithful to the original and at the same time comprehensible to his readers. My intention was that this translation should serve both those who know no Greek, as a substitute for the text, and those who do, as an aid to reading it. This has inevitably led to compromises. On the whole, I have kept closely to the meaning and structure of the Greek, even, on occasion, where this entailed abandoning idiomatic English. But I have usually broken up Ptolemy's enormously long sentences (characteristic of Hellenistic scientific prose) into shorter units more suitable for English, and I have frequently substituted mathematical symbols (\(=\), \(+\) etc.) and a symmetric presentation for the continuous rhetorical exposition of the ancient text. I have been liberal with explanatory additions, which are marked as such by enclosure within square brackets. Wherever the need to be intelligible forced me to a paraphrase, I give the literal translation in a footnote.

It would have made what is an already big book impossibly unwieldy if I had

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1 For full references here and elsewhere see the Bibliography.
Preface

provided a full technical and historical commentary on the Almagest. Fortunately two recent works, by Neugebauer (HAMA) and Pedersen, are excellent guides to the technical content, and the former is also of considerable help on the numerous historical problems which arise from it. I have therefore confined my own commentary to footnotes on points of detail (referring to the above works for expository treatments), and to an introduction giving the minimum of information necessary to understand and use the translation.

In the course of making the translation I recomputed all the numerical results in the text, and all the tables (the latter mostly by means of computer programs). The main purpose of this was to detect scribal errors (in which I have been moderately successful). But my calculations incidentally revealed a number of computing errors or distortions committed by Ptolemy himself. Where these are explicable as the result of rounding in the course of computation they are ignored, since to list some thousands of slightly more accurate results which I have found with modern mechanical aids would invite Ptolemy’s own sardonic remark: ‘Scrupulous accuracy about such a small amount is a sign of vain conceit rather than love of truth’. However, I have noted every computing error of a significant amount, and also those cases where the rounding errors are not random, but seem directed towards obtaining some ‘neat’ result. I hope that this will shed some light on the problem of Ptolemy’s manipulation of his material (both computational and observational) in order to present an appearance of rigor in his theoretical treatment which he could never have found in his actual experience. The problem is an interesting one, which deserves an informed and critical discussion. Unfortunately, the recent book on this subject by R. R. Newton provides nothing of the kind, but rather tends to bring the whole topic into disrepute. The only detailed discussion which is useful is that by Britton [1]. This, however, is confined to certain classes of the observations. My own inferences from the computations tend to confirm Britton’s conclusions about the nature and purpose of Ptolemy’s manipulations of his data.

This book owes much to the help of numerous people and institutions, which I gratefully acknowledge here. The Bibliothèque Nationale, Paris, the Biblioteca Apostolica Vaticana and the Biblioteca de El Escorial provided me with microfilms of various Greek and Arabic manuscripts of the Almagest (detailed on pp. 3-4). I thank my colleague, David Pingree, Prof. Dr. Fuat Sezgin and Prof. Dr. Paul Kunitzsch for providing me with other microfilms and photocopies which I needed. Mr. Colin Haycraft not only gave me the necessary encouragement actually to embark on a project which I had been contemplating for a long time, but also bore patiently with the repeated delays until the book was ready for publication. When B. R. Goldstein, who was already engaged in preparing an English version of the Almagest, heard that I had decided to make this translation, he generously abandoned the project and turned over his materials to me. I owe to these and to him several ideas about format and notation. My pupil, Don Edwards, detected a number of slips and

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2It is regrettable that this has never been formally published. It is available in Xerox copy from University Microfilms International, Ann Arbor, Michigan 48106.
typing errors in my preliminary version, and performed many useful services in comparing the translation with the Greek text. Michele Wilson drew Fig. F for me. Janet Sachs provided invaluable help in preparing the typescript for publication and eliminating numerous mistakes. Several of my footnotes on difficult problems have been influenced by my discussions with Noel Swerdlow. Rather than trying to disentangle his contribution at each place, I here record, with thanks, the stimulus he has given to my thinking. N. G. Wilson answered my questions on points of Greek palaeography and went out of his way to examine manuscripts at my request. My colleague, A. J. Sachs, gave me the benefit of his unrivalled expertise on several points of Babylonian astronomy and Mesopotamian history. To my colleague O. Neugebauer Lowe more than I can express here. Let me say only that it was he who first introduced me to the Almagest more than twenty years ago, that his own investigations of it (only part of which have been published in his monumental *A History of Ancient Mathematical Astronomy*) have been invaluable to me as an aid and as a model, and that many will recognize his draughtsmanship in several of the supplementary diagrams. As an inadequate token I dedicate this book to him.

Providence, 1982

G.J.T.
Introduction

1. Ptolemy

For a detailed discussion of what little is known of the life of the author of the Almagest, and an account of his numerous other works, on astronomy, astrology, geography, optics and other mathematical subjects, I refer the reader to my article in the Dictionary of Scientific Biography (Toomer [5]). Here I mention only that his name was Claudius Ptolemaeus (Klau³ìq̄ǐq Tò³̄eμαί̌oq'), that he lived from approximately A.D. 100 to approximately A.D. 175, and that he worked in Alexandria, the principal city of Greco-Roman Egypt, which possessed, among other advantages, what was probably still the best library in the ancient world.

2. The Almagest

The Almagest is firmly dated to the reign of the Roman emperor Antoninus (A.D. 138-161). The latest observation used in it is from 141 February 2 (IX 7 p. 450), and Ptolemy takes the beginning of the reign of Antoninus as the epoch of his star catalogue (VII 4 p. 340). Although it is clear that Ptolemy had spent much time on it and that it is a work of his maturity (his own observations recorded in it range from A.D. 127 to 141), it has always been considered as his earliest extant work, because of the changes from it and references back to it in other works by him (for details see Toomer [5] p. 187). However, a recent discovery by Norman T. Hamilton (see IV n.51 p. 205) has shown that the 'Canobic Inscription' represents a stage in the development of Ptolemy's astronomical theory earlier than the Almagest. Since Ptolemy erected that dedication in the tenth year of Antoninus (A.D. 146/7), the Almagest can hardly have been published earlier than the year 150.

As is implied by its Greek name, μαθηματικὴ σύνταξις, 'mathematical systematic treatise', the Almagest is a complete exposition of mathematical astronomy as the Greeks understood the term. Whether there were any comparable works (i.e. comprehensive astronomical treatises) before it is not known. In any case, its success contributed to the loss of most of the work of Ptolemy's scientific predecessors, notably Hipparchus, by the end of antiquity, because, being obsolete, they ceased to be copied. Whereas Hipparchus' works are still used by Ptolemy's younger contemporaries, Galen and Vettius Valens,¹

by the early fourth century (and probably much earlier), when Pappus wrote his commentary on it, the Almagest had become the standard textbook on astronomy which it was to remain for more than a thousand years. Thus its importance for us lies not only in its value as a historical source for earlier theories and observations, but also, and perhaps chiefly, in its influence on all later astronomy in antiquity and the middle ages (in both Islamic and Christian areas) down to the sixteenth century. It was dominant to an extent and for a length of time which is unsurpassed by any scientific work except Euclid's Elements.

No attempt can be made here to sketch even an outline of the history of its influence. I mention only some points to which I will make reference in the notes to the translation. The position of the Almagest as the standard textbook in astronomy for ‘advanced students’ in the schools at Alexandria (and no doubt at Athens and Antioch too) in late antiquity is amply demonstrated by the partially extant commentaries on it by Pappus (c. 320) and by Theon of Alexandria (c. 370). In the late eighth and ninth centuries, with the growth of interest in Greek science in the Islamic world, the Almagest was translated, first into Syriac, then, several times, into Arabic. In the middle of the twelfth century no less than five such versions were still available to the amateur ibn aṣ-Ṣalāḥ: a Syriac translation, two versions made under the Caliph al-Ma’mūn (an older one by al-Ḥasan ibn Quraysh, and one dated 827/8 by al-Ḥajjāj), a version by the famous translator Ishāq ibn Hunayn (c. 879-90), and a revision of the latter by Thābit ibn Qurra (d. 901). Two of these translations are still extant, those of al-Ḥajjāj and Ishāq-Thābit. In them we find the title of Ptolemy’s treatise given as ‘al-majṣṭy’ (consonantal skeleton only). This is undoubtedly derived (ultimately) from a Greek form μεγίστη (?sc. συνταξις), meaning ‘greatest [treatise]’, but it is only later that it was incorrectly vocalised as al-majastā, whence are derived the mediaeval Latin ‘almagesti’, ‘almagestum’, the ancestors of the modern title ‘Almagest’. The available evidence has been assembled and discussed by Kunitzsch, *Der Almagest* 115-25, where he makes a good case for supposing that the Arabic form was derived, not directly from the Greek, but from a middle Persian (Pahlavi) translation of the Almagest. There is independent evidence for the existence of the latter, but whether it was made as early as the reign of the Sassanid king Shahpuhr I (241–272), as later Persian accounts maintain, seems very dubious to me.

While Ptolemy’s work in the original Greek continued to be copied and studied in the eastern (Byzantine) empire, all knowledge of it was lost to western

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2 The evidence for the practice of astronomy in the third century is pitifully small, but there exists a fragment of a text from about A.D. 213 which is closely related to the Almagest (see H.I.M. II 948-49), and there are several third-century papyri related to the Handy Tables (ibid. 974-75, 979-80). P. Ryl. 27 (written c. 260) quotes Ptolemy’s solstice and equinox observations from Almagest III 1, and in the late third century Porphyry (Comm. on Harmonica 2, p. 24,15 ff.) quotes Almagest I 2 (H 9, 11-16). The only evidence I have seen for knowledge of the Almagest in the second century, Galen, Commentary on Hippocrates’ *Airs Waters and Places* III (ms. Cairo, Ṭal‘at jībīb 550, p. 73a), where Ptolemy is mentioned at the end of a list of authorities on astronomy, must be an interpolation in the Arabic tradition, since Ptolemy is there characterized as ‘the king of Egypt’.

3 I know of no satisfactory account of this. I gave a very brief sketch, Toomer[5] 202.

4 For a full account of this see Kunitzsch, *Der Almagest*, especially 15-71. Kunitzsch has also published the work of ibn aṣ-Ṣalāḥ (see Bibliography).
Europe by the early middle ages. Although translations from the Greek text into Latin were made in mediaeval times, the principal channel for the recovery of the Almagest in the west was the translation from the Arabic by Gerard of Cremona, made at Toledo and completed in 1175. Manuscripts of the Greek text began to reach the west in the fifteenth century, but it was Gerard's text which underlay (often at several removes) books on astronomy as late as the Peurbach-Regiomontanus epitome of the Almagest (see Bibliography under Regiomontanus). It was also the version in which the Almagest was first printed (Venice, 1515). The sixteenth century saw the wide dissemination of the Greek text (printed at Basel by Hervagius, 1538), and also the obsolescence of Ptolemy's astronomical system, brought about not so much by the work of Copernicus (which in form and concepts is still dominated by the Almagest), as by that of Brahe and Kepler.

3. The translation

The basis of my translation is the Greek text established by Heiberg. I have, however, found it necessary to make several hundred corrections to that text. These are noted at the places in the translation where they occur, and are also listed in Appendix B. In many cases (usually involving numerical computations), my correction consists of adopting the reading of the manuscript D, unjustly spurned by Heiberg as descended from an archetype due to an Alexandrian recension in late antiquity (Prolegomena, in Ptolemy, Opera Minora CXXVI-VII). Whatever the truth about that, and despite the fact that D itself is, as Heiberg says, 'most negligently written', I am convinced on grounds of internal consistency that it represents a sounder tradition than that of the mss. ABC, generally preferred by Heiberg. In many cases its obviously correct readings are shared by all or part of the Arabic tradition. Nevertheless, I have not deviated from Heiberg's text except where it seemed essential for sense or numerical consistency. In making corrections I have referred to photographs of the following manuscripts.

Greek (I use Heiberg's notation)
A Parisinus graecus 2389. Mainly uncial, ninth century
B Vaticanus graecus 1594. Minuscule, ninth century
D Vaticanus graecus 180. Several hands, but not, as Heiberg, Almagest I p. V, of the twelfth century, but rather of the tenth: see the Vatican Catalogue by Mercati and Franchi de' Cavalieri, I p. 206. N. G. Wilson has confirmed this dating for me by personal inspection. (Heiberg himself seems to have changed his opinion later: see Prolegomena LXXIX.)

Arabic (I have used the abbreviations 'Ar' to refer to the consensus of the

6 Kunitzsch, Der Almagest 83-112, gives a valuable account of the evidence for this, and of Gerard's method of work: evidently he used more than one of the Arabic translations.
7 I have acknowledged there all cases known to me where my correction has been anticipated by others, notably Manitius.
Arabic tradition, and ‘Is’ to the consensus of the mss. containing the Ishāq-Thābit version).

L Leiden, or. 680. Eleventh century according to Kunitzsch, *Der Almagest* 38. This is the only surviving manuscript of the version of al-Hajjaj.


E Escorial 914. See Kunitzsch, *Der Almagest* 43–4. The Ishāq-Thābit version, Books V–IX.

F Escorial 915. Completed September 1276. See Kunitzsch, *Der Almagest* 44–5. The Ishāq-Thābit version, allegedly containing Books VII–XIII, but in fact lacking large sections even of these, and bound in such disorder as to be almost useless.

Ger The Latin translation of Gerard of Cremona, for which I have used only the printed edition (Venice, Liechtenstein, 1515). For the complex dependence of this on the various Arabic versions see Kunitzsch, *Der Almagest* 97–104.

I did not undertake a complete collation of any of the above mss. For the Greek mss. that would have been largely useless, since Heiberg’s reports are, as in all his editions, very accurate (to judge from my sporadic verifications; I remarked the rare exceptions in the notes to the translation). To collate the Arabic translation would have delayed this book for several years, with no commensurate gain. I have consulted the above mss. only in passages where I already considered Heiberg’s text wrong or suspect. Therefore no conclusions should be drawn about the readings of the Arabic mss. where I do not explicitly report them.

There are a number of places where, if I were to establish a Greek text, it would differ from Heiberg’s, but which I have not bothered to record in this book. Examples are:

**mere orthography:**

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<tr>
<td>ἰήριςκομεν</td>
<td>εὐρίσκομεν (imperfect)</td>
<td>I 327,15</td>
</tr>
<tr>
<td>Καλλιπος</td>
<td>Κάλλιπος</td>
<td>I 199,5</td>
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<tr>
<td>αμεταπειστον</td>
<td>αμετάπιστον</td>
<td>I 6,18 (cf. Boll, <em>Studien</em> 74)</td>
</tr>
<tr>
<td>κρικος</td>
<td>κρίκος</td>
<td>I 196,8</td>
</tr>
</tbody>
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**changes in form not affecting the sense:**

- αν for εαν I 393,11

**reversals of letters referring to figures:**

- ZK for KZ I 243, 22

**obvious misprints:**

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<tr>
<td>σεληνης</td>
<td>σηληνης</td>
<td>I 406,25</td>
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<tr>
<td>ανωμαλιας</td>
<td>αμωμαλίας</td>
<td>I 462,19</td>
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(less obvious misprints, particularly those involving numbers, are recorded).

During the course of making the translation, I became convinced that the
text contains quite a large number of interpolations, which must go back to antiquity, since they are in the whole manuscript tradition, both Greek and Arabic. I was first led to this conclusion by the discovery that there are places in the text, nonsensical as they stand, which can be made to yield perfect sense by the simple elimination of a clause or sentence, which must have been inserted as 'explanation' by someone who failed to understand Ptolemy's meaning. A notable example is V I (see p. 219 n.5). Cf also V 12, p. 245 with n.41. I later realised that there are whole classes of textual matter which must also be regarded as interpolations. One of these is the totals in the star catalogue (see pp. 16-17). The other is the chapter headings. Some of these (e.g. IX 2) are so inept as descriptions of the actual content of the chapter that it is impossible to attribute them to Ptolemy. In fact I do not believe that Ptolemy himself used any chapter divisions at all. It is obvious that he is responsible for the division into 13 books, both from the summaries that are found at the beginning of most books, and from explicit references such as 'in Book I' (ἐν τῷ πρώτῳ τής συντάξεως, II I p. 75) and 'in the preceding book' (ἐν τῷ πρὸ τούτων συντάξματι, VI 5 p. 283). But he never refers to a chapter division. Furthermore, there is some discrepancy in the manuscript tradition (especially between the branch represented by D and that represented by A) as to the points of division between chapters (e.g. at the beginning of Book III), and it is clear from Pappus' commentary that although a division into chapters already existed in his time, it was very different, at least in Book V, from the present division. If the chapter division and headings are spurious, so too must be the table of contents preceding each book. Nevertheless, since this method of subdividing the text is useful for reference purposes, and appears in all editions, I have retained it, merely marking the character of the chapter headings by enclosing them in brackets thus: { }.

4. What is in the Almagest, and what is not

The order of treatment of topics in the Almagest (outlined in I 2) is completely logical. In Book I, after a brief treatment of the nature of the universe (in so far as it concerns the astronomer), Ptolemy develops the trigonometrical theory necessary for the work as a whole. In Book II he discusses those aspects of spherical astronomy which are related to the observer's position on earth (rising-times, length of daylight, etc.). Book III is devoted to the theory of the sun. This is a necessary preliminary for the treatment of the moon in Book IV, since the use of lunar eclipses there depends on one's ability to calculate the solar position. Book V treats the advanced lunar theory, which is a refinement of that in Book IV, and also lunar and solar parallax. Book VI is on eclipses, and thus requires a knowledge of both solar and lunar theory, and also of parallax. Books VII and VIII treat the fixed stars: since the moon is used as a 'marker' to determine the position of some crucial fixed stars, lunar theory must precede this, and since some planetary observations are made with respect to fixed stars,

8See the note in Rome[1] I p. 106, and cf. (for Theon) II p. 448 n. (1).
the establishment of a star catalogue (VII 5 and VIII 1) must precede the planetary theory. The last five books are devoted to the planets. Books IX–XI develop the theory of their longitudinal motion, Book XII treats retrogradations and greatest elongations (which depend only on longitude), while Book XIII deals with planetary latitude and those phenomena (the ‘phases’) which are partially dependent on it. Ptolemy occasionally anticipates later results for the sake of convenience (see IV 3 p. 179 and IX 3 p. 423, where the mean motion tables of moon and planets incorporate some later corrections), but in general the order of presentation, within books as well as in the treatise as a whole, is dictated by the logic of the didactic method.

There are, however, certain topics which Ptolemy does not discuss either because he takes it for granted that they are already known to his readers, or because it seemed superfluous to go into details (here I am referring especially to chronological matters). He says specifically (I 1 p. 37 with n.13) that the work is for ‘those who have already made some progress in the field’. This means, in practice, that he assumes a knowledge of elementary geometry (‘Euclid’) and ‘logistic’ (thus he does not consider it necessary to explain how to extract a square root), and also of ‘sphersics’. The latter is illustrated by the extant works of Autolycus, Euclid (Phaenomena) and Theodosius (Sphaerica), which deal with the phenomena arising from the rotation of stars and sun about a central, spherical earth, e.g. their risings, settings, first and last visibilities, periods of invisibility etc., using elementary geometry, but arriving mainly at qualitative rather than quantitative results. These results are mostly irrelevant to Ptolemy’s work, but he does use much of the terminology and concepts of sphersics without explanation.

5. What the reader of the Almagest needs to know

The modern reader, too, is likely to be familiar with elementary geometry. So I have not burdened the translation with references to Euclid except where the theorems assumed are not immediately obvious. However, in what follows I give a brief explanation of methods, concepts and facts not explained by Ptolemy which the reader of the Almagest needs to know, but which may be less familiar. On Ptolemy’s mathematical methods in general one may profitably consult Pedersen 47–56.

(a) The sexagesimal system

This was taken over by the Greeks (one may guess by the Hellenistic astronomers) from the Babylonians as a convenient way of expressing fractions and (to a lesser extent) large numbers, and of performing calculations with them. It is the first place-value system in history. In the translation and notes I use the convenient modern ‘comma and semi-colon’ notation, in which

\footnote{For more detail see HAMA II 755–71.}
6,13;10,0,58 represents $6 \times 60 + 13 + 10 \times 60^{-1} + 0 \times 60^{-2} + 58 \times 60^{-3}$. Ptolemy uses the system only for fractions, and represents whole numbers, even when combined with sexagesimal fractions, by the standard Greek (alphabetic) notation. The translation follows this mixed notation (thus the above number would be written 373;10,0,58 in the translation, and τοις εκ τῶν in Greek).

(b) Fractions

Except where it is necessary to be precise, Ptolemy prefers the traditional Greek fractional system to the sexagesimal. In this, although it is possible to express proper fractions as e.g. '4 5ths', preference is given to unit fractions, so that, e.g. '4' is expressed as the sum of $\frac{1}{6}$ and $\frac{1}{6}$ (written $\frac{2}{6}$, i.e. '1'). There is a special sign for $\frac{1}{6}$. In the translation I have usually converted these sums of unit fractions to proper fractions without comment. However, I have always retained the fractional form where Ptolemy has it, since it gives a misleading appearance of precision to convert to sexagesimals (as Manitius often does, putting an exact number of minutes instead of a fraction of a degree). This is particularly true of the star catalogue.

(c) Trigonometry

The sole trigonometrical function used by Ptolemy is the chord. The derivation and structure of his chord table are fully explained in I 10. However, Ptolemy does not give explicit instructions for its use in trigonometrical calculations, although his method is obvious enough from the worked examples. In what follows I give a literal translation, with commentary, of a typical calculation involving trigonometry.

See Fig. A, and, for my conventions, compare the translation pp. 163-4. In the given situation arc $\Theta H$ is 30°, $AD$ is 60°, $AH$ is 2;30°, and it is required to find the angle $ADH$ (the 'equation'). In modern trigonometry we would use the cosine formula. Ptolemy has no equivalent, so he drops the perpendicular $HK$, thus transforming the problem into one of solving only right triangles, which is his standard procedure.**

'Then since arc $\Theta H$ is again 30 degrees, angle $\Theta AH$ would be 30 of those [units] of which 4 right angles are 360, and 60 of those [units] of which 2 right angles are 360. So the arc on $HK$ is 60 of the units of which the circle [circumscribed] about the right-angled [triangle] $HKA$ is 360, and the arc on $AK$ is 120, the supplement making up the semi-circle. And so, of the chords subtended by them, $HK$ will be 60 of the units of which hypotenuse $AH$ is 120, and $AK$ 103;55 of the same [units].'

**He knows the equivalent of the sine formula, namely that in the general triangle the sides are proportional to the chords of the doubles of the opposite angles, but uses it surprisingly infrequently. An example is IX 10 p. 462 (cf. n.96 there).
To solve a right-angled triangle (here HKA), Ptolemy imagines a circle circumscribed about it. Then the hypotenuse of the triangle is the diameter of the circle, and is taken (initially) as 120 parts (R = 60 being the standard on which Ptolemy's chord table is constructed). The two acute angles of the triangle being given, the other two sides can now be expressed in the same units: they are the chords of the arcs of the circumscribed circle, which are the doubles of the angles of the triangle (since they are equal to the angles at the centre). Instead of explicitly doubling these angles, Ptolemy always first expresses them in 'units of which 2 right angles are 360'. (Following the convention invented by B. R. Goldstein, I indicate these 'demi degrees' by the notation °°, reserving ° for the standard degree of which there are 90 in a right angle.) This enables him to switch smoothly from the triangle to the circle (and hence to the chord table, which gives him the actual numbers 60° and 103;55°): an angle of size θ° is 2θ°°, and hence the arc of the circumscribing circle which corresponds to that angle is 2θ°.

'Therefore in those [units] of which line AH is 2;30, and the radius AD is 60, HK will be 1;15 and AK, likewise, 2;10, and KD, the remainder, 57;50.'

The sides of triangle AKH are converted to the norm representing their actual size (AH = 2;30°, hence they are multiplied by 2;30/120). This gives two sides of the next right triangle to be solved, DHK:HK and (by subtraction of AK from the given AD) KD.

'And since the squares on these added together make the square on DH, the
latter will be, in length, approximately 57;51 of the units of which line KH was found to be 1;15.'

Since Ptolemy has no tangent function, he has to use 'Pythagoras' theorem' to find the hypotenuse of the right triangle in question. He uses the word μήκει, 'in length', to indicate that he is taking the square root (considered as the side of a square, hence a line length).

'And so of those units of which hypotenuse DH is 120, line HK will be 2;34 and the arc on it [HK, will be] 2;27 of those [units] of which the circle about DHK is 360. So that angle HDK is 2;27 of those [units] of which 2 right angles are 360, and about 1;14 of those of which 4 right angles are 360.'

The sides of triangle DHK are now converted to the standard in which the hypotenuse is 120°, thus enabling Ptolemy to use the chord table to determine the size of the arc corresponding to the side opposite the angle to be determined, HDK. The latter, being at the circumference of the circumscribed circle, is half the arc. Ptolemy again expresses this relationship by saying that it is the same number of 'demi degrees' as the arc is 'single degrees', and then converting the 'demi degrees' to 'single degrees' by halving. Note that I frequently translate expressions like '30 degrees of the kind of which the great circle is 360' simply as '30°'.

(d) Chronology and calendars

Ptolemy's own chronological system is very simple. He uses the Egyptian year and the era Nabonassar. The Egyptian year is of unvarying length of 365 days, consisting of twelve 30-day months and 5 extra ('epagomenal') days at the end. Ptolemy uses the Greek transliterations of the Egyptian month names. For the reader's convenience, I usually add a Roman numeral indicating the number of the month. The order of the months is:

| I  | Thoth       | VII | Phamenoth   |
| II | Phaophi     | VIII| Pharmouthi |
| III| Athyr       | IX  | Pachon      |
| IV | Choiak      | X   | Payni       |
| V  | Tybi        | XI  | Epiphi      |
| VI | Mechir      | XII | Mesore      |

The reason for choosing the era Nabonassar is given by Ptolemy at III 7 (p. 166: the earliest (Babylonian) observations available to him were from the reign of King Nabonassar. Ptolemy's epoch, Nabonassar 1, Thoth 1 corresponds to −746 February 26 in our reckoning.11

11 Throughout this book I use the 'astronomical' system of dating according to the Christian era, since it is far simpler for calculating intervals than the 'B.C./A.D.' system. In this, year −1 corresponds to 2 B.C., year 0 to 1 B.C., year 1 to A.D. 1, etc.
Even when he refers to other calendars, Ptolemy usually gives the equivalent date in his own system, so there is no uncertainty. Sometimes, however, he gives, not the running date in the era Nabonassar, but only the regnal year of a king. It is clear that there already existed, in some form, a ‘king-list’ enabling one to relate the regnal year of a given king to a standard epoch. Later, in his ‘Handy Tables’, Ptolemy published such a king-list (known as ‘Canon Basileon’), and it survives, in a considerably augmented form, in Byzantine versions of Theon of Alexandria’s revision of the Handy Tables. From these I have excerpted and ‘reconstructed’ the table on p. 11, which makes no historical pretensions, but is intended solely as an aid to readers of this book. The basis of the table is Usener’s edition of the two versions in the manuscript Leidensis gr. 78, in Monumenta Germaniae Historica, Auctores Antiquissimi XIII (Chronica Minora Saec. IV-V-VI-VII, ed. Th. Mommsen), Vol. III, 447-53, supplemented by my own reading of the version in the ms. Vaticanus gr. 1291, 16'-17'. The names of the Babylonian and Assyrian kings are obviously very corrupt, and I have made no attempt to emend them, but have chosen those manuscript variants which seem closest to the forms now known from the cuneiform sources, which are listed in the second column (supplied to me by A. Sachs).

For the purposes of astronomical chronology, an integer number of years is assigned to each reign. As far as can be checked from independent sources, ‘Year 1’ of each reign was assumed to begin on the Thoth 1 preceding the historical date on which the king began to reign. Thus, to use the table to go from a given regnal year to the era Nabonassar, one simply adds the number of the regnal year to the total listed (in the fourth column) for the previous king. E.g. to find the second year of Mardokempad in the era Nabonassar (cf. IV 8 p. 204), we add 2 to the total of 26 given for his predecessor, Ilulai, and get the twenty-eighth year in the era Nabonassar.

Although I supply in the translation the modern equivalent of all dates in the Almagest, I have added, for the use of those readers who wish to check them, a fifth column listing the Julian equivalent of the first day of each king’s reign. If one bears in mind that every Julian year divisible by 4 is a leap-year, while the Egyptian year is constant, this is a sufficient basis for the calculation. However, I recommend as an easier alternative the use of Schram’s Kalendariographische Tafeln: from pp. 182-9 of that one can find the Julian day number of any date in

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12 Papyrus fragments of such king-lists are found in P. Oxy. 1.35 and Sattler, Studien 39-50. These are, however, later than Ptolemy. P. Oxy. 19.2222, a list of the Ptolemies of Egypt, is earlier than the Almagest, but is very different in format from Ptolemy’s king-list.

13 It is not known why these two kings are combined. In cuneiform sources (e.g. the king-list translated in Pritchard, Ancient Near Eastern Texts 272 (iv), they appear consecutively, Ukín-ze'r being assigned 3 years and Pulu 2.

14 This must be a corruption in the Greek tradition of Arses (’Αρσης), the usual form of this king’s name (also known as ’Οαρσης).

15 This was recognised long ago. See Usener, MGH XIII.3 p. 441, with references to older literature in his n.5.

16 In the Handy Tables Ptolemy adopted the ‘era Philip’ (which already occurs in the Almagest as ‘death of Alexander’); hence in the ms. the totals for era Nabonassar go only as far as Alexander the Macedonian (no. 31), and a new totalling system begins with Philip (no. 32). I have converted all these later totals to the era Nabonassar by the addition of 424 to each. Cf. Schram p. 173.
**Introduction: Reconstructed king-list**

<table>
<thead>
<tr>
<th>Ruler</th>
<th>Correct form</th>
<th>Years of reign</th>
<th>Total years to end of reign</th>
<th>Julian date of beginning of reign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Nabonassar</td>
<td>Nabû-nāṣir</td>
<td>14</td>
<td>14</td>
<td>-746 Feb. 26</td>
</tr>
<tr>
<td>2 Nadi</td>
<td>Nādin</td>
<td>2</td>
<td>16</td>
<td>-732 Feb. 23</td>
</tr>
<tr>
<td>3 Chineer and Por</td>
<td>Ukin-ĝer; Pulu</td>
<td>5</td>
<td>21</td>
<td>-730 Feb. 22</td>
</tr>
<tr>
<td>4 Ilulai</td>
<td>Elilai</td>
<td>5</td>
<td>26</td>
<td>-725 Feb. 21</td>
</tr>
<tr>
<td>5 Mardokempad</td>
<td>Marduk-apla-iddin</td>
<td>12</td>
<td>38</td>
<td>-720 Feb. 20</td>
</tr>
<tr>
<td>6 Arkean</td>
<td>Šarru-ukin</td>
<td>5</td>
<td>43</td>
<td>-708 Feb. 17</td>
</tr>
<tr>
<td>7 First interregnum</td>
<td></td>
<td>2</td>
<td>45</td>
<td>-703 Feb. 15</td>
</tr>
<tr>
<td>8 Belib</td>
<td>Bēl-ibni</td>
<td>3</td>
<td>48</td>
<td>-701 Feb. 15</td>
</tr>
<tr>
<td>9 Aparanad</td>
<td>Assur-nādin-sumi</td>
<td>6</td>
<td>54</td>
<td>-698 Feb. 14</td>
</tr>
<tr>
<td>10 Regbel</td>
<td>Nergal-uesîlib</td>
<td>1</td>
<td>55</td>
<td>-692 Feb. 13</td>
</tr>
<tr>
<td>11 Mesesemordak</td>
<td>Mûsîlib-Marduk</td>
<td>4</td>
<td>59</td>
<td>-691 Feb. 12</td>
</tr>
<tr>
<td>12 Second interregnum</td>
<td></td>
<td>8</td>
<td>67</td>
<td>-687 Feb. 11</td>
</tr>
<tr>
<td>13 Asariñin</td>
<td>Assur-aga-iddina</td>
<td>13</td>
<td>80</td>
<td>-679 Feb. 9</td>
</tr>
<tr>
<td>14 Siosdouchin</td>
<td>Šamās-ûma-ukin</td>
<td>20</td>
<td>100</td>
<td>-666 Feb. 6</td>
</tr>
<tr>
<td>15 Kintliadan</td>
<td>Kandalamu</td>
<td>22</td>
<td>122</td>
<td>-646 Feb. 1</td>
</tr>
<tr>
<td>16 Nabopolassar</td>
<td>Nabû-apla-usur</td>
<td>21</td>
<td>143</td>
<td>-624 Jan. 27</td>
</tr>
<tr>
<td>17 Nabokolassar</td>
<td>Nabû-kudurra-usur</td>
<td>43</td>
<td>186</td>
<td>-603 Jan. 21</td>
</tr>
<tr>
<td>18 Ilarqumolam</td>
<td>Amil-Marduk</td>
<td>2</td>
<td>188</td>
<td>-560 Jan. 11</td>
</tr>
<tr>
<td>19 Nergasolassar</td>
<td>Nergal-ûra-usur</td>
<td>4</td>
<td>192</td>
<td>-558 Jan. 10</td>
</tr>
<tr>
<td>20 Nabonadi</td>
<td>Nabû-na'id</td>
<td>17</td>
<td>209</td>
<td>-554 Jan. 9</td>
</tr>
</tbody>
</table>

**Kings of the Persians**

<table>
<thead>
<tr>
<th>Ruler</th>
<th>Correct form</th>
<th>Years of reign</th>
<th>Total years to end of reign</th>
<th>Julian date of beginning of reign</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 Cyrus</td>
<td>Kûrûs</td>
<td>9</td>
<td>218</td>
<td>-537 Jan. 5</td>
</tr>
<tr>
<td>22 Cambyses</td>
<td>Kāmûbu-ziwa</td>
<td>8</td>
<td>226</td>
<td>-528 Jan. 3</td>
</tr>
<tr>
<td>23 Darius I</td>
<td>Darâvava-u</td>
<td>36</td>
<td>262</td>
<td>-520 Jan. 1</td>
</tr>
<tr>
<td>24 Xerxes</td>
<td>Šâsâvarša</td>
<td>21</td>
<td>283</td>
<td>-465 Dec. 23</td>
</tr>
<tr>
<td>25 Artaxerxes I</td>
<td>Artaxā-bâra</td>
<td>41</td>
<td>324</td>
<td>-464 Dec. 17</td>
</tr>
<tr>
<td>26 Darius II</td>
<td>Darâvava-u</td>
<td>19</td>
<td>343</td>
<td>-423 Dec. 7</td>
</tr>
<tr>
<td>27 Artaxerxes II</td>
<td>Artaxā-bâra</td>
<td>46</td>
<td>389</td>
<td>-404 Dec. 2</td>
</tr>
<tr>
<td>28 Ochus</td>
<td>Vâhâuka</td>
<td>21</td>
<td>410</td>
<td>-358 Nov. 21</td>
</tr>
<tr>
<td>29 Artaxerxes</td>
<td>Vâhâuka</td>
<td>2</td>
<td>412</td>
<td>-357 Nov. 16</td>
</tr>
<tr>
<td>30 Darius III</td>
<td>Darâvava-u</td>
<td>4</td>
<td>416</td>
<td>-335 Nov. 15</td>
</tr>
<tr>
<td>31 Alexander the Macedonian</td>
<td>'Alêxânûrûs</td>
<td>8</td>
<td>424</td>
<td>-331 Nov. 14</td>
</tr>
</tbody>
</table>

**Kings of the Macedonians**

<table>
<thead>
<tr>
<th>Ruler</th>
<th>Correct form</th>
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<th>Total years to end of reign</th>
<th>Julian date of beginning of reign</th>
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</thead>
<tbody>
<tr>
<td>32 Philip who succeeded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alexander the founder</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33 Alexander II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34 Ptolemy son of Lagos</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35 Ptolemy Philadelphus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36 Ptolemy Euergetes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37 Ptolemy Philopator</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38 Ptolemy Epiphanes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39 Ptolemy Philometor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 Ptolemy Euergetes II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41 Ptolemy Soter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42 Ptolemy Neos Dionysus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43 Cleopatra</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Kings of the Romans**

<table>
<thead>
<tr>
<th>Ruler</th>
<th>Correct form</th>
<th>Years of reign</th>
<th>Total years to end of reign</th>
<th>Julian date of beginning of reign</th>
</tr>
</thead>
<tbody>
<tr>
<td>44 Augustus</td>
<td>Augustus</td>
<td>43</td>
<td>761</td>
<td>-29 Aug. 31</td>
</tr>
<tr>
<td>45 Tiberius</td>
<td>Tiberius</td>
<td>22</td>
<td>783</td>
<td>-14 Aug. 20</td>
</tr>
<tr>
<td>46 Gaius</td>
<td>Gaius</td>
<td>4</td>
<td>787</td>
<td>-36 Aug. 14</td>
</tr>
<tr>
<td>47 Claudius</td>
<td>Claudius</td>
<td>14</td>
<td>801</td>
<td>-40 Aug. 13</td>
</tr>
<tr>
<td>48 Nero</td>
<td>Nero</td>
<td>14</td>
<td>815</td>
<td>-54 Aug. 10</td>
</tr>
<tr>
<td>49 Vespasian</td>
<td>Vespasianus</td>
<td>10</td>
<td>825</td>
<td>-68 Aug. 6</td>
</tr>
<tr>
<td>50 Titus</td>
<td>Titus</td>
<td>3</td>
<td>828</td>
<td>-78 Aug. 4</td>
</tr>
<tr>
<td>51 Domitian</td>
<td>Domitianus</td>
<td>15</td>
<td>843</td>
<td>-81 Aug. 3</td>
</tr>
<tr>
<td>52 Nerva</td>
<td>Nerva</td>
<td>1</td>
<td>844</td>
<td>-96 July 30</td>
</tr>
<tr>
<td>53 Trajan</td>
<td>Traianus</td>
<td>19</td>
<td>863</td>
<td>-97 July 30</td>
</tr>
<tr>
<td>54 Hadrian</td>
<td>Hadrianus</td>
<td>21</td>
<td>884</td>
<td>-116 July 25</td>
</tr>
<tr>
<td>55 Antoninus</td>
<td>Aelius Antoninus</td>
<td>23</td>
<td>907</td>
<td>-137 July 20</td>
</tr>
</tbody>
</table>
the era Nabonassar in a few seconds, and hence (from his other tables) the equivalent date in any standard calendar.

The only other aspect of Ptolemy's own chronology requiring remark is the 'double dates'. He frequently characterises the day of an observation by expressions like Παχών ζ' είς τήν ιη', translated 'Pachon 17/18', but literally 'Pachon, the seventeenth towards the eighteenth'. Modern commentators have made unnecessarily heavy weather of this. Ptolemy himself uses a noon epoch, but this is an artificial starting-point (the reason for which he explains at III 9 pp. 170-1), and has nothing to do with numbering the day. In antiquity the 'civil epoch' of the day was either dawn (as in Egypt) or sunset (as in Babylon). In either system, an event which took place in the daylight would be on the same 'day', but one which took place in the night would be on 'day n' for those using dawn epoch and 'day n+1' for those using sunset epoch. Hence ambiguity was possible. Ptolemy uses double dates (which are found only for night-time observations) to avoid this ambiguity. The form he uses implies the Egyptian, i.e. dawn epoch (cf. the long form III 1 p. 138, τήν ια' τοῦ Μεσορῆ μετα Βώρας ἐγγίζ τοῦ εἰς τήν ιβ' μεσονυκτίου (literally 'on the eleventh of Mesore, approximately two hours after the midnight towards the twelfth'), but it would be clear even to someone using sunset epoch (who would date the above event to Mesore 12') what day he means.

In using the observations of his predecessors Ptolemy often has occasion to refer to other systems of chronology and calendars. Although in such cases one can always readily derive the equivalent date in Ptolemy's own system (he almost always gives it explicitly), I shall describe them briefly here.

The most frequently mentioned is the Kallippic Cycles. To explain this, we must go back to Meton, who in -431 devised a 19-year 'cycle', i.e. a fixed scheme of intercalation of months containing 6940 days (thus the average length of a year was 365 1/4 days). Since he was an Athenian, he used the month names of the Athenian civil calendar for the months of his artificial 'calendar'. A hundred years later an associate of Aristotle, Kallippos, produced a revision of this, based on the more accurate year-length of 365 1/4 days. In order to achieve this, he eliminated one day from 4 Metonic cycles, thus producing the 'Kallippic cycle' of 76 years and 27759 days. What was later known as the 'First Kallippic Cycle' began at the summer solstice (probably June 28th) of the year -329. In the Almagest we find references also to the Second and Third Kallippic Cycles, which began in -253 and -177 respectively. To judge from the Almagest, this chronological system was the one most used by earlier Hellenistic astronomers. In VII 3 four observations by Timocharis (Alexandria, third century B.C.) are given according to the year of the First Kallippic Cycle and 'Athenian' month and day. On the basis of these, several attempts have been made to reconstruct the whole 'Kallippic calendar', with discrepant results. Since the above constitute the whole evidential basis, apart from the
passage in Geminus, *Eisagoge* VIII, which I regard as fiction, and two dubious equivalences in the Milesian parapegma, any reconstruction is academic. Here I note only that Kallippos evidently retained the peculiar Athenian method of counting the days of the month by decades, and in the last decad counting backwards, so that VII 3 p. 336 τῆς ἑπτάνοιας, literally 'on the sixth [day] of the waning [moon]', means 'the sixth day from the end of the last decad', i.e. the twenty-fifth.

Hipparchus too used the Kallippic cycles for astronomical dating, but combined them, not with Kallippos' 'Athenian' calendar, but with the Egyptian calendar (i.e. he used the cycles simply as a year count), at least as far as we can tell from the Almagest. This seems to have led to ambiguities, since the 'Kallippic' year began at or near the summer solstice, while the Egyptian year is a 'wandering year', which in Hipparchus' time began about the end of September. Thus there arose the possibility of a discrepancy of 1 in the year count, for certain stretches of the year (whether it is +1 or -1 depends on Hipparchus' choice). Such a discrepancy is firmly attested in Almagest IV 11 (see p. 214 n. 72), and cannot plausibly be removed by emendation, though this has been done (by Ideler and others) in the interest of consistency. In fact it is impossible to make all of Hipparchus' 'Kallippic cycle' dates in the Almagest consistent with one another (see p. 224 no. 13), and we must allow for the possibility that Hipparchus used different systems in different works.

Three planetary observations in the Almagest are dated κατὰ Χαλδαῖος, 'according to the Chaldaeans', with a year number and a Macedonian month name and day number. The year numbers show that the era used is that known in modern times as the *Seleucid Era* (dating from the year which Seleucus I counted as the first of his reign, -311/10), which was common throughout the Seleucid empire. Since the observations are undoubtedly Babylonian, the particular epoch used in them is, as one would expect, that known from the surviving Babylonian astronomical texts, 1 Nisan (April) -310 (Greeks under the Seleucid empire commonly used an epoch of autumn -311). The use of Macedonian month names has rightly been taken to show that the Babylonian lunar months were simply called by the names of the Macedonian months by the Greeks under the Seleucid empire: if one computes the date of the first day of the 'Macedonian' month from the equivalent date in the era Nabonassar given by Ptolemy, it coincides (with an error of no more than one day) with the computed day of first visibility of the lunar crescent at Babylon. There is other evidence for the assimilation of the month names, but this is the strongest.

Unattested outside the Almagest is the *Calendar of Dionysius*. This had a

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19 Those who care to may consult Ginzel II 409-19 and Samuel, *Greek and Roman Chronology*, 42-9 for details and literature.

20 For this system see Samuel, *Greek and Roman Chronology* 59-60. I do not know why it is not used for the other three 'Kallippic' dates in which the days are simply numbered consecutively.

21 These are conveniently listed in Parker-Dubberstein.

22 For details see Samuel, *Greek and Roman Chronology* 140-2. However, Samuel is wrong in saying that the Almagest evidence proves that the assimilation was made as early as the date of the earliest observation (Nov. -244). In the cuneiform record from which this was derived the Babylonian names must have been used. It was only when this was translated into Greek (which may have been as much as a century later) that the Macedonian names were substituted.
running year count and months named after the signs of the zodiac (corresponding, at least approximately, to the period of the year when the sun was in the sign in question). The months Tauron (Δ), Didymon (Π), Leonton (Ξ), Parthenon (Μ), Skorpion (Μ), Aigon (Ι) and Hydron (Ξ) are attested. From analysis of the Almagest evidence Böckh, *Sonnenkreise* 286–340, showed that the epoch of the calendar was the summer solstice of -284. Since Thoth 1 (Nov. 2) of -284 is the beginning of the first regnal year of Ptolemy Philadelphos, it is plausibly concluded that Dionysius observed in Egypt. Böckh’s further conclusions, that the calendar was similar to the Egyptian one in having 12 months of 30 days, but was modified by introducing a sixth epagomenal day every four years, cannot be regarded as certain, especially since this requires ‘emending’ some of the Almagest dates. Here, as for the Kallippic calendar, ‘reconstruction’ seems pointless when the evidence is so scanty and the likelihood of verification utterly remote.23

One observation is dated in the Bithynian calendar of the imperial period. Like a number of other contemporary calendars in Asia Minor, this was simply the Julian calendar, with different month-names, and with the first day of the year Augustus’ birthday, Sept. 23. For details and literature see Samuel, *Greek and Roman Chronology* 174–5.

(e) Ptolemy’s star catalogue

The list of the coordinates and magnitudes of the principal fixed stars visible to Ptolemy poses special problems to the translator. In particular, there are numerous manuscript variants in the coordinates, and while one must put some number in the translation, it is often difficult to be certain about one’s choice. The solution I have adopted is (in the star catalogue only) to append an asterisk to any element (longitude, latitude, magnitude, description or identification) where there is reason to suppose that it may be incorrect (i.e. not what Ptolemy wrote or intended),24 either because there is a plausible ms. variant, or because of some gross inconsistency with the astronomical facts. In such cases I give all significant variants known to me in a footnote. I have made no effort to record all variants, since most are obviously wrong. The reader who wishes to go further must still consult Peters-Knobel, on which I have drawn heavily, and which is still the best treatment of the catalogue as a whole, though badly in need of updating and revision in certain respects.25

Ptolemy lists the stars under 48 constellations, and gives for each star (1) a description of its location on the ‘figure’ and (sometimes) of its brightness and colour; (2) its longitude; (3) its latitude and direction (north or south of the ecliptic); and (4) its magnitude. I have followed my predecessors (notably Manitius) in adding to these: (a) an initial column giving a running number to

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23 The interested reader may consult *HAMA* III 1067 n.2 and Samuel, *Greek and Roman Chronology* 50, n.6 for further literature.

24 The lack of an asterisk does not imply that I regard the reading adopted as Ptolemy’s beyond any question, but only that I have no good reason to doubt it.

25 See the strictures of Kunitzsch, *Der Almagest* 46.
the star within its constellation (stars listed at the end of some constellations by Ptolemy as 'outside the constellation', i.e. not part of the imaginary figure, are numbered continuously with those preceding them); (b) a final column giving the modern identification of the star. For those stars which have them, this is the Bayer letter or Flamsteed number. Certain fainter stars have neither; for these I give the number in the Yale Bright Star Catalogue (abbreviated as 'BSC'). From that publication those interested can find the corresponding number in the Durchmusterung and the Henry Draper and Boss General Catalogues. I have abandoned all references to the antiquated Piazzi catalogue (still used by Peters-Knobel).

I have used Roman numerals to number the constellations, and refer to individual stars (throughout the translation) by the combination of Roman and Arabic numerals (thus 'catalogue XXXIX 2' refers to the second star in the thirty-ninth constellation (Canis Minor), namely Procyon).

The star descriptions pose numerous individual problems, only a few of which are touched on in the footnotes. Ideally one should provide a reconstruction of the outline of each constellation as it appears on Ptolemy's star-globe. Unfortunately no one has done the necessary work of assembling and comparing all the literary and iconographic evidence from antiquity and from the derivative Arabic tradition (notably as-Ṣūfī). This would be an interesting and valuable enterprise. Meanwhile, for the reader who needs some visual illustration, I can recommend only the old work of Bayer, l'íanometria, with the warning that in many cases his positioning of the stars on the figures, and the outlines of the figures themselves, are certainly different from Ptolemy's. On the matter of the orientation of the figures, I have satisfied myself that Ptolemy describes them as if they were drawn on the inside of a globe, as seen by an observer at the centre of that globe, and facing towards him. This is in agreement with what Hipparchus says (Comm. in Arat. I 45): ‘for all the stars are described in constellations (ἡστεριστατι) from our point of view, and as if they were facing us, except for such of them as are drawn in profile’ (κατάγραφον, as interpreted by Manitius, whom I follow dubiously). It is in this sense that we must interpret 'left hand', 'right leg', etc. This has to be said, since on the actual star globes the constellations were necessarily drawn on the outside. Hence the orientation of the figures was (at least in some cases) reversed, which could lead to confusion. I have rendered the prepositions used by Ptolemy in indicating the positions of stars with respect to parts of the figures consistently, as follows:

\[
\begin{align*}
\text{in} &= \varepsilon\nu \\
\text{on} &= \varepsilon\pi\tilde{i} \\
\text{over} &= \upsilon\pi\varepsilon\rho
\end{align*}
\]

26 The work of Thiele, Antike Himmelsbilder, is very little help, although I have referred to it to illustrate some particulars.

27 Cf. the scholion on Aratus, Maass, Comm. in Arat. p. 384 no. 251: 'the signs look inward with respect to the heavens . . . but they have their backs to the globe, so that their faces may be seen. Hence, if he says 'right hand' or 'left hand' and we find the opposite on the globe, we should not be confounded.'
On the meaning of the last two terms see below p. 20. Note that ‘rear’ is never used in a sense other than directional. To indicate the back parts of an animal figure I use ‘hind’.

Both longitudes and latitudes are given, not in degrees and minutes, but in degrees and fractions of a degree. I have retained this in the translation (see p. 7). With very few exceptions, the longitudes are not given more accurately than to $\frac{1}{6}^\circ$. (This has been taken to imply that the ecliptic ring of Ptolemy’s instrument was graduated only every $10^\circ$). However, one frequently finds the fractions $\frac{1}{10}$ and $\frac{1}{12}$ for the latitudes.

The latitudes in Ptolemy’s list are preceded by the direction ($\betao = \beta\sigma\rho\epsilon\tau\omicron\sigma\varsigma$, ‘northern’; $\vno = \nu\omicron\tau\omicron\iota\sigma\varsigma$, ‘southern’). I have rendered these by + and − respectively.

The magnitudes range (according to a system which certainly precedes Ptolemy, but is only conjecturally attributed to Hipparchus) from 1 to 6. Ptolemy indicates intermediate magnitudes by adding (after the number) μείζων, ‘greater’ or ἐλάσσων, ‘less’ (abbreviated in the ms.). I have rendered these by > and < (before the number) respectively. One occasionally finds for the magnitude, instead of a number, the remark άμωρός (rendered ‘f.’ for ‘faint’) or νεφελ. (for νεφελοειδής), ‘nebulous’, abbreviated as ‘neb.’

For the identifications, wherever Peters-Knobel and Manitius are in agreement, I have usually been content to adopt their opinion. Where they differ (and even when they agree, in some special cases), I have checked the possibilities as carefully as I could, using the large-scale *Atlas of the Heavens* by Bečvář, and transforming Ptolemy’s coordinates to right ascension and declination at the modern epoch, where necessary. However, I have made no attempt to redo the work of Peters and Knobel, namely to compute the longitude and latitude of the relevant stars for Ptolemy’s time from modern data (in particular using the most up-to-date values for the proper motions). This might be worth while, though I doubt whether the degree of improvement over Peters-Knobel would justify the large amount of computation. In any case, it is unlikely that it would eliminate the doubts that remain about the identification of many of the fainter stars.

At the end of each constellation in the ms. are listed the total number of stars in the constellation, and the sub-totals of each magnitude. These in turn are added up at various intermediate points (the northern segment, the zodiac, and the southern segment), and the grand totals are given at the end. I am

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28 Notably, where Ptolemy describes a star as a ‘nebulous mass’ (νεφελοειδής συστροφή), I have preferred to give the globular cluster (abbreviated ‘CGlo’) or galactic cluster (abbreviated ‘CGal’) rather than some particular star inside it.
convinced that this was not done by Ptolemy (who makes no mention of it in his
description of the catalogue, VII 4 pp. 339-40). Another indication of the
spuriousness of these passages is that no separate count is made in the totals of
the stars which are greater (> ) or less (< ) than a certain magnitude: all are
lumped in with the stars of that magnitude. I have translated the passages in
question, but enclosed them in brackets thus: { }.

(f) *Explanations of special terms*

(i) *Geometrical*

*by subtraction* (λοιπός -ή -ον): literally 'the remaining [part]', 'remainder'
(I have on occasion so rendered it).

*by addition* (διαλος -η -ον): literally 'the total'.

*Crd x*: chord of the angle $x^\circ$ ($R = 60^\circ$). Greek has no word with the specific
meaning 'chord', but uses the generic εὐθεία, 'straight line'. *Crd x* renders $\tau\acute{a}c x$ μοίρας ὑποτείνουσα εὐθεία, 'the straight line subtending $x$ degrees'.

In connection with the Menelaus Theorem (see p. 18), an expression of the
type 'Crd arc 2AB' represents $\eta ύπο τὴν διπλήν τῆς AB περιφέρειας, literally
'the [line] subtended by the double of arc AB'.

*supplement, supplementary arc* (ἡ λειπουσα [λοιπη] εἰς τὸ ἡμικύκλιον περιφέρεια):
literally 'the arc which is the remainder to the semi-circle'.

*complement* (λοιπη εἰς τὸ τεταρτημόριον): literally, 'the remainder to the
quadrant'.

|| literally, 'is similar to'. Used of arcs of different-sized circles. Arc AB|| arc GD
if each arc is the same fraction of its circle.

|| (ἰσογώνιον ἔστι): literally, 'has [all] its angles equal to', i.e. is similar to (used
only of triangles).

≡ (ἰσόπλευρον ἔστι): literally 'has its sides equal to', i.e. is congruent to. Used
only of spherical triangles. Sometimes ἰσογώνιον καὶ ἰσόπλευρον ἔστι, 'has its
angles and sides equal to'.

Q.E.D. (διπερ ἔδει δεῖξαι): literally 'which is what it was required to prove'.

*componendo* (συνθέντι). Expresses the operation of addition of ratios: if
a : b = c : d, then (a + b):b = (c + d):d.

*dividendo* (διελόντι, κατά διαίρεσιν) (1) Usually expresses the operation
of subtraction of ratios: if a : b = c : d, then (a - b) : b = (c - d) : d.
(2) Once, at XII 1 (see p. 558 n.4) διελόντι expresses division of members of ratios. If \( a : b = c : d \), then \( \frac{a}{n} : b = \frac{c}{n} : d \).

Menelaus Configuration and Menelaus Theorem (used only in the footnotes and explanatory additions). Cf. HAMA 26-9. Fig. B represents a Menelaus Configuration. \( m,n,r \) and \( s \) are four great circle arcs on the surface of the sphere, intersecting each other as shown, and divided by the intersections into the parts \( m_1, m_2 \) etc. (thus \( m = m_1 + m_2 \) etc.) In I 10 Ptolemy proves the theorems

\[
\begin{align*}
\text{I} & \quad \frac{\text{Crd } 2m}{\text{Crd } 2m_1} = \frac{\text{Crd } 2r}{\text{Crd } 2r_1} \times \frac{\text{Crd } 2s_2}{\text{Crd } 2s} \\
\text{II} & \quad \frac{\text{Crd } 2r_2}{\text{Crd } 2r_1} = \frac{\text{Crd } 2m_2}{\text{Crd } 2m_1} \times \frac{\text{Crd } 2n}{\text{Crd } 2n_2}
\end{align*}
\]

Since it is known that these were discovered by Menelaus, Neugebauer has named them ‘Menelaus Theorem I’ and ‘Menelaus Theorem II’ respectively, and I follow him, abbreviating to ‘M.T.I.’ and ‘M.T.II’.

(ii) Spherical astronomy

(at) sphaera recta (ἐπ' ὀρθῆς τῆς σφαίρας) and (at) sphaera obliqua (ἐπ' ἐγκεκλιμένης τῆς σφαίρας). These mediaeval Latin terms are the literal translations of the Greek, meaning ‘on the upright sphere’ and ‘on the inclined sphere’ respectively. Probably taken from the use of celestial globes, they refer to the phenomena which occur when the celestial equator is perpendicular to the local horizon (sphaera recta) or inclined to it at an acute angle (sphaera obliqua). In particular, we use rising-time at sphaera recta or right ascension, and rising-time at sphaera obliqua or oblique ascension to designate the arc of the equator which crosses the horizon together with a given arc of the ecliptic (e.g. one
zodiacal sign) at sphaera recta (i.e. at the terrestrial equator), and at sphaera obliqua (i.e. any other terrestrial latitude) respectively.

equator represents ἵσημερινος (κύκλος), literally ‘circle of equal day’, so called for the reason Ptolemy gives in I 8 (pp. 45-6).

meridian represents μεσημβρινος (κύκλος), literally ‘midday circle’ (defined and explained at I 8 p. 47). Meridian passage of a heavenly body is called culmination. The Greek terms for culminate and culmination, μετασυμβουλεύειν, μεσουράνθησις, mean literally ‘being in the middle of the heaven’. upper and lower culmination are expressed by ὑπὲρ γῆν and ὑπὸ γῆν, meaning ‘above the earth’ and ‘below the earth’ respectively, and sometimes so translated.

An altitude circle is any circle drawn through the zenith perpendicular to the horizon. Ptolemy has no special term for this in the Almagest, merely saying ‘the (great) circle drawn through the zenith (through the poles of the horizon)’, e.g. II 12, HI 166, 20-1.

colure. This term is used by Ptolemy only once, at II 6 p. 83. I translate part of Manitius’ note on that passage: Two of the circles of declination through the poles of the equator are named ‘colure’ (κόλουρος): the solstitial colure, which goes through the solstices and hence carries the poles of the ecliptic, and the equinoctial colure. These two colures divide the sphere into four equal parts and divide both ecliptic and equator into four quadrants, so that one quadrant corresponds to each season of the year. Ptolemy counts the solstitial colure as boundary of the daily revolution [I 8 pp. 46–7, where however the term ‘colure’ is not used], but never explicitly mentions the equinoctial colure. Both colures were already defined by Eudoxus (Hipparchus, Comm. in Arat. 117 ff.) The term is explained by Achilles, Isagoge 27 (Maass, Comm. in Arat. 60) as follows: ‘They are called colures because they appear to have their tails cut off as it were (κεκολοδοσθαι ὀσπερ τάς οὐράς), since we cannot see the parts of them beginning at the antarctic, always invisible parallel’.

It is unfortunate that we have to use the same word latitude to refer both to the celestial coordinate (vertical to the ecliptic) and to the unrelated terrestrial coordinate. Ptolemy uses, for the former πλάτος, and for the latter κλίμα, literally ‘inclination’. When necessary I gloss this e.g. as ‘[terrestrial] latitude’. κλίμα, however, does not refer to the coordinate as such (for which Ptolemy uses ἐγκλίμα, HI 68,9, ἐγκλίσις, HI 101,23 or, once, πλάτος, HI 188,4), but to a specific ‘band’ of the earth where the same phenomena (e.g. length of longest daylight) are found. Hence in early Hellenistic times arose the notion of the division of the known world (the οἰκουμένη) into 7 standard climata (see HAMA 334 ff., II 727 ff. and Honigmann, Die sieben Klimata). This is reflected in several places in the Almagest, e.g. in Table II 13. I refer to these seven standard parallels by Roman numerals, e.g. Clima IV = the parallel through Rhodes, longest day 14½ hours.
Referring to the heavenly bodies

As Ptolemy explains in I 8, in his system the whole heavens are conceived as rotating from east to west, making one revolution daily. The direction defined by this motion, and the direction counter to it, are called εἰς τὰ προηγούμενα (‘towards the leading [parts]’) and εἰς τὰ ἐπόμενα (‘towards the following [parts]’) respectively. The corresponding adjectives προηγούμενος and ἐπόμενος are also found, particularly in the star catalogue, and Ptolemy frequently uses the phrases εἰς τὰ προηγούμενα ἐπόμενα τῶν ζῳδίων, ‘towards the leading (following) [parts] of the zodiacal signs’, to indicate the direction of motion in the ecliptic. A modern reader may find this confusing: since the normal motion of bodies in the ecliptic is from west to east, what we regard as forward motion, e.g. of a planet, is described as ‘towards the following [parts]’ (‘towards the rear’ in my translation). No version of these terms in a modern language is satisfactory. One cannot use ‘west’ and ‘east’ because these must be reserved for Ptolemy’s δυσμαί and ἀνατολαί, which are confined to situations where a terrestrial observer is implied. It is a distortion to translate (with Manitius) ‘in the reverse order of the signs’ and ‘in the order of the signs’, since this implies that the terms define ecliptic coordinates, whereas they are in the equatorial system, and while it is usually true that a celestial object which προηγεῖται (‘leads’) another will have a lesser ecliptic longitude, if their latitudes differ greatly the reverse may be true, especially at very high ecliptic latitudes. Precisely this situation occurs in the star catalogue, despite Ptolemy’s own statement at VII 4 p. 340 that the terms in the catalogue define ecliptic coordinates (see n.93 there). Although I am aware that my choice too has its drawbacks, I have settled on in advance for εἰς τὰ προηγούμενα, and towards the rear for εἰς τὰ ἐπόμενα. These always imply ‘with respect to the daily motion from east to west’, with the paradoxical consequence, as remarked above, that in the ecliptic a body which is ‘in advance’ of another has a lesser longitude. However, I have committed an inconsistency in translating the derived noun προήγησις as retrogradation. This is used only for the portion of the courses of the five planets in which they reverse their normal direction of motion, and it would be too confusing to render this by ‘motion in advance’.

Ptolemy never refers to this circle by the term ἐκλειπτικός (which he confines strictly to the meaning ‘having to do with eclipses’). His normal term ὁ διὰ μέσων τῶν ζῳδίων (κύκλος), ‘the (circle) through the middle of the zodiacal signs’ (e.g. HI 18,23-4); more fully, ὁ λόξος καὶ διὰ μέσων τῶν ζῳδίων κύκλος, ‘the inclined circle through the middle of the signs’ (HI 64,4). Occasionally, when the context is clear, simply λόξος κύκλος, ‘inclined circle’ (HI 8,22). However, the latter can be used for other things, notably the moon’s orbit (which is ‘inclined’ to the ecliptic). I normally use ‘ecliptic’ throughout.

The conventional subdivision of the ecliptic into twelve 30° stretches named Aries, Taurus, etc. For this Ptolemy uses, not ζῳδίων (‘animal sign’), but δωδεκατημόριον (‘twelfth’), presumably because he wishes to
distinguish the ecliptic, a notional circle, from the zodiac, a band of actual constellations.

star. The Greek term ἄστήρ really means ‘heavenly body’, and can be used indifferently for a star (in the modern sense), a planet, or even the sun and moon. When Ptolemy wishes to distinguish what we call stars, he says ‘fixed stars’. I have normally translated ἄστήρ according to the context, as ‘planet’, ‘star’ or ‘body’. However, in I 3–8, where Ptolemy uses the term to include all heavenly bodies, I too have used star in this special sense. When naming the five planets, Ptolemy almost always uses the periphrasis ‘star of . . .’, thus ὁ τοῦ Κρόνου ἄστήρ, ‘[star] of Kronos’. I always translate simply ‘Saturn’ etc.

latitude (celestial). πλάτος (literally ‘breadth’) refers not only to ‘the direction orthogonal to the ecliptic’, but to any ‘vertical’ direction, e.g. that normal to the equator. In such cases I use, not ‘latitude’, but another appropriate term (see I 12 p. 63 with n. 74). In VII 3, however, I have been forced to use ‘latitude’ to express the more general meaning of the Greek (see p. 329 n.55).

Ptolemy uses ἐκκεντρός as both adjective and noun. It may be that in the latter case one has always to understand ἐκκεντρός κύκλος, ‘eccentric circle’. However, to avoid ambiguity, I have (following mediaeval usage) consistently denoted the noun by eccentre and the adjective by eccentric. An ‘eccentre’ is simply an eccentric circle. Similarly for concentre and concentric.

I have occasionally used the convenient mediaeval term deferent to denote the circle on which an epicycle is ‘carried’. Ptolemy has no one-word equivalent, but uses phrases like ‘the concentric carrying the epicycle’, ‘the circle carrying it’.

anomaly. As noted e.g. by Pedersen (139 with n.9), ἀνωμαλία in the Almagest has a number of different meanings. Despite the ambiguity, I have generally rendered ἀνωμαλία and the adjective from which it is derived, ἀνώμαλος, by ‘anomaly’, ‘anomalistic’, although where necessary I have translated the latter literally as ‘non-uniform’. Besides referring to non-uniform motion, ‘anomaly’ is also used for the mean (hence uniform) motion of the moon and planets on their epicycles (because the motion on the epicycle produces the appearance of ‘non-uniformity’). For the planets Ptolemy distinguishes between the synodic anomaly (ἡ πρὸς τὸν ἡλιον ἀνωμαλία, ‘the anomaly with respect to the sun’, HII 255,8), which produces the phenomena of retrogradation and varies with the planet’s elongation from the sun, and the ecliptic anomaly (ζῳδιακή ἀνωμαλία, HII 258,11), which varies according to the planet’s position in the ecliptic.

equation. I use this convenient mediaeval term for the angle (or arc) to be applied to a mean motion to ‘correct’ it to account for a particular feature of the geometric model. Ptolemy uses the vaguer terms τὸ διάφορον ‘difference’ (which is also used for many other things) and προσθαφαιρέσις (‘amount to be added
or subtracted'). *equation of anomaly* refers to the correction for the varying position of a body on its epicycle, and *equation of centre* (only in the footnotes, not the text) to the correction due to the eccentricity of a planet’s deferent.

*centrum*. I have occasionally used this mediaeval term in the footnotes to denote the angular distance from apogee (see below) to the centre of the epicycle.

*elongation* (αποχή) is the angular distance along the ecliptic between two bodies or points. It is used particularly, but not exclusively, for the ecliptic distance between sun and moon.

*apogee* and *perigee* are simply transcriptions of άπογειον and περίγειον, literally ‘[point] far from earth’ and ‘[point] near to earth’. These are the usual terms for the points on a body’s orbit which are respectively farthest from and nearest to the terrestrial observer. Ptolemy also uses the superlative forms άπογειότατον (περίγειότατον) σημείον (‘point farthest from (nearest to) earth’), with no obvious difference in meaning. However, in the case of Mercury, translation of both by ‘perigee’ generates an ambiguity. For all other bodies, in Ptolemy’s models, the perigee is diametrically opposite the apogee, but for Mercury the point of closest approach is about 120° from apogee. Ptolemy still refers to the point 180° from apogee as the ‘perigee’ (περίγειον) for Mercury, and when he wants to refer to the point of that planet’s closest approach uses the superlative (περίγειότατος). I have mitigated the ambiguity by translating the latter, not as ‘perigee’, but as ‘closest to earth’ (for Mercury alone).

*phase*. Used for the fixed stars and planets, this is simply a transcription of φάσις, and is a general term including all the significant ‘configurations with respect to the sun’ (listed by Ptolemy at VIII 4 pp. 409-10, and exemplified in his partially extant work φάσεις ἀπλανών ἀστέρων, ‘Phases of the Fixed Stars’), such as first visibility at sunset, or last visibility just before dawn. But the literal meaning of φάσις is ‘appearance’, and Ptolemy also uses it to mean specifically ‘first visibility’ of a body after a period of invisibility. To avoid ambiguity, I have translated the latter case by ‘first visibility’, reserving ‘phase’ for the general term.

(iv) Referring to sun and moon

*conjunction* is a fairly literal rendering of σύνοδος (‘meeting’), but *opposition* renders πανσέληνος (literally ‘full moon’, which occurs when sun and moon are in opposition). *syzygy* is a transcription of the convenient συζυγία (literally ‘yoking together’), a general term to denote either or both conjunction and opposition. In eclipses the partial phases are denoted by *immersion* (ἐμπτωσις, ‘falling in’, the phase from the beginning of the eclipse to totality) and *emersion* (ἀναπλήρωσις, ‘filling up again’, the phase from the end of totality to the end of the eclipse). The total phase is denoted by μονή (‘remaining’) and rendered by *duration* (of totality).
Time-reckoning

Ptolemy often uses the term νυχθήμερον, which combines the Greek words for night and day, to mean the ‘solar day’ of 24 hours. There is no such convenient term in English. I have generally translated it day when no ambiguity is possible, but have occasionally resorted to periphrasis (e.g. II 3 p. 79 = HI 96, 7-9). Since we use clocks, we reckon time by the mean solar day of uniform length, the average time taken by the sun to go from one meridian crossing to the next. In antiquity, where the normal means of telling time was the sundial, it was usually reckoned by the true solar day, of varying length, the time taken by the sun to go from one meridian crossing to the next on a specific day. In III 9 Ptolemy explains why they are different, and how to transform one into the other. He uses the terms ὑμαλὰ νυχθήμερα (‘uniform days’) and ἀνώμαλα νυχθήμερα (‘non-uniform days’) for mean and true solar days respectively. When he is talking about intervals, he often refers to those measured in true solar days as ‘reckoned simply’, and those measured in mean solar days as ‘reckoned accurately’.

The kind of hours normally used in the ancient world were seasonal hours (σωματικοὶ), sometimes known as ‘civil hours’. An hour was 1/24th of the actual length of daylight or night-time at a given place, and hence the length of an hour varied according to terrestrial latitude and time of year, and a day-hour was of different length from a night-hour except at equinox. For astronomical purposes, however, the uniform 1/24th of a day was used; these were known as equinoctial hours (ἀπαντικαὶ ἑσπεριναί), because they were the same length as the seasonal hour at equinox. If an ordinal number is attached to an hour, it indicates a seasonal hour, counted from dawn (or sunset, if specified by ‘of night’ or by the context). Thus ‘the sixth hour’ is the same as noon.

Time-degrees. Another way of measuring time was by the amount of the celestial equator which had passed a bound (horizon or meridian). This was often connected with the rising-times of ecliptic arcs (see pp. 18-19). This measurement was in degrees. Since 360° of the equator cross the meridian in about one day, one ‘time-degree’ equals 1/24th of an equinoctial hour or 4 minutes. The Greek term is χρόνοι ἑσπερινοὶ (‘equatorial times’), sometimes abbreviated to χρόνοι (‘times’).

Other

mean (μέσος) can imply ‘of average length’ (as in ‘mean synodic month’) or ‘uniform’ (as in ‘mean motion in longitude’).

hypothesis. With some hesitation, I have used this to translate ὑπόθεσις, although the connotation in the Almagest never really coincides with the modern one. Whereas we use ‘hypothesis’ to denote a tentative theory which has still to be verified, Ptolemy usually means by ὑπόθεσις something more like ‘model’, ‘system of explanation’, often indeed referring to ‘the hypotheses
which we have demonstrated'. The word still retains much of the etymological meaning of 'basis on which something else is constructed'. The corresponding verbal forms are ὑποτίθεται, ὕποκείται, which I have frequently translated, not only as 'assume', but even as 'it is given'. They are standard terms of Greek geometry in this sense at least as early as Euclid.

6. Editorial procedures

Since the translation is based principally on the Teubner text of Heiberg (see p. 3), it is keyed to that edition by the addition of Heiberg's page numbers in the margin. There and elsewhere references to Heiberg are preceded by 'H'. Thus HI 236.15 means 'Heiberg's edition, Vol. I p. 236 line 15'. Where the context makes it unnecessary the volume number is omitted.

Brackets are used as follows. Square brackets [ ] enclose explanatory additions to or expansions of the Greek text by the translator. Curved brackets { } enclose passages which I believe to be later additions to Ptolemy's original text. Parentheses ( ) are used merely for clarity, better to express the author's sequence of thought.

As explained on p. 5, I believe the list of chapter headings preceding each book to be a later addition. Nevertheless, since these serve a useful purpose, I have grouped them together at the beginning (pp. 27-32) to serve as a table of contents.

I have made no effort to provide a continuous commentary, but refer the reader to the relevant sections in Olaf Pedersen's A Survey of the Almagest (abbreviated 'Pedersen') and O. Neugebauer's A History of Ancient Mathematical Astronomy (abbreviated HAMA). My footnotes are confined to particulars not treated by them, or requiring some elaboration, and to textual corrections. In Appendix A, however, I have provided worked examples of every type of problem (including, where it is not utterly trivial, the use of the tables) which arises in the Almagest, except where Ptolemy himself gives a worked example. Where possible, my example is taken from a calculation or observation actually occurring in the Almagest. Appendix B lists all my corrections to Heiberg's text. Appendix C discusses the problem of the derivation of Ptolemy's planetary mean motions, which has never been adequately treated.

The index includes all proper names occurring in the text, and certain selected topics (mostly taken from the Introduction and footnotes). It also contains all observations recorded in the Almagest, under the topic or body concerned (e.g. 'equinox', 'moon'). For a list of the observations in chronological order the reader is referred to Pedersen's Appendix A.

In drawing the diagrams I have tried to reproduce the manuscript tradition, while at the same time making the figures as clear as possible by marking the points unambiguously. Since there is often considerable variation in the manuscript representations, I have been forced to make many choices; but I have not 'modernized' the figures. Where a figure seemed inadequate, I have not changed it, but have added an explanatory one of my own. Such explanatory (and other supplementary) figures are distinguished by alpha-
betical numbering ('Fig. A' etc.), whereas figures reproduced from the manuscripts are numbered according to the book and the order within that book (thus 'Fig. 3.10' indicates that this is the tenth diagram in Book III; in the manuscripts they are not usually numbered, but where they are, they are numbered separately in each book). I have represented the Greek letters of the figures by the following system:

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7. Other conventional symbols and abbreviations

e  eccentricity
r  radius of epicycle or body
M  length of longest day in hours
m  length of shortest day in hours
R  radius of principal circle (e.g. of deferent)
α  (1) right ascension (see p. 18)
    (2) anomaly (see p. 21)
β  celestial latitude
δ  declination
ε  obliquity of ecliptic
η  elongation
θ  equation
ι  inclination of orbit (of moon or planet)
κ  'centrum', i.e. distance from apogee (see p. 22)
λ  longitude
ρ  (1) oblique ascension (see p. 18)
    (2) geocentric distance
φ  terrestrial latitude
ω  distance from northpoint on orbit

A bar over a letter denotes 'mean', thus \( \bar{\lambda} \) = 'mean longitude'.

The following are used in a raised position (e.g. 2'') to denote units:

d  days
h  equinoctial hours
**Introduction: Conventional symbols**

- **m** months
- **y** years
- **p** 'parts', i.e. the arbitrary units in trigonometrical calculations (see pp. 7-9)
- **°** degrees
- **∞** demi degrees ($2° = 1°$, see p. 8)
- **%** degrees per day

In the star catalogue only, * indicates some doubt about the reading. For other abbreviations particular to the star catalogue see p. 341 n.95.

**Zodiacal signs**

- **♈** Aries
- **♉** Taurus
- **♊** Gemini
- **♋** Cancer
- **♌** Leo
- **♍** Virgo
- **♎** Libra
- **♏** Scorpius
- **♐** Sagittarius
- **♑** Capricornus
- **♒** Aquarius
- **♓** Pisces

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<thead>
<tr>
<th>Symbol</th>
<th>Sign</th>
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<tr>
<td>♈</td>
<td>Aries</td>
<td>0° = 0° in longitude</td>
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<td>♉</td>
<td>Taurus</td>
<td>0° = 30°</td>
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<tr>
<td>♊</td>
<td>Gemini</td>
<td>0° = 60°</td>
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<tr>
<td>♋</td>
<td>Cancer</td>
<td>0° = 90°</td>
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<td>♌</td>
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<td>♍</td>
<td>Virgo</td>
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<td>♎</td>
<td>Libra</td>
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<td>♏</td>
<td>Scorpius</td>
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<td>♐</td>
<td>Sagittarius</td>
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<td>♑</td>
<td>Capricornus</td>
<td>0° = 270°</td>
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<td>♒</td>
<td>Aquarius</td>
<td>0° = 300°</td>
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<tr>
<td>♓</td>
<td>Pisces</td>
<td>0° = 330°</td>
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**Planetary symbols**

- **♃** Saturn
- **♄** Jupiter
- **♂** Mars
- **♀** Venus
- **♂** Mercury
- **☉** Sun
- **☽** Moon

**Other astronomical symbols**

- **⊕** Earth
- **☊** ascending node
- **☋** descending node

On 'sexagesimal' representations such as 6,13; 10,0,58 see pp. 6-7.

For the mathematical symbols || and ||| (both meaning 'is similar to') and ≡ ('is congruent to') see p. 17.

For 'M. T. I' and 'M. T. II' see p. 18.

For manuscript abbreviations see pp. 3-4.
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¹ These lists of the chapter headings are found in the mss. at the beginning of each book preceded by the words 'The following are the contents of Book n of Ptolemy's mathematical treatise'. I believe that the division into chapters and the chapter headings are later additions (see Introduction p. 5).
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2 Most mss., followed by Heiberg, read at H 86.20 κανονίων τῶν κατὰ δεκαμορίαν παράλληλων, which is untranslatable. I read with D κανονίων τῶν κατὰ παράλληλων. Someone who compared the text at II 8 (H 134,1), κανόνιον τῶν κατὰ δεκαμορίαν ἀναφοράν, imported δεκαμορίαν here and tried to combine the two inconsistent descriptions.
10. [Method of] calculation and table of the first, simple anomaly of
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11. That the difference in the size of the lunar anomaly according to
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3. Layout of the tables
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7. Construction of eclipse tables

3 In the text the ‘method of calculation’ is explained at the end of ch. 9, and ch. 10 consists solely of
the table. This variation is perhaps a remnant of a different chapter division. Cf. Introduction p. 5.
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Translation
of the
ALmagest
1. {Preface}^4

The true philosophers, Syrus,^5 were, I think, quite right to distinguish the theoretical part of philosophy from the practical. For even if practical philosophy, before it is practical, turns out to be theoretical,^6 nevertheless one can see that there is a great difference between the two: in the first place, it is possible for many people to possess some of the moral virtues even without being taught, whereas it is impossible to achieve theoretical understanding of the universe without instruction; furthermore, one derives most benefit in the first case [practical philosophy] from continuous practice in actual affairs, but in the other [theoretical philosophy] from making progress in the theory. Hence we thought it fitting to guide our actions (under the impulse of our actual ideas [of what is to be done]) in such a way as never to forget, even in ordinary affairs, to strive for a noble and disciplined disposition, but to devote most of our time to intellectual matters, in order to teach theories, which are so many and beautiful, and especially those to which the epithet ‘mathematical’ is particularly applied. For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology.^7 For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] ‘theology’, since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from

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^4 This ‘philosophical’ preface and its relationship to Ptolemy’s attitude to philosophy is discussed by Boll, Studien 68–76, to which the reader is referred for the relevant passages in ancient literature. The general standpoint is Aristotelian.

^5 Syrus is also the addressee of a number of other works by Ptolemy (see Toomer[5] 187). Nothing is known about him. The name is very common in (but not confined to) Greco-Roman Egypt. The statement in a scholion to the Tetrabiblos (quoted by Boll, Studien 67, n. 2) that some say he was a fictitious person, others that he was a doctor, merely reveals that he was equally unknown in late antiquity.

^6 Theon in his commentary (Rome II 320, 13–14) gives φησι... συμβεβηκέναι τῷ πρακτικῷ τῷ πρώτῳ ... τοῦ θεωρητικοῦ τυγχάνειν. This is a paraphrase rather than a different reading, but shows that he understood the text as I have translated it. By this obscure expression I take Ptolemy to mean that before actually practising virtues one must have some concept of them (even though this is innate rather than taught).

^7 E. g. Metaphysics E I, 1026a 18 ff., ὥστε τρεῖς ἀν ἐ̂ ἴεν φιλοσοφία τῇ θεωρητικῇ, μαθηματικῇ, φυσικῇ, θεολογικῇ.
perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an aethereal® nature, it keeps their unchanging form unchanged.

From all this we concluded:® that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we were able to the whole of it, but especially to the theory concerning divine and heavenly things. For that alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings10 which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. As for physics, mathematics can make a significant contribution. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre. With

®'aethereal' (αἰθέριον) has a precise meaning in Aristotelian physics: everything above the sphere of the moon is composed of an 'incorruptible' substance, unlike anything known on earth in its consistency (very thin) and in its natural motion (circular). See I 3 p. 40. One of the names for this substance is 'aether', another 'fifth essence'. See Campanus IV n. 56. pp. 394-5.

®In this exaltation of mathematics above the other two divisions of philosophy Ptolemy parts company with Aristotle, for whom theology was the most noble pursuit for the human mind.

10The heavenly bodies.
regard to virtuous conduct in practical actions and character, this science, above all things, could make men see clearly; from the constancy, order, symmetry and calm which are associated with the divine, it makes its followers lovers of this divine beauty, accustoming them and reforming their natures, as it were, to a similar spiritual state.

It is this love of the contemplation of the eternal and unchanging which we constantly strive to increase, by studying those parts of these sciences which have already been mastered by those who approached them in a genuine spirit of enquiry, and by ourselves attempting to contribute as much advancement as has been made possible by the additional time between those people and ourselves.11 We shall try to note down12 everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field.13 For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability.

2. [On the order of the theorems]

In the treatise which we propose, then, the first order of business is to grasp the relationship of the earth taken as a whole to the heavens taken as a whole.14 In the treatment of the individual aspects which follows, we must first discuss the position of the ecliptic15 and the regions of our part of the inhabited world and also the features differentiating each from the others due to the [varying] latitude at each horizon taken in order.16 For if the theory of these matters is treated first it will make examination of the rest easier. Secondly, we have to go through the motion of the sun and of the moon, and the phenomena accompanying these [motions];17 for it would be impossible to examine the theory of the stars18 thoroughly without first having a grasp of these matters. Our final task in this way of approach is the theory of the stars. Here too it would be appropriate to deal first with the sphere of the so-called 'fixed stars';19

11 This notion of the advancement of science, and particularly astronomy, by the additional time available is one to which Ptolemy recurs in the epilogue (XIII 11 p. 647), and also, in a specifically astronomical context, at VII 1 p. 321 and VII 3 p. 329.
12 ὑπομνήματα αἰτίας. Ὑπομνήμα is a 'memoir', usually implying summary brevity. Ptolemy recurs to this too in the epilogue (XIII 11 p. 647).
13 Ptolemy assumes that his readers will have a certain competence. See Introduction p. 6.
15 I 12–16. The mathematical section I 10–11 is not specifically mentioned here.
16 Book II.
17 Books III–VI.
18 *Stars* here and throughout chs. 3–8 includes both fixed stars and planets (see Introduction p. 21) and also, sometimes, sun and moon.
19 Books VII–VIII.
and follow that by treating the five 'planets', as they are called. We shall try to provide proofs in all of these topics by using as starting-points and foundations, as it were, for our search the obvious phenomena, and those observations made by the ancients and in our own times which are reliable. We shall attach the subsequent structure of ideas to this [foundation] by means of proofs using geometrical methods.

The general preliminary discussion covers the following topics: the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its centre; in size and distance it has the ratio of a point to the sphere of the fixed stars; and it has no motion from place to place. We shall briefly discuss each of these points for the sake of reminder.

3. (That the heavens move like a sphere)^

It is plausible to suppose that the ancients got their first notions on these topics from the following kind of observations. They saw that the sun, moon and other stars were carried from east to west along circles which were always parallel to each other, that they began to rise up from below the earth itself, as it were, gradually got up high, then kept on going round in similar fashion and getting lower, until, falling to earth, so to speak, they vanished completely, then, after remaining invisible for some time, again rose afresh and set; and [they saw] that the periods of these [motions], and also the places of rising and setting, were, on the whole, fixed and the same.

What chiefly led them to the concept of a sphere was the revolution of the ever-visible stars, which was observed to be circular, and always taking place about one centre, the same [for all]. For by necessity that point became [for them] the pole of the heavenly sphere: those stars which were closer to it revolved on smaller circles, those that were farther away described circles ever greater in proportion to their distance, until one reaches the distance of the stars which become invisible. In the case of these, too, they saw that those near the ever-visible stars remained invisible for a short time, while those farther away remained invisible for a long time, again in proportion [to their distance]. The result was that in the beginning they got to the aforementioned notion solely from such considerations; but from then on, in their subsequent investigation, they found that everything else accorded with it, since absolutely all phenomena are in contradiction to the alternative notions which have been propounded.

For if one were to suppose that the stars' motion takes place in a straight line towards infinity, as some people have thought, what device could one

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20 Books IX-XIII.
21 See Pedersen 36-7.
22 According to Theon's commentary (Rome II 338) this belief was Epicurean, but I know of no other evidence. The only other relevant passage appears to be Xenophanes, Diels-Kranz A41a (the sun really moves towards infinity).
conceive of which would cause each of them to appear to begin their motion from the same starting-point every day? How could the stars turn back if their motion is towards infinity? Of, if they did turn back, how could this not be obvious? [On such a hypothesis], they must gradually diminish in size until they disappear, whereas, on the contrary, they are seen to be greater at the very moment of their disappearance, at which time they are gradually obstructed and cut off, as it were, by the earth's surface.

But to suppose that they are kindled as they rise out of the earth and are extinguished again as they fall to earth is a completely absurd hypothesis. For even if we were to concede that the strict order in their size and number, their intervals, positions and periods could be restored by such a random and chance process; that one whole area of the earth has a kindling nature, and another an extinguishing one, or rather that the same part [of the earth] kindles for one set of observers and extinguishes for another set; and that the same stars are already kindled or extinguished for some observers while they are not yet for others: even if, I say, we were to concede all these ridiculous consequences, what could we say about the ever-visible stars, which neither rise nor set? Those stars which are kindled and extinguished ought to rise and set for observers everywhere, while those which are not kindled and extinguished ought always to be visible for observers everywhere. What cause could we assign for the fact that this is not so? We will surely not say that stars which are kindled and extinguished for some observers never undergo this process for other observers. Yet it is utterly obvious that the same stars rise and set in certain regions [of the earth] and do neither at others.

To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to vary for the same observers during the course of each revolution, since at one time they must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs. For the apparent increase in their sizes at the horizons is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear.

The following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results; furthermore, the motion of the heavenly bodies is the most unhampered and free of all motions, and freest motion belongs among

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23 Theon (Rome II 340) ascribes this to Heraclitus. Otherwise it is attested for Xenophanes (Diels-Kranz A38), and was admitted as one possible explanation by Epicurus (e.g. Letter to Pythocles 92) and his followers.

24 Ptolemy refers to the well-known phenomenon that the sun and moon appear larger when close to the horizon. He gives an incorrect physical and optical explanation here. In a later work (Optics III 60, ed. Lejeune p. 116) he correctly explains it as a purely psychological phenomenon. No doubt instrumental measurement of the apparent diameters had convinced him that the enlargement is entirely illusory.
plane figures to the circle and among solid shapes to the sphere; similarly, since of different shapes having an equal boundary those with more angles are greater [in area or volume], the circle is greater than [all other] surfaces, and the sphere greater than [all other] solids; likewise the heavens are greater than all other bodies.

Furthermore, one can reach this kind of notion from certain physical considerations. E.g., the aether is, of all bodies, the one with constituent parts which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among planes, and the spherical, among three-dimensional surfaces. And since the aether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all aethereal and divine bodies out of shapes which are of like parts and spherical. For if they were flat or shaped like a discus they would not always display a circular shape to all those observing them simultaneously from different places on earth. For this reason it is plausible that the aether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform fashion.

4. [That the earth too, taken as a whole, is sensibly spherical]\(^{27}\)

That the earth, too, taken as a whole,\(^{28}\) is sensibly spherical can best be grasped from the following considerations. We can see, again, that the sun, moon and other stars do not rise and set simultaneously for everyone on earth, but do so earlier for those more towards the east, later for those towards the west. For we find that the phenomena at eclipses, especially lunar eclipses,\(^{29}\) which take place at the same time [for all observers], are nevertheless not recorded as occurring at the same hour (that is at an equal distance from noon) by all observers. Rather, the hour recorded by the more easterly observers is always later than that recorded by the more westerly. We find that the differences in the hour are proportional to the distances between the places [of observation]. Hence one can reasonably conclude that the earth’s surface is spherical, because its evenly curving surface (for so it is when considered as a whole) cuts off [the heavenly bodies] for each set of observers in turn in a regular fashion.

If the earth’s shape were any other, this would not happen, as one can see from the following arguments. If it were concave, the stars would be seen rising first by those more towards the west; if it were plane, they would rise and set

\(^{25}\)These propositions were proved in a work by Zenodorus (early second century B.C., see Toomer[1]) from which extensive excerpts are given by (among others) Theon (Rome II 355–79). There is a good summary in Heath *HGM* II 207–13.

\(^{26}\)The only relevant passage I know is Empedocles, Diels-Kranz A60, who maintained that the moon is disk-shaped.

\(^{27}\)See Pedersen 37–9.

\(^{28}\)‘taken as a whole’: ignoring local irregularities such as mountains, which are negligible in comparison to the total mass.

\(^{29}\)The timings for solar eclipses are complicated by parallax.
simultaneously for everyone on earth; if it were triangular or square or any other polygonal shape, by a similar argument, they would rise and set simultaneously for all those living on the same plane surface. Yet it is apparent that nothing like this takes place. Nor could it be cylindrical, with the curved surface in the east-west direction, and the flat sides towards the poles of the universe, which some might suppose more plausible. This is clear from the following: for those living on the curved surface none of the stars would be ever-visible, but either all stars would rise and set for all observers, or the same stars, for an equal [celestial] distance from each of the poles, would always be invisible for all observers. In fact, the further we travel toward the north, the more of the southern stars disappear and the more of the northern stars appear. Hence it is clear that here too the curvature of the earth cuts off [the heavenly bodies] in a regular fashion in a north-south direction, and proves the sphericity [of the earth] in all directions.

There is the further consideration that if we sail towards mountains or elevated places from and to any direction whatever, they are observed to increase gradually in size as if rising up from the sea itself in which they had previously been submerged: this is due to the curvature of the surface of the water.

5. *That the earth is in the middle of the heavens*

Once one has grasped this, if one next considers the position of the earth, one will find that the phenomena associated with it could take place only if we assume that it is in the middle of the heavens, like the centre of a sphere. For if this were not the case, the earth would have to be either

[a] not on the axis [of the universe] but equidistant from both poles, or
[b] on the axis but removed towards one of the poles, or
[c] neither on the axis nor equidistant from both poles.

Against the first of these three positions militate the following arguments. If we imagined [the earth] removed towards the zenith or the nadir of some observer, then, if he were at *sphaera recta*, he would never experience equinox, since the horizon would always divide the heavens into two unequal parts, one above and one below the earth; if he were at *sphaera obliqua*, either, again, equinox would never occur at all, or, [if it did occur,] it would not be at a position halfway between summer and winter solstices, since these intervals would necessarily be unequal, because the equator, which is the greatest of all parallel circles drawn about the poles of the [daily] motion, would no longer be bisected by the horizon; instead [the horizon would bisect] one of the circles parallel to the equator, either to the north or to the south of it. Yet absolutely everyone agrees that these intervals are equal everywhere on earth, since [everywhere] the increment of the longest day over the equinoctial day at the

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30 Reading πλειόνα (with D) for τὰ πλειόνα at H16.9. Corrected by Manitius.
31 See Pedersen 39–42.
summer solstice is equal to the decrement of the shortest day from the equinoctial day at the winter solstice. But if, on the other hand, we imagined the displacement to be towards the east or west of some observer, he would find that the sizes and distances of the stars would not remain constant and unchanged at eastern and western horizons, and that the time-interval from rising to culmination would not be equal to the interval from culmination to setting. This is obviously completely in disaccord with the phenomena.

Against the second position, in which the earth is imagined to lie on the axis removed towards one of the poles, one can make the following objections. If this were so, the plane of the horizon would divide the heavens into a part above the earth and a part below the earth which are unequal and always different for different latitudes,\(^{32}\) whether one considers the relationship of the same part at two different latitudes or the two parts at the same latitude. Only at \textit{sphaera recta} could the horizon bisect the sphere; at a \textit{sphaera obliqua} situation such that the nearer pole were the ever-visible one, the horizon would always make the part above the earth lesser and the part below the earth greater; hence another phenomenon would be that the great circle of the ecliptic would be divided into unequal parts by the plane of the horizon. Yet it is apparent that this is by no means so. Instead, six zodiacal signs are visible above the earth at all times and places, while the remaining six are invisible; then again [at a later time] the latter are visible in their entirety above the earth, while at the same time the others are not visible. Hence it is obvious that the horizon bisects the zodiac, since the same semi-circles are cut off by it, so as to appear at one time completely above the earth, and at another [completely] below it.

And in general, if the earth were not situated exactly below the [celestial] equator, but were removed towards the north or south in the direction of one of the poles, the result would be that at the equinoxes the shadow of the gnomon at sunrise would no longer form a straight line with its shadow at sunset in a plane parallel to the horizon, not even sensibly.\(^{33}\) Yet this is a phenomenon which is plainly observed everywhere.

It is immediately clear that the third position enumerated is likewise impossible, since the sorts of objection which we made to the first [two] will both arise in that case.

To sum up, if the earth did not lie in the middle [of the universe], the whole order of things which we observe in the increase and decrease of the length of daylight would be fundamentally upset. Furthermore, eclipses of the moon would not be restricted to situations where the moon is diametrically opposite the sun (whatever part of the heaven [the luminaries are in]), since the earth would often come between them when they were not diametrically opposite, but at intervals of less than a semi-circle.

\(^{32}\) The word translated here and elsewhere as '[terrestrial] latitude' is \kappa\lambda\iota\mu\alpha, for the meaning of which see Introduction p. 19.

\(^{33}\) The \textit{caveat} 'sensibly' is inserted because the equinox is not a date but an instant of time. Therefore on the day of equinox the sun does not rise due east and set due west (as is implied by the rising and setting shadows lying on the same straight line). However, the difference would be 'imperceptible to the senses'.
I 6. Earth negligibly small in relation to heavens

6. {That the earth has the ratio of a point to the heavens}^*

Moreover, the earth has, to the senses, the ratio of a point to the distance of the sphere of the so-called fixed stars. A strong indication of this is the fact that the sizes and distances of the stars, at any given time, appear equal and the same from all parts of the earth everywhere, as observations of the same [celestial] objects from different latitudes are found to have not the least discrepancy from each other. One must also consider the fact that gnomons set up in any part of the earth whatever, and likewise the centres of armillary spheres, operate like the real centre of the earth; that is, the lines of sight [to heavenly bodies] and the paths of shadows caused by them agree as closely with the [mathematical] hypotheses explaining the phenomena as if they actually passed through the real centre-point of the earth.

Another clear indication that this is so is that the planes drawn through the observer’s lines of sight at any point [on earth], which we call ‘horizons’, always bisect the whole heavenly sphere. This would not happen if the earth were of perceptible size in relation to the distance of the heavenly bodies; in that case only the plane drawn through the centre of the earth could bisect the sphere, while a plane through any point on the surface of the earth would always make the section [of the heavens] below the earth greater than the section above it.

7. {That the earth does not have any motion from place to place, either}^*

One can show by the same arguments as the preceding that the earth cannot have any motion in the aforementioned directions, or indeed ever move at all from its position at the centre. For the same phenomena would result as would if it had any position other than the central one. Hence I think it is idle to seek for causes for the motion of objects towards the centre, once it has been so clearly established from the actual phenomena that the earth occupies the middle place in the universe, and that all heavy objects are carried towards the earth. The following fact alone would most readily lead one to this notion [that all objects fall towards the centre]. In absolutely all parts of the earth, which, as we said, has been shown to be spherical and in the middle of the universe, the direction and path of the motion (I mean the proper, [natural] motion) of all bodies possessing weight is always and everywhere at right angles to the rigid plane drawn tangent to the point of impact. It is clear from this fact that, if

^* See Pedersen 42-3.

Ptolemy qualifies the traditional terminology for the fixed stars as ‘so-called’ (καλουμένων) because they do in fact, according to him, have a motion (the modern ‘precession’). He develops the point further at VII 1 p. 321, q.v. In general, however, he uses the traditional terminology without qualification.

An example of an armillary sphere (κρικωτή σφαῖρα) is the ‘astrolabe’ described in V 1. For references to the term in other works see LSJ s.v. κρικωτός.

See Pedersen 43-4.

πρόσνευσις, which I have translated ‘the direction of motion’ here, means basically ‘direction in which something points’ (for astronomical usages see V 5 p. 227 n. 19 and VI 11 p. 313 n. 77). Thus it would also include here the direction of a plumb-line (cf. I 12 p. 62).
[these falling objects] were not arrested by the surface of the earth, they would certainly reach the centre of the earth itself, since the straight line to the centre is also always at right angles to the plane tangent to the sphere at the point of intersection [of that radius] and the tangent.

Those who think it paradoxical that the earth, having such a great weight, is not supported by anything and yet does not move, seem to me to be making the mistake of judging on the basis of their own experience instead of taking into account the peculiar nature of the universe. They would not, I think, consider such a thing strange once they realised that this great bulk of the earth, when compared with the whole surrounding mass [of the universe], has the ratio of a point to it. For when one looks at it in that way, it will seem quite possible that that which is relatively smallest should be overpowered and pressed in equally from all directions to a position of equilibrium by that which is the greatest of all and of uniform nature. For there is no up and down in the universe with respect to itself, any more than one could imagine such a thing in a sphere: instead the proper and natural motion of the compound bodies in it is as follows: light and rarefied bodies drift outwards towards the circumference, but seem to move in the direction which is 'up' for each observer, since the overhead direction for all of us, which is also called 'up', points towards the surrounding surface; heavy and dense bodies, on the other hand, are carried towards the middle and the centre, but seem to fall downwards, because, again, the direction which is for all us towards our feet, called 'down', also points towards the centre of the earth. These heavy bodies, as one would expect, settle about the centre because of their mutual pressure and resistance, which is equal and uniform from all directions. Hence, too, one can see that it is plausible that the earth, since its total mass is so great compared with the bodies which fall towards it, can remain motionless under the impact of these very small weights (for they strike it from all sides), and receive, as it were, the objects falling on it. If the earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size: living things and individual heavy objects would be left behind, riding on the air, and the earth itself would very soon have fallen completely out of the heavens. But such things are utterly ridiculous merely to think of.

But certain people, [propounding] what they consider a more persuasive view, agree with the above, since they have no argument to bring against it, but think that there could be no evidence to oppose their view if, for instance, they supposed the heavens to remain motionless, and the earth to revolve from west to east about the same axis [as the heavens], making approximately one revolution each day; or if they made both heaven and earth move by any amount whatever, provided, as we said, it is about the same axis, and in such a

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39 Reading οὐρνον (with D, Is) for οὐρην at H23.1.
40 It is not clear to me whether Ptolemy means the outmost boundary of the universe or merely the surface (of the 'aether') surrounding the earth.
41 Heraclides of Pontos (late fourth century B.C.) is the earliest certain authority for the view that the earth rotates on its axis. See HAMA II 694-6. It was also adopted by Aristarchus as part of his more radical heliocentric hypothesis.
42 'approximately' because one revolution takes place in a sidereal, not a solar day.
way as to preserve the overtaking of one by the other. However, they do not realise that, although there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a notion is quite ridiculous. Let us concede to them [for the sake of argument] that such an unnatural thing could happen as that the most rare and light of matter should either not move at all or should move in a way no different from that of matter with the opposite nature (although things in the air, which are less rare [than the heavens] so obviously move with a more rapid motion than any earthy object); [let us concede that] the densest and heaviest objects have a proper motion of the quick and uniform kind which they suppose (although, again, as all agree, earthy objects are sometimes not readily moved even by an external force). Nevertheless, they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth: neither clouds nor other flying or thrown objects would ever be seen moving towards the east, since the earth’s motion towards the east would always outrun and overtake them, so that all other objects would seem to move in the direction of the west and the rear. But if they said that the air is carried around in the same direction and with the same speed as the earth, the compound objects in the air would none the less always seem to be left behind by the motion of both [earth and air]; or if those objects too were carried around, fused, as it were, to the air, then they would never appear to have any motion either in advance or rearwards: they would always appear still, neither wandering about nor changing position, whether they were flying or thrown objects. Yet we quite plainly see that they do undergo all these kinds of motion, in such a way that they are not even slowed down or speeded up at all by any motion of the earth.

8. \{That there are two different primary motions in the heavens\}\(^{45}\)

It was necessary to treat the above hypotheses first as an introduction to the discussion of particular topics and what follows after. The above summary outline of them will suffice, since they will be completely confirmed and further proven by the agreement with the phenomena of the theories which we shall demonstrate in the following sections. In addition to these hypotheses, it is proper, as a further preliminary, to introduce the following general notion, that there are two different primary motions in the heavens. One of them is that which carries everything from east to west: it rotates them with an unchanging and uniform motion along circles parallel to each other, described, as is obvious, about the poles of this sphere which rotates everything uniformly. The greatest of these circles is called the ‘equator’,\(^{44}\) because it is the only [such

\(^{45}\)See Pedersen 45.

\(^{44}\)‘equator’: ἵσμερινός, literally ‘of equal day’ or ‘equinoctial’. See Introduction p. 19.
parallel circle] which is always bisected by the horizon (which is a great circle), and because the revolution which the sun makes when located on it produces equinox everywhere, to the senses. The other motion is that by which the spheres of the stars perform movements in the opposite sense to the first motion, about another pair of poles, which are different from those of the first rotation. We suppose that this is so because of the following considerations. When we observe for the space of any given single day, all heavenly objects whatever are seen, as far as the senses can determine, to rise, culminate and set at places which are analogous and lie on circles parallel to the equator; this is characteristic of the first motion. But when we observe continuously without interruption over an interval of time, it is apparent that while the other stars retain their mutual distances and (for a long time) the particular characteristics arising from the positions they occupy as a result of the first motion, the sun, the moon and the planets have certain special motions which are indeed complicated and different from each other, but are all, to characterise their general direction, towards the east and opposite to [the motion of] those stars which preserve their mutual distances and are, as it were, revolving on one sphere.

Now if this motion of the planets too took place along circles parallel to the equator, that is, about the poles which produce the first kind of revolution, it would be sufficient to assign a single kind of revolution to all alike, analogous to the first. For in that case it would have seemed plausible that the movements which they undergo are caused by various retardations, and not by a motion in the opposite direction. But as it is, in addition to their movement towards the east, they are seen to deviate continuously to the north and south [of the equator]. Moreover the amount of this deviation cannot be explained as the result of a uniformly-acting force pushing them to the side: from that point of view it is irregular, but it is regular if considered as the result of [motion on] a circle inclined to the equator. Hence we get the concept of such a circle, which is one and the same for all planets, and particular to them. It is precisely defined and, so to speak, drawn by the motion of the sun, but it is also travelled by the moon and the planets, which always move in its vicinity, and do not randomly pass outside a zone on either side of it which is determined for each body. Now since this too is shown to be a great circle, since the sun goes to the north and south of the equator by an equal amount, and since, as we said, the eastward motion of all of the planets takes place on one and the same circle, it became necessary to suppose that this second, different motion of the whole takes place about the poles of the inclined circle we have defined [i.e. the ecliptic], in the opposite direction to the first motion.

If, then, we imagine a great circle drawn through the poles of both the above-mentioned circles, (which will necessarily bisect each of them, that is the equator and the circle inclined to it [the ecliptic], at right angles), we will have four points on the ecliptic: two will be produced by [the intersection of] the

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45 These characteristics of the fixed stars are e.g. dates of first and last visibility. They are unchanged "for a long time" because the effect of precession is very slow.

46 The qualification is inserted here to allow for the retrogradations of the planets.
equator, diametrically opposite each other; these are called 'equinoctial' points. The one at which the motion [of the planets] is from south to north is called the 'spring' equinox, the other the 'autumnal'. Two [other points] will be produced by [the intersection of] the circle drawn through both poles; these too, obviously, will be diametrically opposite each other; they are called 'tropical' [or 'solsticial'] points. The one south of the equator is called the 'winter' [solstice], the one north, the 'summer' [solstice].

We can imagine the first primary motion, which encompasses all the other motions, as described and as it were defined by the great circle drawn through both poles [of equator and ecliptic] revolving, and carrying everything else with it, from east to west about the poles of the equator. These poles are fixed, so to speak, on the 'meridian' circle, which differs from the aforementioned [great] circle in the single respect that it is not drawn through the poles of the ecliptic too at all positions of the latter. Moreover, it is called 'meridian' because it is considered to be always orthogonal to the horizon. For a circle in such a position divides both hemispheres, that above the earth and that below it, into two equal parts, and defines the midpoint of both day and night.

The second, multiple-part motion is encompassed by the first and encompasses the spheres of all the planets. As we said, it is carried around by the aforementioned [first motion], but itself goes in the opposite direction about the poles of the ecliptic, which are also fixed on the circle which produces the first motion, namely the circle through both poles [of ecliptic and equator]. Naturally they [the poles of the ecliptic] are carried around with it [the circle through both poles], and, throughout the period of the second motion in the opposite direction, they always keep the great circle of the ecliptic, which is described by that [second] motion, in the same position with respect to the equator.

9. [On the individual concepts]

Such, then are the necessary preliminary concepts which must be summarily set out in our general introduction. We are now about to begin the individual demonstrations, the first of which, we think, should be to determine the size of the arc between the aforementioned poles [of the ecliptic and equator] along the great circle drawn through them. But we see that it is first necessary to explain the method of determining chords; we shall demonstrate the whole topic geometrically once and for all.

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48 My translation follows the interpretation of Theon (Rome II 447). Maniti (p. 24 n. a) wrongly considers τοῖς γραμματέωσιν δι' αὐτῆς μεγίστου καὶ λοξοῦ κύκλου interpolated, partly because he misinterprets συντηροῦσιν (which is used here in a way similar to συντηροῦσαν at H1 6,10).
49 'chords': literally 'straight lines in a circle'. On this term see Introduction p. 17.
For the user’s convenience, then, we shall subsequently set out a table of their amounts, dividing the circumference into 360 parts, and tabulating the chords subtended by the arcs at intervals of half a degree, expressing each as a number of parts in a system where the diameter is divided into 120 parts. [We adopt this norm] because of its arithmetical convenience, which will become apparent from the actual calculations. But first we shall show how one can undertake the calculation of their amounts by a simple and rapid method, using as few theorems as possible, the same set for all. We do this so that we may not merely have the amounts of the chords tabulated unchecked, but may also readily undertake to verify them by computing them by a strict geometrical method. In general we shall use the sexagesimal system for our arithmetical computations, because of the awkwardness of the [conventional] fractional system. Since we always aim at a good approximation, we will carry out multiplications and divisions only as far as to achieve a result which differs from the precision achievable by the senses by a negligible amount.

First, then, [see Fig. 1.1] let there be a semi-circle ABG about centre D and on diameter ADG. Draw DB perpendicular to AG at D. Let DG be bisected at E, join EB, and let EZ be made equal to EB. Join ZB.

I say that ZD is the side of the [regular] decagon, and BZ the side of the [regular] pentagon.

[Proof:] Since the straight line DG is bisected at E, and a straight line DZ is adjacent to it,

\[ GZ\cdot ZD + ED^2 = EZ^2, \]
\[ \text{But } EZ^2 = BE^2 (EB = ZE), \]
\[ \text{and } EB^2 = ED^2 + DB^2. \]
\[ \therefore GZ\cdot ZD + ED^2 = ED^2 + DB^2. \]

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50 On Ptolemy’s calculation of his chord table see HAMA 21-4, Pedersen 56-63.
51 The principal convenience is that the radius is 60 parts, or 1.0 in the sexagesimal system. Hence in some ways this resembles a sine table with \( R = 1 \).
52 Euclid II 6.
GZ.ZD = DB^2 (subtracting ED^2, common).
GZ.ZD = DG^2.

So ZG has been cut in extreme and mean ratio at D.53

Now since the side of the hexagon and the side of the decagon, when both are inscribed in the same circle, make up the extreme and mean ratios of the same straight line,54 and since GD, being a radius, represents the side of the hexagon,55 DZ is equal to the side of the decagon.

Similarly, since the square on the side of the pentagon equals the sums of the squares on the sides of the hexagon and decagon when all are inscribed in the same circle,56 and, in the right-angled triangle BDZ, the square on BZ equals the sum of the squares on BD, which is the side of the hexagon, and on DZ, which is the side of the decagon, it follows that BZ equals the side of the pentagon.

Since, then, as I said, we set the diameter of the circle as 120 parts, it follows from the above that

\[
\begin{align*}
DE &= 30^\circ \text{ (DE half the radius)} \\
\text{and } DE^2 &= 900^\circ; \\
BD &= 60^\circ \text{ (BD a radius)} \\
\text{and } BD^2 &= 3600^\circ.
\end{align*}
\]

And EZ^2 = EB^2 = 4500^\circ, the sum [of DE^2 and BD^2]
\[
\therefore EZ \approx 67;4,55^\circ
\]

and by subtraction [of DE from EZ], DZ = 37;4,55^\circ.

So the side of the decagon, which subtends 36^\circ, has 37;4,55^\circ where the diameter has 120^\circ.

Again, since DZ = 37;4,55^\circ.

\[
\begin{align*}
DZ^2 &= 1375;4,15^\circ; \quad \text{57} \\
\text{and } DB^2 &= 3600^\circ.
\end{align*}
\]

so BZ^2 = DZ^2 + DB^2 = 4975;4,15^\circ.
\[
\therefore BZ \approx 70;32,3^\circ.
\]

Therefore the side of the pentagon, which subtends 72^\circ, contains 70;32,3^\circ where the diameter has 120^\circ.

It is immediately obvious that the side of the [inscribed] hexagon, which subtends 60^\circ and is equal to the radius, contains 60^\circ.

Similarly, since the side of the [inscribed] square, which subtends 90^\circ, is equal, when squared, to twice the square on the radius, and since the side of the [inscribed] triangle, which subtends 120^\circ, is equal, when squared, to three times the square on the radius, and the square on the radius is 3600^\circ, we compute that

the square on the side of the square is 7200^\circ

and the square on the side of the triangle is 10800^\circ.

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53 Euclid VI Def. 3 states that 'a straight line has been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less'; i.e. here ZG:DG = DG:ZD.
54 Euclid XIII 9.
55 Euclid IV 15 porism.
56 Euclid XIII 10.
57 The reading 14 (for 15) occurs as a marginal variant, in the Greek mss., here and at related places (see apparatus at H34.16; 34.18; 36.4 and 36.7), and, in the Arabic, in T, and was adopted in Hajjāj's translation. It is more accurate, but makes no difference to the final result.
10. Chord of the supplement; Ptolemy's Theorem

We can, then, consider the above chords as established individually by the above straightforward procedures. It will immediately be obvious that if any chord be given, the chord of the supplementary arc is given in a simple fashion, since the sum of their squares equals the square on the diameter. For instance, since the chord of 36° was shown to be 37°4,55'', and the square of this is 1375°4,15'', and the square on the diameter is 14400°, the square on the chord of the supplementary arc (which is 144°) will be the difference, namely 13024°55,45, and so

\[ \text{Crd } 144° \approx 114°7;37''. \]

Similarly for the other chords of the supplements.

We shall next show how the remaining individual chords can be derived from the above chords, first of all setting out a theorem which is extremely useful for the matter at hand.

[See Fig. 1.2.] Let there be a circle with an arbitrary quadrilateral ABGD inscribed in it. Join AG and BD.

![Fig. 1.2]

We must prove that

\[ AG.BD = AB.DG + AD.BG. \]

[Proof:] Make \( \angle ABE = \angle DBG \).

Then, if we add \( \angle EBD \) common,

\[ \angle ABD = \angle EBG. \]

58 Reading ωτόδεν (with D) for ἐτοδεν at H35, 18.

59 This proposition, commonly known as 'Ptolemy's Theorem', is not in fact attested before him. It remains uncertain whether any of the earlier chord tables (e.g. Menelaus') used any geometrical basis beyond the half-angle theorem (see n. 60 and Toomer[2] 18-19).
I 10. Chord of the difference

But $\angle BDA = \angle BGE$ also, since they subtend the same segment.
\[ \therefore \text{triangle } ABD \parallel \text{triangle } BGE. \]
\[ \therefore \frac{BG}{GE} = \frac{BD}{DA}. \]
\[ \therefore \frac{BG}{AD} = \frac{BD}{GE}. \]

Again, since $\angle ABE = \angle DBG$,
and $\angle BAE = \angle BDG$,
\[ \text{triangle } ABE \parallel \text{triangle } BGD. \]
\[ \therefore \frac{BA}{AE} = \frac{BD}{DG}. \]
\[ \therefore \frac{BA}{DG} = \frac{BD}{AE}. \]

But it was shown that
\[ \frac{BG}{AD} = \frac{BD}{GE}. \]

Therefore, by addition, $\frac{AG}{BD} = \frac{AB}{DG} + \frac{AD}{BG}$.

Q.E.D.

Having established this preliminary theorem, we draw [Fig. 1.3] semi-circle $ABGD$ on diameter $AD$, and draw from $A$ two chords, $AB$, $AG$, each given in size in terms of a diameter of $120^\circ$. Join $BG$.

I say that $BG$ too is given.

[Proof:] Join $BD, GD$.

\[ \begin{align*}
\text{Fig. 1.3}
\end{align*} \]

Then, clearly, $BD$ and $GD$ too will be given, since they are chords of arcs supplementary [to the arcs of the given chords $AB$ and $AG$].

Now since $ABGD$ is a cyclic quadrilateral,
\[ \frac{AB}{GD} + \frac{AD}{BG} = \frac{AG}{BD}. \]

But $\frac{AG}{BD}$ and $\frac{AB}{GD}$ are given.
\[ \therefore \frac{AD}{BG} \text{ is given by subtraction.} \]

And $AD$ is a diameter.

Therefore chord $BG$ is given.

And we have shown that, if two arcs and the corresponding chords are given, the chord of the difference between the two arcs will also be given.

It is obvious that by means of this theorem we shall be able to enter [in the table] quite a few chords derived from the difference between the individually calculated chords, and notably the chord of $12^\circ$, since we have those of $60^\circ$ and $72^\circ$. 

H37

H38

H39
Let us now consider the problem of finding the chord of the arc which is half that of some given chord.

Let Fig. 1.4 ABG be a semi-circle on diameter AG. Let GB be a given chord. Bisect arc GB at D, join AB, AD, BD, DG, and drop perpendicular DZ from D on to AG.

![Fig. 1.4 Diagram]

I say that

\[ ZG = \frac{1}{2}(AG - AB). \]

[Proof:] Let AE = AB, and join DE.

Then since (in the triangles ABD, ADE)

- \( AB = AE \), and AD is common,
- the two pairs of sides \( AB, AD, \text{ and } AE, AD \) are equal.

Furthermore \( \angle BAD = \angle EAD \).

- Since base BD = base DE.
- But BD = DG [by construction]
- \( \therefore DG = DE. \)

So, since, in the isosceles triangle DEG, perpendicular DZ has been drawn from apex to base

\[ EZ = ZG. \]

But \( EG = [AG - AE = AG - AB. \]

\[ \therefore ZG = \frac{1}{2}(AG - AB). \]

Now, if the chord of arc BG is given, the supplementary chord AB is immediately given.

Therefore \( ZG, \) which is \( \frac{1}{2}(AG - AB), \) is also given.

But, since, in the right-angled triangle AGD, the perpendicular DZ has been drawn,

triangle ADG \( \parallel \) triangle DGZ (both right-angled).

\[ \therefore AG:GD = GD:GZ. \]

\[ \therefore AG.GZ = GD^2. \]

60 Although Ptolemy's formula for the chord of the half-angle can easily be derived from his general theorem (see Toomer[2] 16-17), he introduces instead another theorem, which goes back to Archimedes (see HAMA 23-4). It is a plausible inference that this is because the latter theorem was the sole basis of earlier chord tables, notably Hipparchus', as I have argued, Toomer[2] 18-19.

61 Euclid VI 8.
But AG.GZ is given.
Therefore GD² is given, and so chord GD, which subtends an arc half of [the arc of the given chord] BG, is also given.

By means of this theorem too a large number of chords will be derived by halving [the arcs of] the previously determined chords, and notably, from the chord of 12°, the chords of 6°, 3°, 1½° and ½°. By calculation we find the chord of 1½° to be approximately 1;34,15° where the diameter is 120°, and the chord of ½° to be approximately 0;47,8° in the same units.

Again, [see Fig. 1.5] let there be a circle ABGD on diameter AD, with centre Z. From A let there be cut off in succession two given arcs, AB, BG. Join the corresponding chords AB, BG; they too will be given.

Fig. 1.5

I say, that if we join AG, that [chord] too will be given.

[Proof:] Draw through B diameter BZE, and join BD,DG,GE,DE. It is immediately clear that from BG one can derive GE, and from AB one can derive BD and DE [all as chords of the supplementary arc]. By an argument similar to the preceding [p. 51], since BGDE is a cyclic quadrilateral, in which BD and GE are diagonals, the product of the diagonals will be equal to the sum of the products of the opposite sides [i.e. BD.GE = BG.DE + BE.GD]. Therefore, since (BD.GE) and (BG.DE) are both given, (BE.GD) is also given. But BE also is given, being a diameter: therefore the remaining 62 part, GD, will also be given, and hence GA, the [chord of the] supplement.

Therefore, if two arcs and the corresponding chords are given, the chord corresponding to the sum of these two arcs will be given by means of this theorem.

It is obvious that by combining [in this way] the chord of 1½° with all the chords we have already obtained, and then computing successive chords, we will be able to enter [in the table] all chords [of arcs] which when doubled are

62 Reading η λοιπή (with A) at H42,1 for λοιπή ("by subtraction").
divisible by three (i.e. multiples of $1^\circ$). Then the only chords remaining to be
determined will be those between the $1^\circ$ intervals, two in each interval, since
our table is made at $\frac{1}{10}$ intervals. If, therefore, we can find the chord of $1^\circ$, this
will enable us to complete [the table with] all the remaining intermediate
chords, by finding the sum or difference [of $1^\circ$] from the given chords at either
end of the [$1^\circ$] intervals. Now, if a chord, e.g. the chord of $1^\circ$, is given, the
chord corresponding to an arc which is one-third of the previous one cannot be
found by geometrical methods.\(^3\) (If this were possible, we should immediately
have the chord of $1^\circ$). Therefore we shall first derive the chord of $1^\circ$ from those of
$\frac{1}{10}$ and $\frac{3}{10}$. [We shall do this] by establishing a lemma which, though it cannot
in general exactly determine the sizes [of chords], in the case of such very small
quantities can determine them with a negligibly small error.

I say, then, that if two unequal chords be given, the ratio of the greater to the
lesser is less than the ratio of the arc on the greater to the arc on the lesser.

[See Fig. 1.6] Let there be a circle ABGD, in which there are drawn two
unequal chords. the lesser AB and the greater BG.

\[ \text{Fig. 1.6} \]

I say that

\[ GB:BA < \text{arc BG: arc BA}. \]

[Proof:] Let $\angle ABG$ be bisected by [chord] BD. Join AEG, AD and GD. Then,
since $\angle ABG$ is bisected by chord BED,

\[ GD = AD \]

and $GE > EA$.\(^4\)

\(^3\) This is true: the problem of finding Crd $\alpha$ from given Crd $3\alpha$ can be reduced to a cubic equation
of the kind which cannot (except for a few particular values of $\alpha$) be solved by Euclidean geometry

\(^4\) Derivable from Euclid VI 3, which states that the bisector of the angle at the apex of a triangle
divides the base in the ratio of the two sides enclosing the angle. Here, since $BG > BA$, $GE > EA$.\(^4\)
So drop perpendicular DZ from D on to AEG.

Then, since AD > ED and ED > DZ, a circle drawn on centre D with radius DE will cut AD and pass beyond DZ. Let it be drawn as HEΘ, and let DZ be produced to Θ. Now, since sector DEΘ is greater than triangle DEZ, and triangle DEA is greater than sector DEH, triangle DEZ: triangle DEA < sector DEΘ: sector DEH.

But triangle DEZ: triangle DEA = EZ:EA, and sector DEΘ: sector DEH = \angle ZDE: \angle EDA.

So, componendo,

\[ \frac{ZA}{EA} < \frac{ZDA}{ADE}. \]

And, doubling the first members [of the ratios],

\[ \frac{GA}{AE} < \frac{GDA}{EDA}. \]

Then, dividendo,

\[ \frac{GE}{EA} < \frac{GDE}{EDA}. \]

But GE:EA = GB:BA, and \( \angle GDB: \angle BDA = \text{arc GB}:\text{arc BA} \).

\[ \therefore GB:BA < \text{arc GB}:\text{arc BA}. \]

Having established this, let us draw [Fig. 1.7] circle ABG, and in it two chords, AB and AG. Let us suppose, first, that AB is the chord of \( 3^{\circ} \) and AG the chord of \( 1^{\circ} \). Then, since

\[ AG:BA < \text{arc AG}:\text{arc AB} \]

and \( \text{arc AG} = \frac{4}{3} \text{arc AB} \),

\[ GA < \frac{4AB}{3}. \]

\[ ^{65} \text{Euclid VI 1.} \]

\[ ^{66} \text{Euclid VI 3.} \]
But, in units of which the diameter contains 120, we showed that
\[ AB = 0;47,8^\circ. \]
\[ \therefore GA < 1;2,50^\circ \text{ (for } 1;2,50 \approx 1;0.47,8). \]

H46 Again, using the same figure, let us set \( AB \) as the chord of \( 1^\circ \) and \( AG \) as the chord of \( 1\frac{1}{2}^\circ \). By the same argument, since
\[ \text{arc } AG = \frac{3 \text{ arc } AB}{2}, \]
\[ \therefore GA < \frac{3BA}{2}. \]

But, in units of which the diameter contains 120, we showed that
\[ AG = 1;34,15^\circ. \]
\[ \therefore AB > 1;2,50^\circ \text{ (for } 1;34,15 = \frac{1}{2}1;2,50). \]

Therefore, since the chord of \( 1^\circ \) was shown to be both greater and less than the same amount, we can establish it as approximately \( 1;2,50^\circ \) where the diameter is \( 120^\circ \). By the preceding propositions we can also establish the chord of \( \frac{1}{2}^\circ \), which we find to be approximately \( 0;31,25^\circ \). The remaining intervals can [now] be completed, as we said [p. 54]. For example, in the first \( [1\frac{1}{2}^\circ] \) interval, we can calculate the chord of \( 2^\circ \) by using the addition formula for the chord of \( 1^\circ \) applied to the chord of \( 1\frac{1}{2}^\circ \), while the chord of \( 2\frac{1}{2}^\circ \) is given by using the difference formula for [the chord of \( 1^\circ \)] applied to the chord of \( 3^\circ \). Similarly for the remaining chords.

Such, then, I think, is the easiest way to undertake the calculation of the chords. But, as I said, in order that we may have the actual amounts of the chords readily available for every occasion, we shall set out tables [for that purpose] below. They will be arranged in sections of 45 lines\(^{67}\) to achieve a symmetrical appearance. The first column [in each section] will contain the arcs tabulated at intervals of \( \frac{1}{10}^\circ \), the second the corresponding chords in units of which the diameter contains 120, and the third the thirtieth part of the increment in the chord for each interval. [This last] is so that we may have the average increment corresponding to one minute [of arc], which will not be sensibly different from the true increment [for each minute]. Thus we can easily calculate the amount of the chord corresponding to fractions which fall between the [tabulated] half-degree intervals.

It is easy to see that, if we suspect some scribal corruption in one of the values for the chord in the table, the same theorems which we have already set out will enable us to test and correct it easily, either by taking the chord of double the arc [of that] of the chord in question, or from the difference with some other given chord, or from the chord of the supplement.

The layout of the table is as follows.

11. \{Table of Chords\}\(^{68}\)

\(^{67}\) 45 lines is the standard height of tables throughout the \textit{Almagest}. It is presumably chosen to conform to some standard height of papyrus roll (on papyrus standards see \textit{Lewis, Papyrus in Classical Antiquity}, 36-9, 56. on Pliny \textit{NH} 13, 78). Various consequences flow from it, notably the 18-year interval in mean motion tables (see III 1 p. 140 with n. 28).
TABLE OF CHORDS

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*Ptolemy’s chord table has been recomputed, using a computer program which reproduces, as far as possible, Ptolemy’s own methods of calculation, by Glowatzki and Göttche. Although much of their book is superfluous (see my review, Toomey14), it contains some interesting results, notably that Ptolemy must have carried out his calculations to five sexagesimal places to achieve the*
accuracy he does in the third place. The book also enables one to make a number of corrections of scribal errors in the table. Before seeing it I had already made those given below. None of the other corrections (all of 1 in the last place) suggested by the authors seem likely to me, although some are possible.

Corrections to Heiberg’s text:

Crd 9°, seconds, v6 (with D, Ar) for va (51) at H48.20 (corrected by Hultsch, Sehentafeln 52)
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Crd 72°, seconds, γ (with all mss. except D) for δ (4) at H54,10 (cf. H35,1 and p. 81 n. 19; corrected by Manitius)

Crd 88°, minutes, μδ (with Ar) for μα (41) at H55,43.

Crd 97°, seconds, γδ (with D, Ar) for κζ (27) at H56,15

Crd 108°, seconds, νε (with D, Ar) for ψ (56) at H57,37

Crd 118°, seconds, μδ (with Ar) for μα (41) at H58,13

Crd 143°, seconds, νε (with D, Ar) for κζ (26) at H60,17.
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Now that we have tabulated the chords, our first task, as we said, is to determine the inclination of the ecliptic to the equator, that is, the ratio of the great circle through the poles of both to the arc intercepted between the poles. It is obvious that this is equal to the distance from the equator to either of the solstitial points. This quantity can be determined directly by an instrumental method, using the following simple apparatus.\(^7^0\) [See Fig. C.]

We make a bronze ring of a suitable size, turned on the lathe so that its surface is accurately squared off [i.e. has a rectangular cross-section]. We use this as a meridian circle, by dividing it into the normal 360° of a great circle, and subdividing each degree into as many parts as [the size of the instrument] allows. Then we take another smaller ring, and fit it inside the first in such a manner that its center coincides with the center of the larger circle. This smaller ring is then made to rotate [about the common axis] so that its plane is parallel to the plane of the equator. The inclined plane is then seen through the opening in the large circle, and its position is indicated by the position of the ring. The number of degrees between the two planes is then determined by the number of divisions of the smaller ring that are visible in the opening of the large ring. This number is then multiplied by the number of degrees of the larger circle, and the result gives the inclination of the ecliptic to the equator.

\(^6^9\) On Ptolemy's determination of the obliquity of the ecliptic see Britton[2].

\(^7^0\) On the instruments described by Ptolemy here see Price, Precision Instruments, 587-9. There is a very detailed ancient description of the construction and use of the ring instrument by Proclus, Hypotyposis III 5-27 (ed. Manitius pp. 42-52).
manner that the lateral faces of both are in the same plane, while the smaller ring can rotate freely inside the larger, with a north-south motion, [always] in the same plane. At two diametrically opposite points on one lateral face of the smaller ring we fix [two] little plates, of equal size, pointing towards each other and the centre of the rings, and exactly in the middle of the width of each plate we fix small pointers, which graze the surface of the larger, graduated ring. To serve all the necessary purposes we fix this ring firmly on a pillar of appropriate size, and set it up in the open air, so that the base of the pillar is on a foundation which is not inclined to the plane of the horizon. We take care that the [lateral] plane of the rings is perpendicular to the plane of the horizon and parallel to the plane of the meridian. The first of these [desiderata] is achieved by suspending a plumb-line from a point [on the outer ring] chosen as zenith, and adjusting supporting elements until the plumb-line points towards the point diametrically opposite [the zenith-point]. The second is achieved by marking a meridian line clearly in the plane below the pillar and moving the rings laterally until one can sight their [lateral] plane as parallel to that line. Having set the instrument up in that way, we observed the sun's movement towards the north and south by turning the inner ring at noon until the lower plate was completely enshadowed by the upper one. When this was the case, the tips of the pointers indicated to us the distance of the sun from the zenith in degrees.

We found an even handier way of making this kind of observation by constructing, instead of the rings, a plaque [see Fig. D] of stone or wood, square
and rigid, with one of its faces smooth and accurately squared off. On this we drew a quadrant, using as centre a point near one of the corners, and drew from the centre to the inscribed arc the lines enclosing the right angle forming the quadrant. We divided the arc, as we had [the other instrument], into 90 degrees and subdivisions of those degrees. Next, on that line which was chosen to be perpendicular to the plane of the horizon and towards the south, we fixed two small cylindrical pegs, with their sides at right angles to their bases and exactly circular, machined to be of equal size: one of them we fixed on the centre-point itself, positioning the mid-point of the peg precisely on it, the other at the lower end of the line. Then we set this inscribed face of the plaque up along the meridian line which we had drawn on the foundation-plane, so as to be parallel to the plane of the meridian, and, using a plumb-line suspended between the pegs, set up the line between them precisely at right angles to the plane of the horizon, again correcting any deficiency by adjusting thin supporting elements underneath. In the same way as before, we observed the shadow cast at midday by the peg at the centre. In order to determine its position more accurately, we placed some object on the inscribed arc [where the shadow crossed it]. Marking the mid-point of the shadow, we took that division of the quadrant as indicating the position of the sun on the meridian in the north-south direction.74

From observations of this kind, and especially from comparing observations near the actual solstices, which revealed that, over a number of returns [of the sun], the distance from the zenith was in general the same number of degrees of the meridian circle at the [same] solstice, whether summer or winter, we found that the arc between the northernmost and southernmost points, which is the arc between the solstitial points, is always greater than 47° and less than 47°. From this we derive very much the same ratio as Eratosthenes, which Hipparchus also used. For [according to this] the arc between the solstices is approximately 11 parts where the meridian is 83.75

From the preceding kind of observation it is easy to derive immediately the latitude of the region in which the observation is made, wherever it is: one takes the point halfway between the two extrema; this point lies on the equator; then one takes the distance between this point and the zenith, which is the same, obviously, as the distance of the poles from the horizon.

74κατά πλάτος, literally 'in latitude'. Ptolemy, following common Greek usage, uses πλάτος for any 'vertical' direction, including that normal to the equator, as here. See Introduction p. 21.

75If of $360° = 47°42',29' = 2\epsilon$, hence $\epsilon = 23°31',20''$, which is what Ptolemy actually adopts (his $2\epsilon$ lies between 47°40' and 47°45', but is not the mean). The text could equally well mean, not that Eratosthenes and Hipparchus used the ratio 11:83, but that the ratio 11:83 is Ptolemy's value, which is close to the actual ratio used by them [namely 2:15, i.e. $\epsilon = 24°$]. That interpretation has the advantage of agreeing with the only value otherwise attested for Eratosthenes (in his Geography, see Berger Fr. II B 23, Strabo 2.5.7) and Hipparchus (in his Geography and in his Commentary on Aratus, ed. Manitius p. 96,20; cf. HAMA 303, 335). It was proposed by Berger, Eratosthenes 131, followed by Heath, Aristarchus 131 n. 4. I prefer the traditional interpretation, since I find it inconceivable that Ptolemy would not mention what the ratio was to which his own was close, and also because of his expression at I 14 (p. 70). Eratosthenes' peculiar ratio is due not to a perverse division of the circle into 83rds, as Theon supposes (Rome II 529), but to a pre-trigonometrical derivation from gnomon measurements, as I shall show elsewhere.
13. (Preliminaries for spherical proofs)\textsuperscript{76}

Our next task is to demonstrate the sizes of the individual arcs cut off between the equator and the ecliptic along a great circle through the poles of the equator. As a preliminary we shall set out some short and useful theorems which will enable us to carry out most demonstrations involving spherical theorems in the simplest and most methodical way possible.

H69 [See Fig. 1.8.] Let two straight lines, BE and GD, which are drawn to meet two straight lines, AB and AG, cut each other at point Z.

I say that

\[ \frac{GA}{AE} = \frac{GD}{DZ} \cdot \frac{ZB}{BE}. \]

[Proof:] Let EH be drawn through E parallel to GD. Then, since GD and EH are parallel,

\[ \frac{GA}{AE} = \frac{GD}{EH}. \]

If we bring ZD in [as auxiliary],

\[ \frac{GD}{EH} = \frac{GD}{DZ} \cdot \frac{DZ}{HE}. \]

\[ \therefore \frac{GA}{AE} = \frac{GD}{DZ} \cdot \frac{DZ}{HE}. \]

But \( \frac{DZ}{HE} = \frac{ZB}{BE} \) (EH parallel to ZD).

\[ \therefore \frac{GA}{AE} = \frac{GD}{DZ} \cdot \frac{ZB}{BE}. \]

In the same way, dividendo, we shall prove that

\[ \frac{GE}{EA} = \frac{GZ}{DZ} \cdot \frac{DB}{BA}. \]

\textsuperscript{76} On the spherical trigonometry in this chapter see HAMA 26–30, Pedersen 72–8.

\textsuperscript{77} Literally (here and in general) this kind of ratio is expressed as 'the ratio of GA to AE is combined from (συνήπται εκ, συγκεῖται εκ) the ratio of GD to DZ and the ratio of ZB to BE'.
[See Fig. 1.9.] Draw a line through A parallel to EB and produce GD to cut it at H. Again, since AH is parallel to EZ,
\[ \text{GE:EA} = \text{GZ:ZH}. \]

But, if we bring in ZD [as auxiliary],
\[ \text{GZ:ZH} = \left(\frac{GZ}{ZD}\right) \cdot \left(\frac{DB}{BA}\right). \]

But DZ:ZH = DB:BA (BA and ZH drawn to meet the parallel lines AH and ZB).

\[ \therefore \text{GZ:ZH} = \left(\frac{GZ}{DZ}\right) \cdot \left(\frac{DB}{BA}\right). \]

\[ \text{GE:EA} = \left(\frac{GZ}{DZ}\right) \cdot \left(\frac{DB}{BA}\right). \]  \[\text{[13.2]}\]

Q.E.D.

Again [Fig. 1.10] on circle ABG, with centre D, take any three points A, B, G, on the circumference, provided that each of the arcs AB and BG is less than a semi-circle (let the same condition be understood to apply to all subsequent arcs we take). Draw AG and DEB.

I say that
\[ \text{Crd arc 2AB:Crd arc 2BG} = \text{AE:EG}. \]

[Proof:] Drop perpendiculars AZ and GH from points A and G on to DB. Then, since AZ is parallel to GH, and they meet line AEG,
\[ \text{AZ:GH} = \text{AE:EG}. \]

But AZ:GH = Crd arc 2AB : Crd arc 2BG
(for AZ = \( \frac{1}{2} \) Crd arc 2AB and GH = \( \frac{1}{2} \) Crd arc 2BG).

\[ \therefore \text{AE:EG} = \text{Crd arc 2AB:Crd arc 2BG}. \]  \[\text{[13.3]}\]

Q.E.D.
It immediately follows that if we are given the whole of arc AG and the ratio \((\text{Crd arc } 2\text{AB} : \text{Crd arc } 2\text{BG})\), both arc AB and arc BG will be given.

For, repeating the same figure [see Fig. 1.11], join AD, and drop perpendicular DZ from D onto AEG.

It is obvious that, if arc AG be given, \(\angle ADZ\), which subtends half arc AG, will be given, and hence the whole triangle ADZ.\(^{78}\) Now, since the whole chord AG is

\(^{78}\) For one already knows \(\angle AZD\), a right angle, and AD, a radius.
given, and \((\text{AE:EG})\) is given (for it equals \((\text{Crd arc } 2\text{AB:}\text{Crd arc } 2\text{BG})\)), \(\text{AE}\) will be given,\(^{79}\) and so will \(\text{ZE}\), by subtraction [of \(\text{AZ}\) from \(\text{AE}\)]. Hence, since \(\text{DZ}\) too is given, in the right-angled triangle \(\text{EDZ}\), \(\angle \text{EDZ}\) will be given, and hence the whole angle \(\angle \text{ADB}\). Hence arc \(\text{AB}\) will be given and (by subtraction) arc \(\text{BG}\). Q.E.D.

Again [see Fig. 1.12] on circle \(\text{ABG}\) with centre \(\text{D}\) take three points on the circumference, \(\text{A,B,G}\).\(^{80}\) Join \(\text{DA}\) and \(\text{GB}\) and produce them to meet at \(\text{E}\).

![Fig. 1.12](image)

I say that
\[
\text{Crd arc } 2\text{GA} : \text{Crd arc } 2\text{AB} = \frac{\text{GE}}{\text{EB}}.
\]

By a similar argument to the previous theorem, if we drop perpendiculars \(\text{BZ}\) and \(\text{GH}\) from \(\text{B}\) and \(\text{G}\) on to \(\text{DA}\), since they are parallel,
\[
\frac{\text{GH}}{\text{BZ}} = \frac{\text{GE}}{\text{EB}}.
\]
\[
\therefore \text{Crd arc } 2\text{GA} : \text{Crd arc } 2\text{AB} = \frac{\text{GE}}{\text{EB}}. \tag{13.4}
\]
Q.E.D.

In this case too it follows immediately that if we are given just the arc \(\text{GB}\) and the ratio \((\text{Crd arc } 2\text{GA}:\text{Crd arc } 2\text{AB})\), arc \(\text{AB}\) will also be given.

For, if we repeat the same figure [see Fig. 1.13], and join \(\text{DB}\) and drop \(\text{DZ}\) perpendicular to \(\text{BG}\), then \(\angle \text{BDZ}\), which subtends half arc \(\text{BG}\), will be given. Hence the whole of the right-angled triangle\(^{81}\) \(\text{BDZ}\) will be given. Now, since the ratio \((\text{GE:EB})\) and line \(\text{GB}\) are given, \(\text{EB}\) will be given, and hence, by addition, line \(\text{EBZ}\). So, since \(\text{DZ}\) is given, in the right-angled triangle \(\text{EDZ}\),

\(^{79}\) Euclid \textit{Data} 7 (if a given magnitude is divided in a given ratio, each part is given).

\(^{80}\) Omitting (with \(\text{D, Is}\)), at H72, 13-15, \(\text{ὅστε ἐκατέραν τῶν \text{AB}, \text{ΑΓ} \ περιφερείων ἑλάσσωνα ἐῖναι ἡμικυκλίου. καὶ ἐπὶ τῶν ἐξής δὲ λαμβανομένων περιφερείων τὸ δήμου ὑπακούεσθαι, which is an otiose repetition of H70, 21-5.

\(^{81}\) Here (H74,3) and elsewhere (e.g. H74,7) \(\text{D}\) has the fuller form \(\text{ὅρθογώνιον} \text{τρίγωνον}\) for Heiberg's \(\text{ὅρθογώνιον}\). This may be right, but I have not recorded it as a correction, following the principle enunciated \textit{Introduction} p. 4.
$\angle EDZ$ is given, and, by subtraction [of the given $\angle BDZ$] $\angle EDB$ is given. Hence arc $AB$ will be given.

Having established these preliminary theorems, let us draw [Fig. 1.14] (Fig. 1.14)

the following arcs of great circles on a sphere: $BE$ and $GD$ are drawn to meet $AB$ and $AG$, and cut each other at $Z$. Let each of them be less than a semi-circle (and let the same condition be understood to apply to all the figures).

I say that

$$\text{Crd arc } 2GE: \text{Crd arc } 2EA =$$

$$(\text{Crd arc } 2GZ: \text{Crd arc } 2ZD). (\text{Crd arc } 2DB: \text{Crd arc } 2BA).$$

[Proof:] Let us take the centre of the sphere, $H$, and draw from it to the intersections of the circles, $B, Z, E$, lines $HB, HZ, HE$. Join $AD$ and produce it to meet $HB$, also produced, at $\Theta$. Similarly, join $DG$ and $AG$, and let them cut $HZ$ and $HE$ at points $K$ and $L$.

---

For an adaptation of this figure useful in visualizing the various planes involved see HAMA Fig. 17 p. 1213.

Reading τα . . . σημεία (with D) at H75,2 for τα . . . σημείων. Corrected by Manitius.
Then Θ, K and L lie on a straight line, since they all lie simultaneously in two planes, the plane of triangle AGD, and the plane of circle BZE.

Draw this line [ΘKL]. The result will be that there are two straight lines, ΘL and GD, drawn to meet two straight lines, ΘA and GA, and intersecting each other at K.

\[ GL:LA = (GK:KD):(DΘ:ΘA).\] [from 13.2]

But GL:LA = Crd arc 2GE:Crd arc 2EA [from 13.3]
and GK:KD = Crd arc 2GZ:Crd arc 2ZD [from 13.3]
and DΘ:ΘA = Crd arc 2DB:Crd arc 2BA. [from 13.4]

\[ \therefore \text{Crd arc 2GE:Crd arc 2EA =} \]
\[ (\text{Crd arc 2GZ:Crd arc 2ZD})(\text{Crd arc 2DB:Crd arc 2BA}). \] [13.5] H76

In the same way, corresponding to the straight lines in the plane figure [Fig. 1.8], it can be shown that

Crd arc 2GA:Crd arc 2EA =
\[ (\text{Crd arc 2GD:Crd arc 2DZ})(\text{Crd arc 2DB:Crd arc 2BA}). \] [13.6]

Q.E.D.

14. {On the arcs between the equator and the ecliptic}

Having set out this preliminary theorem, we shall first of all demonstrate the amounts of the arcs we set ourselves to determine, as follows.

[See Fig. 1.15.] Let the circle through both poles, that of the equator and that of the ecliptic, be ABGD; let the semi-circle representing the equator be AEG, and that representing the ecliptic BED, and let point E be the intersection of the two at the spring equinox, so that B is the winter solstice and D the summer solstice. On arc ABG take the pole of the equator AEG: let it be point Z. Cut off arc EH on the ecliptic: let us suppose it to be 30°, and draw through Z and H an arc of a great circle ZHΘ. Our problem, obviously, is to determine HΘ. Let us take for granted both here and in general for all such demonstrations (to avoid repeating ourselves on each occasion), that when we speak of the sizes of arcs or chords in terms of 'degrees' or 'parts' we mean (for arcs) those degrees of which the circumference of a great circle contains 360, and (for chords) those parts of which the diameter of the circle contains 120.

Now since, in the figure, the two great circle arcs ZΘ and EB are drawn to meet the two great circle arcs AZ and AE, and intersect each other at H,

\[ \text{Crd arc 2ZA:Crd arc 2AB =} \]
\[ (\text{Crd arc 2ΘZ:Crd arc 2ΘH})(\text{Crd arc 2HE:Crd arc 2EB}). \] [M.T.1]

The theorem connecting six great circle arcs on the surface of the sphere in a Menelaus Configuration (see Introduction p. 18), of which the enunciations 13.5 and 13.6 are examples, is due to Menelaus, whom Ptolemy mentions in the Almagest only as an observer (see index s.v.). It appears (in both forms) as Prop. III 1 of his Sphaerica (ed. Krause pp. 194–7). These two forms have been labelled by Neugebauer (HAMA 28) as Theorem I (= 13.6), where four inner parts of the Menelaus Configuration are related to two outer parts, and Theorem II (= 13.5), where four outer parts are related to two inner parts. We shall use this terminology in what follows (M.T. I and M.T. II for brevity).

See HAMA 30–1, Pedersen 95–6.

Reference back to I 13 p. 64.
But arc 2ZA = 180°, so Crd arc 2ZA = 120°.
and arc 2AB = 47°42',40" (according to the ratio 11:83, with
which we agreed [p. 63]).

so Crd arc 2AB = 48°31',55".

Again, arc 2HE = 60°, so Crd arc 2HE = 60°.
and arc 2EB = 180°, so Crd arc 2EB = 120°.

\[ \therefore \text{Crd arc 2Z\theta:Crd arc 2\theta H} = (120 : 48°31',55")/(60 : 120) = 120 : 24°15',57". \]

And arc 2Z\theta = 180°, so Crd arc 2Z\theta = 120°.

\[ \therefore \text{Crd arc 2\theta H} = 24°15',57". \]

\[ \therefore \text{arc 2\theta H} = 23°19',59". \]

and arc \theta H = 11°40' (p. 67).

Again, let arc EH be taken as 60°. Then the other magnitudes will remain unchanged, but

\[ \text{arc 2EH} = 120°, \text{so Crd arc 2EH} = 103°55',23". \]

\[ \therefore \text{Crd arc 2Z\theta:Crd arc 2\theta H} = (120 : 48°31',55")/(103°55',23 : 120) = 120 : 42°1',48". \]

But Crd arc 2Z\theta = 120°.

\[ \therefore \text{Crd arc 2\theta H} = 42°1',48". \]

\[ \therefore \text{arc 2\theta H} = 41°0',18". \]

and arc \theta H = 20°30',9".

Q.E.D.

H79 In the same way we shall compute the sizes of [the other] individual arcs, and
set out a table giving for each degree of the quadrant the arc corresponding to
those computed above. The table is as follows.
I 16. Calculation of right ascensions

15. {Table of Inclination}^{87} [See p. 72.]

16. {On rising-times at sphaera recta}^{88}

Our next task is to show how to compute the size of an arc of the equator determined by a circle drawn through the poles of the equator and a given point on the ecliptic. In this way we can find how long, in equinoctial time-degrees, it takes a given section of the ecliptic to cross the meridian at any point on earth and the horizon at sphaera recta (for only in that situation does the horizon pass through the poles of the equator).

Repeat the previous figure [see Fig. 1.16]. Let the ecliptic arc EH again be given, first as 30°. We have to find arc EΘ of the equator.

![Fig. 1.16](image)

By the same argument as the preceding,

\[
\text{Crd arc } 2ZB : \text{Crd arc } 2BA = \\
(\text{Crd arc } 2ZH : \text{Crd arc } 2H\Theta). (\text{Crd arc } 2\Theta E : \text{Crd arc } 2E A). \quad \text{[M.T.II]}
\]

But arc \( 2ZB = 132;17,20° \),

so \( \text{Crd arc } 2ZB = 109;44,53° \).

---

^{87} Corrections to Heiberg in Table I 15:
45°, seconds, α (with D, Ar) for κ (20) at H81,50 (computed: 2).
69°, seconds, α (with D, Ar) for ια (11) at H81,29 (computed: 10,59 for 11,1).
Possible emendations are:
27°, seconds μξ (47) for νξ (57) (computed: 48). No ms. authority.
51°, seconds ε (5) for ιε (15) (computed: 7). No ms. authority.
59°, seconds α (1) for δ (4) (computed: 0). Only variant is '0' in L.

^{88} See HAMA 31-2, Pedersen 97-9.
### TABLE OF INCLINATION

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And arc $2BA = 47;42,40^\circ$, 
so Crd arc $2BA = 48;31,55^\circ$.

Again, arc $2ZH = 156;40,1^\circ$ [180° – arc $2\Theta H$, p. 70]
so Crd arc $2ZH = 117;31,15^\circ$, 
and arc $2\Theta H = 23;19,59^\circ$,
so Crd arc $2\Theta H = 24;15,57^\circ$.

∴ Crd arc $\Theta E$::Crd arc $2EA = (109;44,53 : 48;31,55)/(117;31,15 : 24;15,57)$
= $54;52,26 : 117;31,15 = 56;1,53 : 120$.

But arc $2EA = 180^\circ$, so Crd arc $2EA = 120^\circ$.

∴ Crd arc $2\Theta E = 56;1,53^\circ$.

So arc $2\Theta E \approx 55;40^\circ$ and arc $\Theta E \approx 27;50^\circ$.

Again, let arc $EH$ be taken as 60°. Then the other magnitudes will remain unchanged, but

arc $2ZH = 138;59,42^\circ$, [180° – arc $2\Theta H$, p. 70]
so Crd arc $2ZH = 112;23,56^\circ$.
And arc $2\Theta H = 41;0,18^\circ$.
so Crd arc $2\Theta H = 42;1,48^\circ$.

∴ Crd arc $2\Theta E$::Crd arc $2EA = (109;44,53 : 48;31,55)/(112;23,56 : 42;1,48)$
= $95;2,40 : 112;23,56$
= $101;28,20 : 120$.

But Crd arc $2EA = 120^\circ$.

∴ Crd arc $2\Theta E = 101;28,20^\circ$
∴ arc $2\Theta E \approx 115;28^\circ$.
∴ arc $\Theta E \approx 57;44^\circ$.

Thus it has been shown that the first sign of the ecliptic, counted from the equinox, rises in the aforementioned manner [i.e. at $sphaera recta$] in the same time as 27;50° of the equator; and that the second sign rises with 29;54° (for the sum of both arcs was shown to be 57;44°). It is obvious that the third sign will rise at $sphaera recta$ in the same time as 32;16° (which is the complement of 57;44°), since each whole quadrant of the ecliptic rises in the same time as the corresponding quadrant of the equator as defined by circles drawn through the poles of the equator.

Following the same method as demonstrated above, we calculated the arc of the equator which rises in the same time as each 10-degree section of the ecliptic. (The [true] rising times of arcs smaller than 10° are not noticeably different from those derived by linear interpolation [from those of 10° arcs]). We shall set these too out, then, in order to be able to reckon conveniently the time which each arc takes, as we said, to cross the meridian at any point on earth and the horizon at $sphaera recta$. We begin with the 10° arc starting at [either] equinoctial point.

---

89 Here and just above (H83,13 and 10) Heiberg's text gives 56;1,25 (κθ for ϑ). The correct reading is given by D and Is.
90 From considerations of symmetry, it makes no difference which equinox one starts from.
91 A 'quadrant' here is understood to start at equinox or solstice.
**I 16. Rising-times at sphaera recta**

<table>
<thead>
<tr>
<th>1st</th>
<th>ten-degree section rises in</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td>9;10°</td>
</tr>
<tr>
<td>3rd</td>
<td>9;15°</td>
</tr>
<tr>
<td></td>
<td>9;25°</td>
</tr>
</tbody>
</table>

For 1st sign sum is 27;50°.

| 4th  | 9;40°                       |
| 5th  | 9;58°                       |
| 6th  | 10;16°                      |

For 2nd sign sum is 29;54°.

| 7th  | 10;34°                      |
| 8th  | 10;47°                      |
| 9th  | 10;55°                      |

For 3rd sign, ending at either solstice, sum is 32;16°. The sum for the whole quadrant is 90°, as it should be.\(^2\)

It is immediately obvious that the arrangement [of the rising-times] is the same for the other [three] quadrants, since the same relationships hold in each at sphaera recta, that is when the equator has no inclination to the horizon [i.e. is vertical to it].

---

\(^2\) These data are repeated in tabular form in the table of rising-times, II 8.
1. {On the general location of our part of the inhabited world}

In Book I of our treatise we discussed such preliminary notions about the situation of the universe as had to be summarily disposed of, and such theorems concerning sphaera recta as might be thought useful for the investigations which we propose. In what follows we shall try to develop the more important theorems concerning sphaera obliqua too, in the most convenient way possible.

On that topic, then, we must first make the following general introductory remark. If one considers the earth to be divided into four quarters by the equator and a circle drawn through the poles of the equator, our part of the inhabited world is approximately bounded by one of the two northern quarters. The main proof of this in the case of latitude (that is in the north-south direction) is that the noon shadows of gnomons at equinox always point towards the north and never towards the south. In the case of longitude (that is in the east-west direction) the main proof is that observations of the same eclipse (especially a lunar eclipse) by those at the extreme western and extreme eastern regions of our part of the inhabited world (which occur at the same [absolute] time), never differ by more than twelve equinoctial hours [in local time], and the quarter [of the earth] contains a twelve-hour interval in longitude, since it is bounded by one of the two halves of the equator.

The individual points [concerning sphaera obliqua] which might be considered most appropriate to study for the subject we have undertaken are the more important phenomena which are particular to each of the northern parallels to the equator and to the region of the earth directly beneath each. These are [1] the distance of the poles of the first motion [i.e. the equator] from the horizon, or [in other words] the distance of the zenith from the equator, measured along the meridian;¹

¹So one must translate καθ’ ἣμας οἰκουμένην: καθ’ ἣμας can mean ‘in our neighbourhood’ or ‘in our time’. Manitius takes the expression to be temporal (e.g. here, 58,17 ‘des zurzeit bewohnten Gebietes der Erde’). This implausible interpretation is contradicted by V16 (p. 294) where Ptolemy talks about ‘different parts of the inhabited world’ (ἐπὶ διαφόρους οἰκουμήνες, H498,2), and mentions the ‘so-called antipodes’ (τῶν ἀντιπόδων καλουμένων). In using the expression he is implicitly allowing the possibility of an inhabited zone in the southern hemisphere. On the meaning and history of the concept οἰκουμένη see Campanus 396-7.

²’differ’: literally ‘are earlier or later’.

³One should not infer that Ptolemy possessed records of lunar eclipses observed simultaneously at eastern and western ends of the known world. In fact it seems probable that the only eclipse observed at places widely separated in longitude for which he had records of both observations was that of – 330 Sept. 20 (cf. HAMA 668 n.30), observed at Arbela and Carthage.

⁴In modern terms, the terrestrial latitude, in antiquity usually known as ἔξαρμα τοῦ πόλου, ‘elevation of the pole’.
II 1. Topics to be dealt with

[2] for those regions where the sun reaches the zenith, when and how often this occurs;
[3] the ratios of the equinoctial and solsticial noon shadows to the gnomon;
[4] the size of the difference of the longest and shortest day from the equinoctial day; and all other additional phenomena which are [commonly] studied concerning
[5] the individual increases and decreases in the length of the days and nights;
[6] and the arcs of the equator which rise or set with [given] arcs of the ecliptic;
[7] and the particulars and quantities of angles between the more important great circles.

2. {Given the length of the longest day, how to find the arcs of the horizon cut off between the equator and the ecliptic}

Let us take as a general basis for our examples the parallel circle to the equator through Rhodes, where the elevation of the pole is 36°, and the longest day 14\textperthousand equinoctial hours. Let [Fig. 2.1] ABGD represent the meridian, BED the eastern half of the horizon, AEG, likewise, the [eastern] half of the equator, with its south pole at Z. Let us suppose that the winter solstice on the ecliptic is rising at H. Draw through Z and H the great circle quadrant ZHO.

Fig. 2.1

\[5\text{Details of [1] to [4] are given for numerous parallels in II 6.}\]
\[\text{See II 9.}\]
\[\text{See II 7-8.}\]
\[\text{See II 10-13.}\]
\[\text{On chapters 2 and 3 see H.I.A. 37-8, Pedersen 101-4.}\]
First of all let the length of the longest day be given, and let the problem be to find arc EH of the horizon.\(^\text{10}\)

Now, since the revolution of the [heavenly] sphere takes place about the poles of the equator, it is obvious that points H and Θ will be on the meridian ABGD at the same time. Thus the time from the rising of H to its upper culmination is given by the equatorial arc ΘA, and the time from its lower culmination to its rising is given by [the equatorial arc] GΘ. It follows that the length of daylight is twice the time corresponding to arc ΘA, and the length of night twice the time corresponding to arc GΘ. For every parallel circle to the equator has both sections alike, that above the earth and that below it, bisected by the meridian.

Therefore arc EΘ, which is half the difference between longest or shortest day and equinoctial day, is 11/4° at the parallel in question, or 18;45 time-degrees. Hence its complement, arc ΘA, is 71;15 time-degrees.

Then since, in accordance with the previous theorems, the two great circle arcs EB and ZΘ have been drawn to meet the two great circle arcs AE and AZ, and intersect each other at H,

\[
\text{Crd arc } 2ΘA : \text{Crd arc } 2AE = \left( \text{Crd arc } 2ΘZ : \text{Crd arc } 2ZH \right) \cdot \left( \text{Crd arc } 2BZ : \text{Crd arc } 2ZA \right). \text{ [M.T.I]} \]

But arc 2ΘA = 142.30°,
so Crd arc 2ΘA = 113;37.54°
and arc 2AE = 180°.
so Crd arc 2AE = 120°.

Again, arc 2ΘZ = 180°, so Crd arc 2ΘZ = 120°,
and arc 2ZH = 132;17,20°, so Crd arc 2ZH = 109;44.53°.

\[\therefore \text{Crd arc } 2HB : \text{Crd arc } 2BE = (113;37.54 : 120)/(120 : 109;44.53) = 103;55.26 : 120.\]

But arc 2BE = 120°, since arc BE is a quadrant.
\[\therefore \text{Crd arc } 2HB = 103;55.26°.\text{[11]} \]
\[\therefore \text{arc } 2HB \approx 120°,\]
and arc HB \approx 60°.
\[\therefore \text{arc } HE, \text{its complement, is } 30° \text{ where the horizon is } 360°.\]

Q.E.D.

3. \text{[If the same quantities be given, how to find the elevation of the pole, and vice versa]}\)

Next let the problem be, given the same quantity [i.e. the length of the longest day] again, to find the elevation of the pole, that is arc BZ of the meridian [in Fig. 2.1]. Now, in the same figure,

\[
\text{Crd arc } 2ΘΘ : \text{Crd arc } 2ΘA = \left( \text{Crd arc } 2EH : \text{Crd arc } 2HB \right) \cdot \left( \text{Crd arc } 2BZ : \text{Crd arc } 2ZA \right). \text{ [M.T.II]} \]

\(^{10}\)In modern terms, arc EH is the ortive amplitude of the sun.

\(^{11}\)Here and just above (H92,11 and 8) Heiberg's text gives 103:55.23 (κγ for κξ). The correct reading is given by ACDAr at H92,8 and by all mss. at H92,11. Heiberg prefers the reading '23' because it is given by all mss. at H93,10. But the comparison is illegitimate, since there the amount is taken from the chord table, whereas here it is derived by calculation.
II 3. Computation of φ from M and M from φ

But arc 2EΘ = 37;30°,
so Crd arc 2EΘ = 38;34,22°,
and arc 2ΘA = 142;30°,
so Crd arc 2ΘA = 113;37,54°.

Furthermore arc 2EH = 60°,
so Crd arc 2EH = 60°,
and arc 2HB = 120°,
so Crd arc 2HB = 103;55,23°.

\[ \text{Crd arc } 2BZ : \text{Crd arc } 2ZA = \left(\frac{38;34,22}{113;37,54}\right)/\left(\frac{60}{103;55,23}\right) \approx 70;33 : 120. \]

And again, Crd arc 2ZA = 120°,
so Crd arc 2BZ = 70;33°.

\[ \text{arc } 2BZ = 72;1° \quad \text{and arc } BZ \approx 36°. \]

To do the reverse, in the same figure again [Fig. 2.1] let BZ, the arc of the pole’s elevation, be given, having been observed to be 36°. Let the problem be to find the difference between the shortest or longest day and the equinoctial day, i.e. arc 2EΘ.

Now, from the same considerations,
Crd arc 2ZB:Crd arc 2BA =
(Crd arc 2ZH:Crd arc 2ΘH) \cdot (Crd arc 2ΘE:Crd arc 2EA). \quad [\text{M.T.II}]

But arc 2ZB = 72°,
so Crd arc 2ZB = 70;32,3°.
and arc 2BA = 108°,
so Crd arc 2BA = 97;4,56°.

Furthermore arc 2ZH = 132;17,20°,
so Crd arc 2ZH = 109;44,53°,
and arc 2ΘH = 47;42,40°,
so Crd arc 2ΘH = 48;31,55°.

\[ \text{Crd arc } 2ΘE : \text{Crd arc } 2EA = \left(\frac{70;32,3}{97;4,56}\right)/\left(\frac{109;44,53}{48;31,55}\right) \approx 38;34 : 120. \]

But Crd arc 2EA = 120°,
\[ \text{Crd arc } 2EΘ = 38;34°. \quad \therefore \text{arc } 2EΘ \approx 37;30°, \text{ or } 2\frac{1}{2} \text{ equinoctial hours}.^{12} \]

Q.E.D.

In the same way arc EH of the horizon can be determined. For
Crd arc 2ZA:Crd arc 2AB =
(Crd arc 2ZΘ:Crd arc 2ΘH) \cdot (Crd arc 2HE:Crd arc 2EB), \quad [\text{M.T.I}]
and (Crd arc 2ZA:Crd arc 2AB) is a given ratio,
and so is (Crd arc 2ZΘ:Crd arc 2ΘH),
so, since arc EB is given, so is the amount of arc EH.

It is obvious that if we suppose H to be, instead of the place of the winter solstice, any other degree of the ecliptic, by similar reasoning both of the arcs

\[^{12}\text{There has been selective rounding at different stages of this calculation to achieve this nice result. Accurate calculation of arc } 2EΘ \text{ would give (to the nearest minute) } 37;29°.\]
EΘ and EH will be given, since we have already set out, in the 'Table of Inclination', the arc of the meridian intercepted between ecliptic and equator for every degree of the ecliptic: this arc\textsuperscript{13} corresponds to HΘ [in Fig. 2.1].

It immediately follows that points on the ecliptic cut by the same parallel circle, i.e. points equidistant from the same solstice, cut off [between ecliptic and equator] arcs of the horizon which are equal and on the same side of the equator. They also make the length of the day equal to that of the day [at the corresponding point], and the length of the night equal to that of the [corresponding] night.

It likewise follows that points [on the ecliptic] cut by equal parallel circles, that is points equidistant from the same equinox, cut off arcs of the horizon which are equal, but on opposite sides of the equator. They also make the length of the day equal to the length of the night at the opposite [corresponding] point, and the length of the night equal to that of the [corresponding] day.

For, in the figure already drawn [see Fig. 2.2], we put K as the point in which the parallel circle equal to the parallel through H cuts the semi-circle BED of the horizon; we draw in arcs HL and KM of the parallel circles: these will, clearly, be equal and opposite. We draw through K and the north pole the [great circle] quadrant NKX. Then

\[ \text{arc } \Theta A = \text{arc } XG \text{ (arc } \Theta A \parallel \text{ arc } LH, \text{ and arc } XG \parallel \text{ arc } MK) \].

\[ \therefore \text{arc } E\Theta = \text{arc } EX \text{ (complements [of arc } \Theta A \text{ and arc } XG])} \].

Then, in the two similar spherical triangles\textsuperscript{14} EHΘ and EKX we have two pairs of corresponding sides equal. EΘ to EX, and HΘ to KX,\textsuperscript{15} and both of the angles at Θ and X are right, so the base EH equals the base KE.

\[ \text{Reading προεκτεθεμένων } (\text{with D) for προεκτεθεμένων at H95.18, and περιεφερείων } (\text{with DL, adopted by Manitius), for περιεφερεία at H95.22.} \]
II 4. How to compute when the sun reaches zenith

4. \{How to compute for what regions, when, and how often the sun reaches the zenith\}\(^\text{16}\)

Once the above quantities are given, it is a straightforward computation to determine for what regions, when, and how often the sun reaches the zenith. For it is immediately obvious that for those beneath a parallel which is farther away from the equator than the 23.51,20° (approximately), which represents the distance of the summer solstice [from the equator], the sun never reaches the zenith at all, while for those beneath the parallel which is exactly that distance [from the equator], it reaches the zenith once [a year], precisely at the summer solstice. It is furthermore clear that for those beneath a parallel less far [from the equator] than the above-mentioned amount the sun reaches the zenith twice [a year]. The time when this happens is readily supplied from the Table of Inclination which we have set out [I 15]. For we take the distance from the equator, in degrees, of the parallel in question (which must, obviously, lie within the [parallel of the] summer solstice), and enter with it the second set of columns; we take the corresponding argument, in degrees from 1° to 90°, in the first set of columns; this gives us the distance of the sun from each of the equinoxes towards the summer solstice when it is in the zenith for those beneath the parallel in question.

5. \{How one can derive the ratios of the gnomon to the equinoctial and solsticial noon shadows from the above-mentioned quantities\}\(^\text{17}\)

The required ratios of shadow to gnomon\(^\text{18}\) can be found quite simply once one is given the arc between the solstices and the arc between the horizon and the pole; this can be shown as follows.

[See Fig. 2.3.] Let the meridian circle be ABGD, on centre E. Let A be taken as the zenith, and draw the diameter AEG. At right angles to this, in the plane of the meridian, draw GKZN: clearly, this will be parallel to the intersection of horizon and meridian. Now, since the whole earth has, to the senses, the ratio of a point and centre to the sphere of the sun, so that the centre E can be considered as the tip of the gnomon, let us imagine GE to be the gnomon, and line GKZN to be the line on which the tip of the shadow falls at noon. Draw through E the equinoctial noon ray and the [two] solsticial noon rays: let BEDZ represent the equinoctial ray, HEΘK the summer solsticial ray, and LEMN the winter solsticial ray. Thus GK will be the shadow at the summer solstice, GZ the equinoctial shadow, and GN the shadow at the winter solstice.

Then, since arc GD, which is equal to the elevation of the north pole from the horizon, is 36° (where meridian ABG is 360°) at the latitude in question, and

\(^{16}\)The word Ptolemy uses for 'spherical triangle', τριγωνον, was, according to Pappus, Synagoge VI 2, Hultsch p. 476, 16-7, the term used by Menelaus.

\(^{17}\)Arc HΘ = arc KX because they are the declinations of points equidistant from an equinox.

\(^{18}\)See Pedersen 104-5 and Appendix A, Example 1a.
both arc $\Theta D$ and arc $DM$ are $23;51,20^\circ$, by subtraction arc $G\Theta = 12;8,40^\circ$, and by addition arc $GM = 59;51,20^\circ$.

Therefore the corresponding angles

$\angle KEG = 12;8,40^\circ$

$\angle ZEG = 36^\circ$  

$\angle NEG = 59;51,20^\circ$  

and

$\angle KEG = 24;17,20^\circ$  

$\angle ZEG = 72^\circ$  

$\angle NEG = 119;42,40^\circ$  

Therefore in the circles about right-angled triangles $KEG$, $ZEG$, $NEG$, $\text{H100}$

arc $GK = 24;17,20^\circ$

and arc $GE = 155;42,40^\circ$ (supplement),

arc $GZ = 72^\circ$

and arc $GE = 108^\circ$, similarly [as supplement],

arc $GN = 119;42,40^\circ$

and arc $GE = 60;17,20^\circ$ (again as supplement).

Therefore where $\text{Crd arc } GK = 25;14,43^\circ$, $\text{Crd arc } GE = 117;18,51^\circ$,

and where $\text{Crd arc } GZ = 70;32,4^\circ$, $\text{Crd arc } GE = 97;4,56^\circ$,

and where $\text{Crd arc } GN = 103;46,16^\circ$, $\text{Crd arc } GE = 60;15,42^\circ$.

Therefore, where the gnomon $GE$ has $60^\circ$, in the same units

the summer [solstitial] shadow, $GK \approx 12;55^\circ$,

the equinoctial shadow, $GZ \approx 43;36^\circ$

and the winter [solstitial] shadow, $GN \approx 103;20^\circ$.

The chord table gives, for $72^\circ$, $70;32,3^\circ$ (wrongly changed to $70;32,4^\circ$ by Heiberg on the basis of this passage). All mss. (including the Arabic tradition, except for Gerard, who has 3) have 4 here. The inconsistency probably goes back to Ptolemy. It has no effect on the final result. Cf. p. 93.
It is immediately clear that the reverse process is possible. That is, provided only that any two of the three above ratios of the gnomon GE to the shadow be given, the elevation of the pole and the arc between the solstices are determined. For if any two of the angles at E are given, so is the third, since arcs $\Theta D$ and $D M$ are equal. However, in so far as accuracy of the observation is concerned, the former quantities [elevation of the pole and $2\varepsilon$] can be exactly determined in the way we explained; but the ratios of the shadows in question to the gnomon cannot be determined with equal accuracy, since the moment of the equinoxes is, in itself, somewhat indeterminate, and the tip of the shadow at winter solstice is hard to discern.

6. *Exposition of the special characteristics, parallel by parallel*\(^{20}\)

By the same method we also found the above-mentioned general characteristics for the other parallels [to the equator]. We calculated for latitudes at intervals of 1-hour [of longest daylight], considering that sufficient. Before we deal with particulars,\(^{21}\) we shall set out these general characteristics.

1. We begin with the parallel beneath the equator itself, which forms, approximately, the southern boundary of the [earth's] quarter which comprises our part of the inhabited world. This is the only parallel which has every day equal to every night, since only in that case [i.e. at the equator] are all parallel circles bisected by the horizon, so that every section above the earth is an arc of the same size, and is equal to the corresponding section below the earth. This does not occur at any other latitude:\(^{22}\) [elsewhere] only the equator is bisected at every place on earth by the horizon, so that it makes the night sensibly equal to the day [when the sun is] in it. For the equator too is a great circle. All the other [parallels] are divided [by the horizon] into unequal parts.\(^{23}\) As the sphere is inclined in our part of the inhabited world, parallels south of the equator make the sections above the earth smaller than those below the earth, and the days shorter than the nights, while the northern [parallels], on the contrary, make the sections above the earth larger, and the days longer.

This parallel [of the equator] also has the shadow going both ways:\(^{24}\) the sun

\(^{20}\) The information given in this chapter is a gesture towards the traditional topics of Hellenistic geography. Most of it is irrelevant to the rest of the Almagest and is never mentioned or used again. In particular, the definition of latitude by the gnomon-shadow ratio at equinox or solstices is known to have been much used in earlier works (see HAMA II 746-8), and, to judge from Sanskrit astronomical works, had important applications in earlier Hellenistic astronomy, but is a mere fossil in the Almagest (although Ptolemy probably introduced the norm of 60" for the gnomon).

\(^{21}\) By ‘particulars’ he refers to rising-times at *sphaera obliqua* and other matters treated in the latter part of Book II.

\(^{22}\) At any other latitude: literally ‘at any of the inclinations’. See Introduction p. 19.

\(^{23}\) Proved Theodosius, *Sphaerica* II 19.

\(^{24}\) δυρόσκιος, meaning that the noon shadow is to the south for part of the year. This term, and the corresponding ἐτρόσκιος and περίοσκιος (see p. 85 n.36 and p. 89 n.67) were used by Posidonius (early first century B.C.) in his geographical work (Edelstein-Kidd frs. 49, 44-8 and 208) as reported
comes into the zenith twice [a year] for those living beneath it, when it reaches
the intersections of ecliptic and equator; only at those [two times] do the
gnomons cast no shadow at noon; while the sun is traversing the northern semi-
circle [of the ecliptic] the shadows of the gnomons point towards the south, and
while it is traversing the southern semi-circle they point towards the north. In
that region a gnomon of $60^\circ$ has a shadow of $26^\circ$ at both summer and winter
solstices. (When we say 'shadow' we mean, in general, the noon shadow; it
makes no significant difference that the equinoxes and solstices do not, in
general, take place exactly at noon.)

For those who live beneath the equator those stars come into the zenith which
revolve on the equator itself, but all stars are seen to rise and set, since the poles
of the sphere are exactly on the horizon, and thus it is impossible for any of the
parallel circles to appear always visible or always invisible, or for any meridian
to be a colure [i.e. always partly invisible]. It is said that the regions beneath
the equator could be inhabited, since the climate must be quite temperate. For
the sun does not stay long in the neighbourhood of the zenith, since its motion in
declination is swift round about the equinoctial points, and hence the summer
would be temperate; furthermore, it is not very far from the zenith at the
solstices, so the winter would not be harsh. But what these inhabited regions are
we have no reliable grounds for saying. For up to now they are unexplored by
men from our part of the inhabited world, and what people say about them must
be considered guesswork rather than report. In any case, such, in sum, are the
characteristics of the parallel beneath the equator.

As for the other parallels, which, according to some authorities, comprise the
inhabited regions, we shall make the following general observations, to avoid
repeating ourselves in every case. For each of them in order those stars come
into the zenith whose distance from the equator, measured along the circle
through the poles of the equator, is equal to the distance of the parallel in question
[from the equator]. Furthermore the circle which has the north pole of the
equator as its pole, and the elevation of the pole [at that parallel] as its radius, is
always visible, and all stars within that circle are always visible. [Likewise], the
circle which has the south pole as its pole, and the same radius [as the former], is
always invisible, and the stars within it are always invisible.

2. The second is the parallel with a longest day of $12\frac{1}{2}$ equinoctial hours. This is
$4^\circ$ from the equator, and passes through the island Taprobane. 26 This too is one
of the parallels with the shadow going both ways: the sun comes into the zenith
for those beneath it twice [a year], and makes the gnomons shadowless at noon,
when it is $79^\circ$ distant from the summer solstice on either side. Thus while it is
traversing these $15^\circ$, the gnomon shadows point towards the south; and while

by Strabo 2.2.3 and 2.5.43. Whether Posidonius actually coined the terms, as Strabo implies
(ἐξάλεξον, wrongly denied by me, Toomer[3] 146) seems improbable, but we have no earlier
attestation.

25 On this term see Introduction p. 19.

26 Ceylon. For this and the rest of the geographical data in this chapter help is provided by
Kiepert's reconstruction of Ptolemy's world map, 'Orbis Terrarum secondum Cl. Ptolemaeum',
Formae Orbis Antiquae no. XXXVI, 1911.
it is traversing the other 201°, they point towards the north. In this region, for a
gnomon of 60°, the equinoctial shadow is 42°, the summer [solstitial] shadow
21°, and the winter [solstitial] shadow 39°.

3. The third is the parallel with a longest day of 12\(\frac{1}{2}\) equinoctial hours. This is
8°25' from the equator and goes through the Avalite gulf.\(^{27}\) This too is one of the
parallels with the shadow going both ways: the sun comes into the zenith for
those beneath it twice [a year], and makes the gnomons shadowless at noon,
when it is 69° distant from the summer solstice on either side. Thus while it is
traversing these 138°, the gnomon shadows point towards the south; and while
it is traversing the other 222°, they point towards the north. In this region, for a
gnomon of 60°, the equinoctial shadow is 85°, the summer [solstitial] shadow
16\(\frac{2}{3}\)°,\(^{28}\) and the winter [solstitial] shadow 37\(\frac{9}{10}\).

4. The fourth is the parallel with a longest day of 12\(\frac{1}{2}\) equinoctial hours. This is
12\(\frac{1}{2}\)° from the equator, and goes through the Adulitic gulf.\(^{29}\) This too is one of the
parallels with the shadow going both ways: the sun comes into the zenith twice [a year] for those beneath it, and makes the gnomons shadowless at noon,
when it is 57° from the summer solstice on either side. Thus while it is
traversing these 115° the gnomon shadows point towards the south, and while
it is traversing the remaining 244° they point towards the north. In this region,
for a gnomon of 60°, the equinoctial shadow is 13\(\frac{1}{2}\), the summer [solstitial] shadow 12°, and the winter [solstitial] shadow 44\(\frac{1}{2}\).

5. The fifth is the parallel with a longest day of 13 equinoctial hours. This is
16:27° from the equator, and goes through the island of Meroe.\(^{30}\) This too is one of the
parallels with the shadow going both ways: the sun comes into the zenith for
those beneath it twice [a year], and makes the gnomons shadowless at noon,
when it is 45° from the summer solstice on either side. Thus while it is traversing
these 90° the gnomon shadows point towards the south, and while it is
traversing the remaining 270° they point towards the north. In this region,
for a gnomon of 60°, the equinoctial shadow is 17\(\frac{2}{3}\), the summer [solstitial] shadow
7\(\frac{3}{4}\), and the winter [solstitial] shadow 51°.\(^{31}\)

6. The sixth is the parallel with a longest day of 13\(\frac{1}{2}\) equinoctial hours. This is

\(^{27}\) Avalites was a trading-post on the African coast just outside the mouth of the Red Sea. It is
identified with the mediaeval and modern Zeila, just south of Djibouti. The 'Avalite gulf' is surely
the nearby Gulf of Tajura, rather than the Gulf of Aden, as asserted by Tomaschek (R-E s.v.
Avalites).

\(^{28}\) Reading \(\gamma'\) (with IS) for \(\gamma\) (16°) at H105.13. Computed: 16:34:28.

\(^{29}\) Adulis or Adulis was a town on the Aethiopic coast of the Red Sea. The gulf is the modern
Gulf of Zula (formerly Annesley Bay).

\(^{30}\) Meroe is not an island in the modern sense, but was so called by the Greek geographers because
it was roughly bounded by the rivers Nile, Atbara (ancient Astaboras), Blue Nile (ancient Astopus)
and possibly some of their tributaries. Cf. Ptolemy, Geography IV 7 20 (\(\nu\gamma\sigma\sigma\rho\sigma\pi\omicron\omicron\upsilon\tau\alpha\upsilon\) Meroe,
bounded by Nile to the west and Astaboras to the east), and the confused account of Strabo, 17.2.2.

\(^{31}\) Computed: 50:53:4. 51 is probably correct as a rounding to the nearest whole number, but one
might consider D's 50:51 or T's 50 (H106.18).
II 6. Characteristics of parallels $M = 13\frac{1}{2}$ to 14

20;14° from the equator, and goes through Napata. This too is one of the parallels with the shadow going both ways: the sun comes into the zenith for those beneath it twice [a year], and makes the gnomons shadowless at noon, when it is 31° from the summer solstice on either side. Thus while it is traversing these 62° the gnomon shadows point towards the south, and while it is traversing the remaining 298° they point towards the north. In this region, for a gnomon of 60°, the equinoctial shadow is 22°, the summer [solstitial] shadow 31°, and the winter [solstitial] shadow 58°.

7. The seventh is the parallel with a longest day of 13° equinoctial hours. This is 23;51° from the equator and goes through Soene. This is the first of the so-called 'one-way-shadow' parallels. For in this region the noon shadows of the gnomon never point towards the south. Only at the actual summer solstice does the sun come into the zenith for those beneath this parallel, so that the gnomons appear shadowless. For they are exactly the same distance from the equator as the summer solstice is. At every other time the shadows of the gnomons point towards the north. In this region, for a gnomon of 60°, the equinoctial shadow is 26°, the winter [solstitial] shadow is 65°, and the summer [solstitial] shadow is zero. Furthermore, all parallels north of this up to the northern boundary of our part of the inhabited world have the shadows going one way. For in those regions the gnomons at noon neither become shadowless nor point their shadows towards the south: they always point them towards the north, since the sun never comes into the zenith for them, either.

8. The eighth is the parallel with a longest day of 13° equinoctial hours. This is 27;12° from the equator, and goes through Ptolemais in the Thebaid, which is called Ptolemais Hermeiou. In this region, for a gnomon of 60°, the summer [solstitial] shadow is 3°, the equinoctial shadow 30°, and the winter [solstitial] shadow 74°.

9. The ninth is the parallel with a longest day of 14 equinoctial hours. This is 30;22° from the equator, and goes through lower Egypt. In this region, for a gnomon of 60°, the summer [solstitial] shadow is 6°, the equinoctial shadow 35°, and the winter [solstitial] shadow 83;12°.

32 Napata is the modern Gebel Barkal, near Merowe in the Sudan.
33 Computed: 22;6.7 for the equinoctial shadow, and 58;5.55 for the winter solstitial shadow. One would expect 6 instead of 5 in both places. Perhaps one should interpret ζ' as ζ, i.e. 6 minutes, but this would normally be written as an aliquot fraction (1').
34 Computed: 23;48.20. The discrepancy is interesting, because it is due, not to rounding, but to the desire to make the parallel with $M = 13\frac{1}{2}$ exactly coincide with the parallel with a latitude equal to the obliquity of the ecliptic, i.e. where the sun is in the zenith at summer solstice. The difference is negligible, but instead of saying so Ptolemy fudges the result.
35 Also known as Syene: the modern Assuan in upper Egypt.
36 Κερόσκιος, the opposite of Διφλάσκιος; see p. 82 n.24.
37 Literally 'shadowless'.
38 Reading ξ' ζ' γ' (with D, Is) for ζζ' ζ' γ' (36') at H108.13. Computed: 30;48.36.
39 Reading Ψυ' β' (with L) for Ψυ' β' (i.e. 12 minutes instead of 1°) at H108.20. Computed: 83;10.39. Ptolemy does not often use the aliquot fraction ε' (1).
II 6. Characteristics of parallels $M = 14 \frac{1}{2}$ to $15 \frac{1}{2}$

10. The tenth is the parallel with a longest of $14 \frac{1}{2}$ equinoctial hours. This is $33;18^\circ$ from the equator, and goes through the middle of Phoenicia. In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $10^\circ$, the equinoctial shadow $39^\circ$, and the winter [solstitial] shadow $93\frac{1}{2}^\circ$.\(^{40}\)

11. The eleventh is the parallel with a longest day of $14 \frac{1}{2}$ equinoctial hours. This is $36^\circ$ from the equator, and goes through Rhodes. In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $12\frac{1}{2}^\circ$, the equinoctial shadow $43\frac{1}{2}^\circ$,\(^{41}\) and the winter [solstitial] shadow $103\frac{1}{2}^\circ$.

12. The twelfth is the parallel with a longest day of $14 \frac{1}{2}$ equinoctial hours. This is $38;35^\circ$ from the equator, and goes through Smyrna. In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $18\frac{1}{2}^\circ$, the equinoctial shadow $52\frac{1}{2}^\circ$, and the winter [solstitial] shadow $127\frac{1}{2}^\circ$.

13. The thirteenth is the parallel with a longest day of $15$ equinoctial hours. This is $40;56^\circ$ from the equator, and goes through the Hellespont. In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $18\frac{1}{2}^\circ$, the equinoctial shadow $52\frac{1}{2}^\circ$, and the winter [solstitial] shadow $127\frac{1}{2}^\circ$.\(^{42}\)

14. The fourteenth is the parallel with a longest day of $15 \frac{1}{2}$ equinoctial hours. This is $43;1^\circ$ from the equator, and goes through Massalia.\(^{43}\) In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $20\frac{1}{2}^\circ$, the equinoctial shadow $55\frac{1}{2}^\circ$,\(^{44}\) and the winter [solstitial] shadow $140\frac{1}{2}^\circ$.

15. The fifteenth is the parallel with a longest day of $15 \frac{1}{2}$ equinoctial hours. This is $45;1^\circ$ from the equator, and goes through the middle of Pontus.\(^{45}\) In this region, for a gnomon of $60^\circ$, the summer [solstitial] shadow is $23\frac{1}{2}^\circ$, the equinoctial shadow $60^\circ$, and the winter [solstitial] shadow $155\frac{1}{2}^\circ$.\(^{46}\)

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\(^{40}\) All the values for the shadow at this parallel are rather inaccurate. For $M = 14 \frac{1}{2}$ one finds $9;57;43$, $39;23;11$ and $92;52;51$. Ptolemy's figures fit a latitude of $33\frac{1}{2}$ much better.

\(^{41}\) Reading $\phi^2 \gamma^2 \varepsilon^2$ (with Ar) for $\phi^2 \gamma^2 \delta^2$ at $H109.9$. Corrected by Manitius. Cf. 43;36 at II 5 p. 81.

\(^{42}\) There is a strange discrepancy here. For $M = 15^\circ$, one finds $\varphi = 40;52;21^\circ$. However, the shadow lengths fit neither $M = 15^\circ$ nor $\varphi = 40;56^\circ$, but $\varphi = 41^\circ$. Computations:

<table>
<thead>
<tr>
<th>$M = 15^\circ$</th>
<th>$\varphi = 40;56^\circ$</th>
<th>$\varphi = 41^\circ$</th>
<th>text</th>
</tr>
</thead>
<tbody>
<tr>
<td>summer shadow</td>
<td>18;21;47</td>
<td>18;25;58</td>
<td>18;30</td>
</tr>
<tr>
<td>equinoctial shadow</td>
<td>51;55;23</td>
<td>52;2;5</td>
<td>52;9;26</td>
</tr>
<tr>
<td>winter shadow</td>
<td>127;5;30</td>
<td>127;26;32</td>
<td>127;49;41</td>
</tr>
</tbody>
</table>

The parallel through the Hellespont is Clima V in the traditional '7 climata' (see Introduction p. 19). Possibly, an older round number for the latitude underlies Ptolemy's values here.

\(^{43}\) Reading $\mu^2 \delta^2$ for $\mu^2 \theta^2$ (43;4) at $H110.3$. Although not supported by any ms. reading (Ar has $43\frac{1}{2}$), $43;1$ is confirmed by the values for the shadow lengths. Furthermore, $\gamma^2$ would normally be written as an aliquot fraction, $\gamma^2$ (but cf. $H111.6$ where $50;4$ is certainly correct, and is written $\nu \delta^2$. i.e. $50;4$ and not $50;4$).

\(^{44}\) Modern Marseilles.

\(^{45}\) Reading $\mu^2 \delta^2$ (with BCIs) for $\mu^2 \delta$ (144) at $H110.6$. Computed: 140;31.31. One might also consider $\mu^2 \delta$ (141), as a rounding to the nearest whole number, but this has no ms. support.

\(^{46}\) The Black Sea.

\(^{47}\) Computed: 155;10.32. Possibly one should read 155;12 (with L, $\gamma^2$ for $\tau^2$). Cf. p. 85 n.39.
16. The sixteenth is the parallel with a longest day of 15\frac{1}{2} equinoctial hours. This is 46;51° from the equator and goes through the sources of the river Istros.\textsuperscript{48} In this region, for a gnomon of 60°, the summer [solsticial] shadow is 25\frac{1}{2}°, the equinoctial shadow 63\frac{1}{2}°, and the winter [solsticial] shadow 171\frac{1}{2}°.

17. The seventeenth is the parallel with a longest day of 16 equinoctial hours. This is 48;32° from the equator, and goes through the mouths of the Borysthenes.\textsuperscript{49} In this region, for a gnomon of 60°, the summer [solsticial] shadow is 271\frac{1}{2}°, the equinoctial shadow 67\frac{1}{2}°, and the winter [solsticial] shadow 188\frac{1}{2}°.\textsuperscript{50}

18. The eighteenth is the parallel with a longest day of 16\frac{1}{2} equinoctial hours. This is 50;4° from the equator, and goes through the middle of the Maiotic lake.\textsuperscript{51} In this region, for a gnomon of 60°, the summer [solsticial] shadow is 29\frac{1}{2}°, the equinoctial shadow 71\frac{1}{2}°, and the winter [solsticial] shadow 208\frac{1}{2}°.\textsuperscript{52}

19. The nineteenth is the parallel with a longest day of 16\frac{1}{4} equinoctial hours. This is 51\frac{3}{4}° from the equator and goes through the southernmost parts of Brittania. In this region, for a gnomon of 60°, the summer [solsticial] shadow is 31\frac{1}{2}°, the equinoctial shadow 75\frac{1}{2}°, and the winter [solsticial] shadow 229\frac{1}{2}°.\textsuperscript{53}

20. The twentieth is the parallel with a longest day of 16\frac{1}{2} equinoctial hours. This is 52;50° from the equator, and goes through the mouths of the Rhine. In this region, for a gnomon of 60°, the summer [solsticial] shadow is 33\frac{1}{2}°, the equinoctial shadow 79\frac{1}{2}°, and the winter [solsticial] shadow 253\frac{1}{2}°.\textsuperscript{54}

21. The twenty-first is the parallel with a longest day of 17 equinoctial hours. This is 54;1° from the equator,\textsuperscript{55} and goes through the mouths of the Tanais.\textsuperscript{56} In this region, for a gnomon of 60°, the summer [solsticial] shadow is 34\frac{1}{2}°, the equinoctial shadow 82\frac{1}{2}°, and the winter [solsticial] shadow 278\frac{1}{2}°.\textsuperscript{57}

\textsuperscript{48}The Danube.
\textsuperscript{49}The modern river Dnieper.
\textsuperscript{50}These shadow lengths accord better with a latitude of 48\frac{1}{2}°. However, \(\varphi = 48;32°\) is abundantly attested for this parallel, which is Clima VII of the 7 climata. There are variants 188\frac{1}{2} (T) and 188\frac{3}{8} (= 188:38, L) for the winter shadow. Computed: 188:44:49.
\textsuperscript{51}Modern Sea of Azov.
\textsuperscript{52}Modern Sea of Azov.
\textsuperscript{53}Modern Sea of Azov.
\textsuperscript{54}Modern Sea of Azov.
\textsuperscript{55}Modern Sea of Azov.
\textsuperscript{56}Modern Sea of Azov.
\textsuperscript{57}Modern Sea of Azov.

For the great error in the latitude assigned to this region here and in the Geography see Toomer[3] 148.
22. The twenty-second is the parallel with a longest day of $17\frac{1}{2}$ equinoctial hours. This is $55^\circ$ from the equator and goes through Brigantium in Great Brittania. In this region, for a gnomon of $60^\circ$, the summer [solsticial] shadow is $36\frac{1}{4}''$, the equinoctial shadow is $85\frac{3}{4}''$, and the winter [solsticial] shadow is $304\frac{1}{4}''$.

23. The twenty-third is the parallel with a longest day of $17\frac{1}{2}$ equinoctial hours. This is $56^\circ$ from the equator, and goes through the middle of Great Brittania. In this region, for a gnomon of $60^\circ$, the summer [solsticial] shadow is $37\frac{3}{4}''$, the equinoctial shadow is $92\frac{3}{4}''$, and the winter [solsticial] shadow is $335\frac{3}{4}''$.

24. The twenty-fourth is the parallel with a longest day of $17\frac{1}{2}$ equinoctial hours. This is $57^\circ$ from the equator, and goes through Caturactonium in Brittania. In this region, for a gnomon of $60^\circ$, the summer [solsticial] shadow is $39\frac{1}{2}''$, the equinoctial shadow is $96''$, and the winter [solsticial] shadow is $372''$.

25. The twenty-fifth is the parallel with a longest day of $18$ equinoctial hours. This is $58^\circ$ from the equator and goes through the southern part of Little Brittania. In this region, for a gnomon of $60^\circ$, the summer [solsticial] shadow is $40''$, the equinoctial shadow is $96''$, and the winter [solsticial] shadow is $419\frac{1}{4}''$.

26. The twenty-sixth is the parallel with a longest day of $18\frac{1}{2}$ equinoctial hours. This is $59^\circ$ from the equator, and goes through the middle of Little Brittania. From here on we no longer used $\frac{1}{2}$-hour increments, since [at intervals of $\frac{1}{2}$-hour for the longest daylight] the parallels are now close together, and the difference in the elevation of the pole is no longer as much as a whole degree. Furthermore, for the points even further north there is not the same need for detail. Hence we considered it superfluous to list the ratios of the shadows to the gnomon, as if it were for some well-defined place.

27. The parallel where the longest day is $19$ equinoctial hours is $61^\circ$ from the equator and goes through the northern parts of Little Brittania.

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58 Computed: 55;7,16. From here on the roundings become much more drastic.
59 By 'Great Brittania' and 'Little Brittania' Ptolemy refers to the two principal islands of the British isles, namely modern 'Great Britain' (England, Wales and Scotland) and Ireland. None of the places called Brigantium were in Britain. However, there was in Britain a tribe of Brigantes, whose kingdom was sometimes known as Brigantia (which was further to the north than this latitude would imply). Ptolemy presumably made an error here. He seems to have corrected it by the time he came to write the Geography, which does mention the Brigantes, but no Brigantium in Britain.
60 Modern Catterick in Yorkshire. The usual Latin form is 'Cataractonium'.
61 Reading $\lambda\beta\cdot\gamma'$ (with D, Is) for $\lambda\beta\cdot\gamma'$ (394) at H113.4. Computed for $\varphi = 57^\circ$: 39;10,48.
62 Reading $\tau\delta\cdot\tau\beta'$ (with B1D2, Ar) for $\tau\delta\cdot\tau\beta'$ (3724) at H113.5. Computed: for $\varphi = 59^\circ$: 372;4,27.
63 Ireland: see above n.59.
64 Computed for $\varphi = 58^\circ$: 419;15,1. Perhaps one should emend to $419\frac{1}{2}$ (5' for $\tau\beta'$ at H113,11). Cf. '1191', Ger.
28. The parallel where the longest day is 19½ equinoctial hours is 62° from the equator and goes through the islands called ‘Eboudae’.

29. The parallel where the longest day is 20 equinoctial hours is 63° from the equator and goes through the island Thule.

30. The parallel where the longest day is 21 equinoctial hours is 64½° from the equator and goes through unknown Scythian peoples.

31. The parallel where the longest day is 22 equinoctial hours is 66° from the equator.

32. The parallel where the longest day is 23 equinoctial hours is 66½° from the equator.

33. The parallel where the longest day is 24 equinoctial hours is 66;8,40° from the equator. This is the first of the [parallels] where the shadow goes full circle. For on that parallel, at the summer solstice (and then only), the sun does not set, so the shadow of the gnomon points towards every part of the horizon [in turn]. There the parallel of the summer solstice is ever-visible, and the parallel of the winter solstice is ever-invisible, since both are tangent to the horizon, on opposite sides. And the ecliptic coincides with the horizon when the spring equinoctial point on it is rising.

If, purely theoretically, one were to investigate some of the general characteristics of the latitudes even farther north, one would find the following.

34. Where the elevation of the north pole is about 67°, the 15° of the ecliptic on either side of the summer solstice do not set at all. So the longest day and the period when the shadow turns to point in all directions on the horizon is about a month long. This too can easily be seen from the Table of Inclination set out above. For we take a parallel, e.g. the parallel which cuts off [a segment of the ecliptic] 15° either side of the solstice (at which point it is either ever-visible or ever-invisible). The distance from the equator corresponding to that segment of the ecliptic will, obviously, give the amount by which the elevation of the north pole differs from the 90° of the quadrant.

35. Thus, where the elevation of the pole is 69½°, one would find that the 30° on either side of the summer solstice do not set at all. So the longest day and the

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65 By this name (which possibly ought to be aspirated, as ‘Hebudae’ in Pliny, VII 4.30) Ptolemy refers to the Hebrides, which he supposed to lie north of Ireland.

66 By ‘Thule’ Ptolemy refers to the modern Shetlands, as is clear from his Geography (II 32). It has been a matter of great dispute to what place (if any) the man who first introduced the name ‘Thule’ to the Greek world, Pytheas of Massalia, was referring. For ancient information on Pytheas’ voyage to Thule, a discussion of its identification and references to modern literature see Hennig, Terrae Incognitae I 119-24, 129-35.


68 See Appendix A, Example 16.
II 7. *Rising-times at sphaera obliqua*

period when the gnomons throw shadows in all directions last about two months.

36. Where the elevation of the pole is $73\frac{1}{2}^\circ$, one would find that the $45^\circ$ on either side of the summer solstice do not set at all. So the longest day and the period when the gnomons throw shadows in all directions last about three months.

37. Where the elevation of the pole is $78\frac{1}{2}^\circ$, one would find that the $60^\circ$ on either side of the same solstice do not set at all. So the longest day and the period when the shadow turns through a full circle would last about four months.

38. Where the elevation of the pole is $84^\circ$, one would find that the $75^\circ$ on either side of the summer solstice do not set at all. So in this case the longest day would be about five months long, and the gnomon would throw shadows in all directions for the same period.

39. Where the north pole is elevated from the horizon through the $90^\circ$ of the complete quadrant, the whole semi-circle of the ecliptic which is north of the equator never goes below the earth, and the whole semi-circle south of it never comes above the earth. Therefore every year contains only one day and one night, each about six months long, and the gnomons always throw shadows in all directions. Further special characteristics of this latitude are that the north pole is in the zenith, and that the equator coincides with the position of the ever-visible circle, and also with that of the ever-invisible circle and with the horizon; thus the whole hemisphere north of the equator is always above the earth, and the whole hemisphere south of the equator is always below the earth.

7. *On simultaneous risings of arcs of the ecliptic and equator at sphaera obliqua*69

After we have thus set out the general characteristics which can be theoretically deduced for the [various] latitudes, our next task is to show how to calculate, for each latitude, the arcs of the equator, measured as time-degrees, which rise together with [given] arcs of the ecliptic. From this we shall systematically derive all the other special characteristics [of the climata]. We shall use the names of the signs of the zodiac for the twelve [30°-] divisions of the ecliptic, according to the system in which the divisions begin at the solsticial and equinoctial points.70

We call the first division, beginning at the spring equinox and going towards the rear with respect to the motion of the universe, ‘Aries’, the second ‘Taurus’, and so on for the rest, in the traditional order of the 12 signs.

We shall first prove that arcs of the ecliptic which are equidistant from the same equinox always rise with equal arcs of the equator.

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70 I.e. the spring equinox defines ‘Aries 0°’, etc. This specification was necessary because other norms existed in antiquity, notably those where the spring equinox was at $9^\circ$ and $10^\circ$ (derived from Babylonian practice). See *HAMA* II.594–8.
II 7. Symmetries of rising-times

[See Fig. 2.4.] Let ABDG be a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZH and ΘK two arcs of the ecliptic such that points Z and Θ are each supposed to be the spring equinox, and equal arcs have been cut off on opposite sides of [that equinox]: these are arcs ZH and ΘK, which are rising at points K and H [respectively]. I say, that the arcs of the equator which rise with them, namely ZE and ΘE respectively, are equal.

[Proof.] Let points L and M represent the poles of the equator, and draw through them the great-circle arcs LEM, LΘ, LK, ZM and MH. Then since

 arc ZH = arc ΘK, and arc LK = arc MH because the parallels through K and H are equidistant from the equator on opposite sides,

 and arc EK = arc EH [spherical triangle] LKΘ ≡ [spherical triangle] MHZ

 and [spherical triangle] LEK ≡ [spherical triangle] MEH.

 ∴ ∠KLE = ∠HME, and ∠KLO = ∠HMZ.

 Therefore, by subtraction, ∠ELΘ = ∠EMZ.

 ∴ Θ = EZ, bases [of congruent triangles ELΘ, EMZ].

 Q.E.D.

Again, we shall prove that if two arcs of the ecliptic are equal and are equidistant from the same solstice, the sum of the two arcs of the equator which

71 Cf. II 3 (p. 79).
II 7. Symmetries of rising-times

rise with them is equal to the sum of the rising-times [of the same two arcs of the ecliptic] at sphaera recta.

[See Fig. 2.5.] Let ABDG be a meridian, and let semi-circle BED represent the horizon, and semi-circle AEG the equator. Draw two arcs of the ecliptic, equal and equidistant from the winter solstice, ZH (where Z is taken as the autumnal equinox) and ΘH (where Θ is taken as the spring equinox).

Thus H is the point on the horizon which is common to the rising of both, since arcs ZH and ΘH are both bounded by the same parallel circle to the equator. Therefore, obviously, arc ΘE rises with arc ΘH, and arc EZ with arc ZH. Then it is immediately obvious that the whole arc ΘEZ is equal to the sum of the rising-times of arc ZH and arc ΘH at sphaera recta.

[Proof.] For if we take K as the south pole of the equator, and draw through it and H the great-circle quadrant KHL, which represents the horizon at sphaera recta, then ΘL is the arc which rises with arc ΘH at sphaera recta, and similarly LZ is the arc which rises with arc ZH. Thus the sum of the arcs (ΘL + LZ) equals the sum of the arcs (ΘE + EZ), and both are comprised in the arc ΘZ.

Q.E.D.

From the above we have shown that, if we can calculate the individual rising-times at any latitude for just a single quadrant, we will simultaneously have solved the problem for the remaining three quadrants as well.

This being the case, let us again take as a paradigm the parallel through Rhodes, where the longest day is $14\frac{1}{2}$ equinoctial hours, and the elevation of the north pole from the horizon is $36^\circ$.

[See Fig. 2.6.] Let ABDG be a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZHΘ the semi-circle of the ecliptic, positioned so that H represents the spring equinox. Take K as the north pole of
II 7. Calculation of rising-time at sphaera obliqua

the equator, and draw through K and L, which is the intersection of the ecliptic and the horizon, the great-circle quadrant KLM.

Let the problem be, given arc HL, to find the arc of the equator which rises with it, that is arc EH.

First let arc HL comprise the sign of Aries.

Then since, in the diagram, the two great-circle arcs ED and KM are drawn to meet the two great-circle arcs EG and GK, and intersect each other at L, Crd arc 2KD : Crd arc 2DG = (Crd arc 2KL : Crd arc 2LM) : (Crd arc 2ME : Crd arc 2EG). [M.T. II] H122

But arc 2KD = 72°, so Crd arc 2KD = 70;32,4°.72
arc 2GD = 108°, so Crd arc 2GD = 97;4,56°.

And arc 2KL = 156;40,1°,73 so Crd arc 2KL = 117;31,15°;
arc 2LM = 23;19,59°, so Crd arc 2LM = 24;15,57°.

\[ \text{Crd arc 2ME} : \text{Crd arc 2EG} = \frac{70;32,4 : 97;4,56}{117;31,15 : 24;15,57} = 18;0,5 : 120. \]

And Crd arc 2EG = 120°.

\[ \therefore \text{Crd arc 2ME} = 18;0,5° \]
\[ \therefore \text{arc 2ME} \approx 17;16° \]
and arc ME = 8;38°

And since the whole arc HM rises with the whole arc HL at sphaera recta, it is 27;50°, as was shown above. [p. 73.]

Therefore, by subtraction, EH is 19;12°.

We have simultaneously proved that the sign Pisces rises in the same time (in

72Here (H122,4) and at H122,10 and H123,13 the Greek and Arabic ms. traditions give 70;32,4° as the chord of 72°, whereas in the chord table it is 70;32,3° (found here only in Ger.). Is this an indication that there was an earlier version of the chord table? Cf. p. 81 n.19.
73Reading \(\pi\pi\) (with B.Is) for \(\pi\pi\) at H122,7. Corrected by Manitius.
degrees of 19;12°, and that each of the signs Virgo and Libra rises in 36;28°, which is the remainder [of 19;12° taken] from twice the rising-time at sphaera recta.

Q.E.D.

Secondly, let arc HL comprise the 60° of the two signs Aries and Taurus. Then, from our assumptions, the other quantities will remain the same, but arc 2KL = 138;59,42°, so Crd arc 2KL = 112;23,56°, and arc 2LM = 41;0,18°, so Crd arc 2LM = 42;1,49°.

\[ \therefore \text{Crd arc } 2\text{ME:Cr}d \text{ arc } 2\text{EG} = \frac{(70;32,4 : 97;4,56)}{(112;23,56 : 42;1,48)} = 32;36,4 : 120. \]

And Crd arc 2EG = 120°.

\[ \therefore \text{Crd arc } 2\text{ME} = 32;36,4^\circ. \]

\[ \therefore \text{arc } 2\text{ME} \approx 31;32^\circ, \]

and arc ME \( \approx 15;46^\circ. \)

But the whole arc MH was previously shown to be 57;44° [p. 73.]

Therefore, by subtraction, arc HE = 41;58°.

Therefore the combined signs of Aries and Taurus rise in 41;58 time-degrees, of which 19;12° was shown to belong to the rising-time of Aries. Therefore the sign of Taurus by itself rises in 22;46 time-degrees.

By the same reasoning as before, the sign of Aquarius will rise in the same time of 22;46°, and each of the signs of Leo and Scorpio in 37;2°, which is the remainder [of 22;46° taken] from twice the rising-time at sphaera recta.

Now since the longest day is 14\frac{1}{2} equinoctial hours, and the shortest 9\frac{1}{2} equinoctial hours, it is obvious that the semi-circle [of the ecliptic] from Cancer to Sagittarius will rise with 217;30° of the equator, and the semi-circle from Capricorn to Gemini with 142;30°. Therefore each of the quadrants on either side of the spring equinox will rise in 71;15 time-degrees, and each of the quadrants on either side of the autumnal equinox will rise in 108;45 time-degrees. Therefore the remaining signs [in each quadrant], Gemini and Capricorn, will each rise in 29;17 time-degrees, which is the difference [of 19;12° + 22;46°] from the 71;15° in which the quadrant rises, and the remaining signs Cancer and Sagittarius will each rise in 35;15 time-degrees, which is the difference [of 36;28° + 37;2°] from the 108;45° in which that quadrant rises.

It is obvious that we could also calculate the rising-times of smaller arcs of the ecliptic [than whole signs] by exactly the same method. But we can also compute them by another easier and more practical procedure, as follows.

[See Fig. 2.7.] First let ABGD represent a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZEH the semi-circle of the ecliptic, with the intersection E taken as the spring equinox. Cut off an arbitrary arc E0 on [the ecliptic], and draw the segment 0K of the parallel to the equator through 0. Taking L as the [south] pole of the equator, draw through it the great-circle quadrants LΩM, LKN and LE.
II 7. Ascensional difference

Then it is immediately obvious that the segment $E\Theta$ of the ecliptic rises with arc $EM$ of the equator at $sphaera recta$, and with $NM$ at $sphaera obliqua$, since arc $K\Theta$ of the parallel circle, with which segment $E\Theta$ rises [at $sphaera obliqua$], is similar to arc $NM$ of the equator and similar arcs of parallel circles rise in equal times everywhere. Therefore arc $EN$ is the difference between the rising-times of segment $E\Theta$ at $sphaera obliqua$ and at $sphaera recta$. Thus we have shown that, for arcs of the ecliptic bounded by point $E$ and the parallel circle through $K$, in every case, if the great-circle arc corresponding to $LKN$ is drawn, segment $EN$ will comprise the difference between that arc's rising-times at $sphaera recta$ and at $sphaera obliqua$.\(^{76}\)

Having established this as a preliminary, let us draw [see Fig. 2.8] a diagram containing only the meridian and the semi-circles of the horizon [BED] and of the equator [AEG]; through $Z$, the south pole of the equator, let us draw the two great-circle quadrants $ZH\Theta$ and $ZKL$. Let us take $H$ as the intersection of the horizon with the parallel circle through the winter solstice, and $K$ as the intersection [of the horizon] with the parallel circle through, e.g., the beginning of Pisces, or any other given point on the quadrant [from the beginning of Capricorn to the end of Pisces].

Then, again, the great-circle arcs $ZKL$ and $EKH$ are drawn to meet the great-circle arcs $ZH\Theta$ and $E\Theta$, and intersect each other at $K$. Therefore

$$\text{Crd arc } 2\Theta H : \text{Crd arc } 2ZH = \left( \text{Crd arc } 2\Theta E : \text{Crd arc } 2EL \right) \cdot \left( \text{Crd arc } 2KL : \text{Crd arc } 2KZ \right).$$\(^{76}\)\(^{76}\) \text{M.T. II}

But at every latitude arc $2\Theta H$ is given and is the same, since it is the arc between the solstices. Hence arc $2HZ$, its supplement, is also given. Similarly,

\(^{76}\)This arc $EN$ is known in mediaeval astronomy as the 'ascensional difference'. See $HAMA$ 36 and 980-2, and Neugebauer-Schmidt.
II 7. Computation of rising-time tables

for the same arc of the ecliptic, arc 2LK is the same at all latitudes, and is given from the Table of Inclination [I 15]; and thence again its supplement, arc 2KZ, is given. Therefore, by division [of the above members], (Crd arc 2OE:Crd arc 2EL) is found to be the same at all latitudes (for the same arc of that quadrant [of the ecliptic]).

Since this is so, we take the different values of arc KL at every 10° [of the ecliptic] through the quadrant from the spring equinox to the winter solstice (for subdivision down to arcs of this size [10°] will be sufficient for practical purposes). Then in every case

\[\text{arc } 2\Theta H = 47;42,40^\circ, \text{ and Crd } 2\Theta H = 48;31,55^\circ,\]
\[\text{arc } 2HZ = 132;17,20^\circ, \text{ and Crd } 2HZ = 109;44,53^\circ.\]

Then, for the 10° [of the ecliptic] from the spring equinox towards the winter solstice,

\[\text{arc } 2KL = 8;3,16^\circ, \text{ and Crd } 2KL = 8;25,39^\circ,\]
\[\text{arc } 2KZ = 171;56,44^\circ, \text{ and Crd } 2KZ = 119;42,14^\circ.\]

For the arc 20° from the equinox

\[\text{arc } 2KL = 15;54,6^\circ, \text{ and Crd } 2KL = 16;35,56^\circ,\]
\[\text{arc } 2KZ = 164;5,54^\circ, \text{ and Crd } 2KZ = 118;50,47^\circ.\]

For the arc 30° from the equinox

\[\text{arc } 2LK = 23;19,58^\circ, \text{ and Crd } 2LK = 24;15,56^\circ,\]
\[\text{arc } 2KZ = 156;40,2^\circ, \text{ and Crd } 2KZ = 117;31,15^\circ.\]

For the arc 40° from the equinox

\[\text{arc } 2LK = 30;8,8^\circ, \text{ and Crd } 2LK = 31;11,43^\circ,\]
\[\text{arc } 2KZ = 149;51,52^\circ, \text{ and Crd } 2KZ = 115;52,19^\circ.\]

For the arc 50° from the equinox

\[\text{arc } 2LK = 36;5,46^\circ, \text{ and Crd } 2LK = 37;10,39^\circ,\]
\[\text{arc } 2KZ = 143;54,14^\circ, \text{ and Crd } 2KZ = 114;5,44^\circ.\]

For the arc 60° from the equinox
arc $2LK = 41;0,18^\circ$, Crd arc $2LK = 42;1,48^\circ$,
arc $2KZ = 138;59,42^\circ$, Crd arc $2KZ = 112;23,57^\circ$.

For the arc 70° from the equinox
arc $2LK = 44;40,22^\circ$, Crd arc $2LK = 45;36,18''$
arc $2KZ = 135;19,38^\circ$, Crd arc $2KZ = 110;59,47''$.

For the arc 80° from the equinox
arc $2LK = 46;56,32^\circ$, Crd arc $2LK = 47;47,40''$
arc $2KZ = 133;3,28^\circ$, Crd arc $2KZ = 110;4,16''$.

From the above we find that if we divide the ratio (Crd arc $2\Theta H$ : Crd arc $2HZ$), namely (48;31,55 : 109;44,53), by the ratio (Crd arc $2LK$ : Crd arc $2KZ$), as given above, at each of the 10° intervals, we will get the ratio (Crd arc $2\Theta E$ : Crd arc $2EL$), which is the same at all latitudes.

For the 10° arc it is 60 : 9;33
for the 20° arc 60 : 18;57
for the 30° arc 60 : 28;1
for the 40° arc 60 : 36;33
for the 50° arc 60 : 44;12
for the 60° arc 60 : 50;44
for the 70° arc 60 : 55;45
and for the 80° arc 60 : 58;55.

It is immediately obvious that for each latitude we will have arc $2\Theta E$ as a given arc, since it is, in degrees, the difference in time-degrees of the equinoctial day from the shortest day. Hence, from Crd arc $2\Theta E$ and the ratio (Crd arc $2\Theta E$ : Crd arc $2EL$), Crd arc $2EL$ will be given, and hence $2EL$. We will subtract half of this, namely arc $EL$, which comprises the above-mentioned difference [between rising-times at *sphaera recta* and *sphaera obliqua*], from the rising-time of the ecliptic arc in question at *sphaera recta*, and thus obtain the rising-time of the same arc at the given latitude.

As an example, let us again take the latitude of the parallel through Rhodes.

Here

$arc\ 2\Theta E = 37;30^\circ$, so Crd arc $2\Theta E = 38;34^\circ$.

Then since $60 : 38;34 = 9;33 : 6;8$

$= 18;57 : 12;11$
$= 28;1 : 18;0$
$= 36;33 : 23;29^77$
$= 44;12 : 28;25$
$= 50;44 : 32;37$
$= 55;45 : 35;52^79$
$= 58;55 : 37;52$,

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77 Computed from Ptolemy’s figures: 36;31,42. For the arc 40° above, a more accurate value for Crd arc $2KZ$ would be 115;52,26°. However, substituting that leads to 36;31,40 here. In either case, 36;32 would be the correct result to the nearest minute. This is the reading of Ger. but the rest of the tradition is unanimous for 36;33.

78 Accurate computation with 36;33 here gives 23;29,36, while 36;32 (see n.77) gives 23;28,58. This speaks in favour of the reading 36;32, but not decisively.

79 Computed: 35;50,6. However 35;52 is guaranteed by 17;24 for the seventh 10° arc below (35;50 leads to 17;23°).
II.7. Computation of rising-time tables

and since Crd arc 2EL equals the above amount [6;8°, etc.] at each of the above-mentioned 10° intervals, half of the arc it subtends, namely arc EL, will assume the following values:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Crd EL</th>
</tr>
</thead>
<tbody>
<tr>
<td>for the first 10°</td>
<td>2;56°</td>
</tr>
<tr>
<td>up to the end of the second</td>
<td>5;50°</td>
</tr>
<tr>
<td>up to the end of the third</td>
<td>8;38°</td>
</tr>
<tr>
<td>up to the end of the fourth</td>
<td>11;17°</td>
</tr>
<tr>
<td>up to the end of the fifth</td>
<td>13;42°</td>
</tr>
<tr>
<td>up to the end of the sixth</td>
<td>15;46°</td>
</tr>
<tr>
<td>up to the end of the seventh</td>
<td>17;24°</td>
</tr>
<tr>
<td>up to the end of the eighth</td>
<td>18;24°</td>
</tr>
<tr>
<td>up to the end of the ninth, obviously,</td>
<td>18;45°</td>
</tr>
</tbody>
</table>

Since the corresponding rising-times at sphaera recta are as follows:

<table>
<thead>
<tr>
<th>Interval</th>
<th>R.T.</th>
</tr>
</thead>
<tbody>
<tr>
<td>for the first 10°</td>
<td>9;10°</td>
</tr>
<tr>
<td>up to the end of the second</td>
<td>18;25°</td>
</tr>
<tr>
<td>up to the end of the third</td>
<td>27;50°</td>
</tr>
<tr>
<td>up to the end of the fourth</td>
<td>37;30°</td>
</tr>
<tr>
<td>up to the end of the fifth</td>
<td>47;28°</td>
</tr>
<tr>
<td>up to the end of the sixth</td>
<td>57;44°</td>
</tr>
<tr>
<td>up to the end of the seventh</td>
<td>68;18°</td>
</tr>
<tr>
<td>up to the end of the eighth</td>
<td>79;5°</td>
</tr>
<tr>
<td>and up to the end of the ninth</td>
<td>90°</td>
</tr>
</tbody>
</table>

H132

it is clear that by subtracting the difference, given by the arc EL, from the corresponding rising-time at sphaera recta in each case, we get the rising-times of the same arcs at the latitude in question. These are

<table>
<thead>
<tr>
<th>Interval</th>
<th>R.T.</th>
</tr>
</thead>
<tbody>
<tr>
<td>for the first 10°</td>
<td>6;14°</td>
</tr>
<tr>
<td>up to the end of the second</td>
<td>12;35°</td>
</tr>
<tr>
<td>up to the end of the third</td>
<td>19;12°</td>
</tr>
<tr>
<td>up to the end of the fourth</td>
<td>26;13°</td>
</tr>
<tr>
<td>up to the end of the fifth</td>
<td>33;46°</td>
</tr>
<tr>
<td>up to the end of sixth</td>
<td>41;58°</td>
</tr>
<tr>
<td>up to the end of the seventh</td>
<td>50;54°</td>
</tr>
<tr>
<td>up to the end of the eighth</td>
<td>60;41°</td>
</tr>
<tr>
<td>up to the end of the ninth</td>
<td>71;15°</td>
</tr>
<tr>
<td>(i.e. for the whole quadrant)</td>
<td>(which corresponds to the length of half of the [shortest] day).</td>
</tr>
</tbody>
</table>

The ten-degree segments will rise in the following time-degrees:

1st | 6;14°
2nd | 6;21°
3rd | 6;37°
4th | 7;1°
5th | 7;33°
6th | 8;12°
7th | 8;56°
II 7. Computation of rising-time tables

8th 9;47°
9th 10;34°.

Once we have established the above, the corresponding rising-times of the remaining quadrants will immediately be established on the same basis, by means of the theorems set out above.

In the same way we calculated the rising-times at every 10° interval for all other parallels which one might come upon in actual practice. For future use we shall set these out in tabular form, beginning with the parallel directly beneath the equator, and going as far as the parallel with a longest day of 17 hours. The parallels are taken at intervals of ½-hour [of longest day], since the difference [of exact computations] from results derived from linear interpolation [between half-hour intervals] is negligible. In the first column we put the 36 ten-degree intervals of the circle, in the next the corresponding time-degrees of the rising-time of that 10-degree arc at the latitude in question, and in the third the accumulated sum, as follows.

8. [Table of rising-times at ten-degree intervals] 80

[See pp. 100-3.]

9. [On the particular features which follow from the rising-times] 81

Now that we have set out the rising-times in the above manner, all the other problems associated with this subject will be easily soluble, and we shall not need to go through geometrical proofs or construct special tables to solve each problem. This will become clear from the actual methods described below.

First, one can find the length of a given day or night as follows. Take the rising-times of the appropriate latitude; for the day, count from the degree in which the sun is to the degree diametrically opposite, going towards the rear through the signs; for the night, count from the degree opposite the sun to the sun's degree. Form the sum of the rising-times [of the relevant 180°], and divide by 15: this will give the relevant interval in equinoctial hours. If we take 1/12th [of the sum of the rising-times] we will have the length of the seasonal hour of that interval [i.e. day or night] in time-degrees.

One can also find the length of the [seasonal] hour more conveniently by taking, from the above Table of Rising-times [II 8], the total rising-time corresponding to the sun's degree for the day (or the degree opposite the sun for the night) both at the parallel beneath the equator [i.e. sphaera recta] and at the relevant latitude, and forming the difference. Take 1/12th of the latter, and add it to the 15 time-degrees of one equinoctial hour for points on the northern semicircle [of the ecliptic], or subtract it from 15° for points on the southern semicircle: the result will be the length of the relevant seasonal hour in time-degrees. 82

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80 Correction to text: at H138,2 (latitude for \( \mu = 16° \)) read \( \mu \nabla \lambda \) (with Ar) for \( \mu \nabla (48°) \). Cf. II 6 p. 87.
81 See HAMA 40-3 (with worked examples) and Pedersen 113-15.
82 See Appendix A, Example 2.
### TABLE OF RISING-TIMES AT 10° INTERVALS

<table>
<thead>
<tr>
<th>SIGNS</th>
<th>10° Intervals</th>
<th>SPHAERA RECTA</th>
<th>AVALITE GULF</th>
<th>MEREO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12° 0°</td>
<td>12° 13°</td>
<td>13° 15°</td>
<td>16° 17°</td>
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<td>Accumulated</td>
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Next, one can convert seasonal hours for a given date into equinoctial hours by multiplying them by the length in time-degrees of the hour of the day in question at the relevant latitude (if they are hours of the day), or by the length in time-degrees of the hour of the night in question (if they are hours of the night). Then division of that product by 15 will give the total of equinoctial hours. *Vice versa*, one can convert equinoctial hours to seasonal by multiplying by 15 and dividing by the length of the hour of the relevant interval in time-degrees.\(^\text{83}\)

Furthermore, given a date and any time whatever, expressed in seasonal hours, on that date, we can find, first, the degree of the ecliptic rising at that moment. We do this by multiplying the number of hours, counted from sunrise by day, and from sunset by night, by the relevant length of the [seasonal] hour in time-degrees. We add this product to the rising-time at the latitude in question of the sun's degree by day (or the degree opposite the sun by night): the degree [of the ecliptic] with rising-time corresponding to the total will be rising at that moment.\(^\text{84}\)

[Secondly], if we want to find the point at upper culmination [at the given moment], we take in every case [i.e. for both day and night] the total of seasonal hours from the last midday to the given time, multiply it by the appropriate length(s) of the hour(s) in time-degrees, and add the product to the rising-time at *sphaera recta* of the sun's degree: the degree [of the ecliptic] with rising-time at *sphaera recta* equal to the total will be at upper culmination at that moment.\(^\text{85}\)

Similarly, we can find the culminating point from the rising point as follows: find from the table of rising-times for the relevant latitude the cumulative rising-times corresponding to the degree which is rising. Subtract from it, in every case, the 90° of the quadrant [of the equator between horizon and meridian]. The degree corresponding to the result in the column for rising-times at *sphaera recta* will be at upper culmination at that moment.\(^\text{86}\) *Vice versa*, one can find the rising point from the culminating point by taking the degree corresponding to the culminating point in the column for rising-times at *sphaera recta*, adding to it, in every case, the above 90°, and finding the degree corresponding to the result in the column for rising-times for the latitude in question: this degree will be rising at that moment.

It is also obvious that for those living beneath the same meridian the sun is the same distance from noon or midnight, counted in equinoctial hours, while for those living beneath different meridians the sun's distance from noon or midnight differs by an amount, counted in time-degrees, equal to the distance of one meridian from the other in degrees.

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\(^{83}\) See Appendix A, Example 3.

\(^{84}\) This sentence, like the corresponding one in the next problem, is a paraphrase giving the sense of Ptolemy's ambiguous expression. Literally "we count off this product towards the rear through the signs, beginning from the sun's degree... by night, according to the rising-times of the latitude in question: we say that whatever degree this amount reaches is the degree rising at that moment". See Appendix A, Example 4.

\(^{85}\) See Appendix A, Example 5.

\(^{86}\) See Appendix A, Example 6.
The remaining topic in the present theory is the discussion of angles formed at the ecliptic. We must first make clear that we define an angle between [two] great circles as follows: we say that [two] great circles form a right angle when a circle having as pole the intersection of the great circles and as radius any distance whatever has [exactly] a quadrant intercepted between the segments of the great circles forming the angle; in general, whatever ratio the intercepted arc of a circle described in the above manner bears to the whole circle is the same as the ratio of the angle between the planes [of the two great circles] to 4 right angles. Thus, since we set the circumference of the circle as 360°, the angle subtending the intercepted arc will contain the same number of degrees as the arc, in the system where one right angle contains 90°.

For the purposes of our present investigation, the most useful of the angles at the ecliptic are those formed by

1. the intersection of the ecliptic and the meridian,
2. the intersection of the ecliptic and the horizon for all positions [of the ecliptic], and
3. the intersection of the ecliptic and a great circle drawn through the poles of the horizon [i.e. an altitude circle];

the process of finding the latter will also produce the arc of this [altitude] circle cut off between its intersection with the ecliptic and the pole of the horizon, i.e. the zenith. Computation of each of the above angles, besides being a most suitable topic for the theory proper, also plays a very important part in the requirements for lunar parallax: it is impossible to make any progress in that subject without having first understood how to compute these angles.

Now there are four angles at the intersection of the two circles (I mean the ecliptic and any of the [above] circles meeting it). Since we shall [always] discuss only one of these, which always occupies the same relative position, we must make the following preliminary definition. In general, when we demonstrate in what follows the characteristics and size of an angle, we refer to that angle [of the four possible] which lies to the rear of the intersection of the circles and to the north of the ecliptic.®®

The computation of the angles between the meridian and the ecliptic is simpler, so we shall start with that, and first we shall show that points on the ecliptic equidistant from the same equinox produce angles of the above kind equal to each other.

[See Fig. 2.9.] Let ABG be an arc of the equator, DBE an arc of the ecliptic, and Z the pole of the equator. Cut off equal arcs, BH and BΘ, on opposite sides of the equinox B, and draw through pole Z and points H, Θ the meridian arcs ZKH and ZΘL. I say that

\[ \angle KHB = \angle ZΘE. \]  \[ 10.1 \]

[Proof:] This is immediately obvious. For the spherical triangle BHK has all its
angles equal to the angles of spherical triangle $B\Theta L$, since the three corresponding sides in each triangle are equal, $HB$ to $B\Theta$, $HK$ to $\Theta L$, and $BK$ to $BL$. All this has been proven previously.\(^{89}\)

Therefore $\angle KHB = \angle B\Theta L = \angle Z\Theta E$. Q.E.D.

Secondly, we must prove that the sum of the angles between ecliptic and meridian at points on the ecliptic equidistant from the same solstice is equal to two right angles.

[See Fig. 2.10.] Let $ABG$ be an arc of the ecliptic, with $B$ taken as solstice. Let equal arcs, $BD$ and $BE$, be taken on opposite sides of it, and draw through $Z$, the pole of the equator, and points $D$, $E$ the meridian arcs $ZD$ and $ZE$. I say that $\angle ZDB + \angle ZEG = 2$ right angles \(^{[10.2]}\)

[Proof:] This too is immediately obvious. For since points $D$ and $E$ are equidistant from the same solstice,

\[ \text{arc } DZ = \text{arc } ZE. \]

\[ \therefore \angle ZDB = \angle ZEB. \]

But $\angle ZEB + \angle ZEG = 2$ right angles.

\[ \therefore \angle ZDB + \angle ZEG = 2 \text{ right angles.} \]

Q.E.D.

Having established these preliminary theorems, let us draw [Fig. 2.11] the meridian circle $ABGD$ and the semi-circle of the ecliptic $AEG$ (taking $A$ as the winter solstice); then with pole $A$ and radius the side of the [inscribed] square draw semi-circle $BED$. Then, since meridian $ABGD$ goes through the poles of $AEG$ and the poles of $BED$, arc $ED$ is a quadrant.\(^{90}\)

\(^{89}\) $HB = B\Theta$ by construction; $HK = \Theta L$, declinations of points equidistant from an equinox (cf. p. 80 n.15); $BK = BL$, cf. II 7 (arc $E\Theta = \text{arc } EZ$ p 91).

\(^{90}\) Derivable from Theodosius Sphaerica II 9.
Therefore $\angle DAE$ is right.

And the angle at the summer solstice is also right, from the previous theorem [10.2].

Q.E.D.

Again, [see Fig. 2.12] let $ABGD$ be a meridian circle, $AEG$ a semi-circle of the equator, and $AZG$ a semi-circle of the ecliptic in such a position that $A$ is the autumnal equinox. Then with pole $A$ and radius the side of the [inscribed] square draw semi-circle $BZED$. 

H150
II 10. Angles between ecliptic and meridian

By the same reasoning [as above], since ABGD goes through the poles of [circles] AEG and BED, AZ and ED are quadrants. Hence point Z is the winter solstice, and

\[ \text{arc } ZE \approx 23;51^\circ, \text{ as was shown previously [I 12 p. 63].} \]

Therefore, by addition, \[ \text{arc } ZED = 113;51^\circ \]

and \[ \angle DAZ = 113;51^\circ \text{ where one right angle } = 90^\circ. \]

And again, from the previous theorem [10.2], the angle at the spring equinoctial point is the supplement, 66;9°.

Again [see Fig. 2.13] let ABGD be a meridian circle, AEG a semi-circle of the equator, and BZD a semi-circle of the ecliptic in such a position that point Z is
the autumnal equinox, and arc $BZ$ is (first of all) the length of one sign, that of Virgo; thus point $B$, obviously, is the beginning of Virgo. Again, with pole $B$ and radius the side of the [inscribed] square, draw semi-circle $H\Theta EK$.

Let the problem be to find $\angle KB\Theta$.

Now since meridian $ABGD$ goes through the poles of circles $AEG$ and $HEK$, arc $BH$, arc $B\Theta$ and arc $EH$ are all quadrants.

And, from the figure,
$$\text{Crd arc } 2BA: \text{Crd arc } 2AH = (\text{Crd arc } 2BZ: \text{Crd arc } 20Z). (\text{Crd arc } 20E: \text{Crd arc } 2EH).$$

[M.T. II]

But, as was shown previously, $^91$ arc $2BA = 23;20^\circ$, so $\text{Crd arc } 2BA = 24;16^\prime$, arc $2AH = 156;40^\circ$, so $\text{Crd arc } 2AH = 117;31^p$, and arc $2ZB = 60^\circ$, so $\text{Crd arc } 2ZB = 60^p$,
$$\text{arc } 2Z\Theta = 120^\circ, \text{ so } \text{Crd arc } 2Z\Theta = 103;55,23^p.$$ \[H152\]

$$\therefore \text{Crd arc } 2\Theta E: \text{Crd arc } 2EH = (24;16 : 117;31)/(60 : 103;55,23) \approx 42;58 : 120.$$ \[H153\]

$$\text{But } \text{Crd arc } 2EH = 120^p.$$ \[H154\]

$$\therefore \text{Crd arc } 2\Theta E \approx 42;58^p$$ \[H155\]

$$\therefore \text{arc } 2\Theta E \approx 42^\circ$$ \[H156\]

and arc $\Theta E \approx 21^\circ. ^92$

Therefore, by addition [of a quadrant] arc $\Theta EK = \angle KB\Theta = 111^\circ$, and the angle at the beginning of Scorpius is also $111^\circ$, and the angles at the beginning of Taurus and Pisces are each $69^\circ$, the supplement, by the theorems proved above $[10.1$ and $10.2].$

Q.E.D.

Next, in the same figure $[2.13]$, let arc $ZB$ represent two signs, so that point $B$ is the beginning of Leo. Then, with the [other] quantities remaining the same,
$$\text{arc } 2BA = [2\Theta (60^\circ) = ] 41^\circ, \text{ so } \text{Crd arc } 2BA = 42;2^p$$ \[H157\]

and arc $2AH = 139^\circ$, so $\text{Crd arc } 2AH = 112;24^p$;
$$\text{furthermore arc } 2ZB = 120^\circ, \text{ so } \text{Crd arc } 2ZB = 103;55,23^p$$ \[H158\]

and arc $2Z\Theta = 60^\circ$, so $\text{Crd arc } 2Z\Theta = 60^p$.
$$\therefore \text{Crd arc } 2\Theta E: \text{Crd arc } 2EH = (42;2 : 112;24)/(103;55,23 : 60) = 25;53 : 120.$$ \[H159\]

$$\therefore \text{Crd arc } 2\Theta E = 25;53^p$$ \[H160\]

$$\therefore \text{arc } 2\Theta E \approx 25^\circ$$ \[H161\]

and arc $\Theta E \approx 121^\circ. ^93$

Therefore, by addition, arc $\Theta EK = \angle KB\Theta = 1021^\circ$.

Therefore the angle at the beginning of Sagittarius is also $1021^\circ$, and the angle at both the beginning of Gemini and the beginning of Aquarius is the supplement, $77\frac{1}{2}^\circ$.

We have [thus] calculated what we set out to do. It is sufficient for practical use to display [the results] for each sign, although the same procedure would apply to even smaller sections of the ecliptic.

$^91$ Reference to II 7 p. 93. The quantities are rounded here.

$^92$ Accurate computation would give $20;58^\circ$ to the nearest minute.

$^93$ Accurate computation would give $12;28^\circ$ to the nearest minute.
Next we shall show how to calculate, for any given latitude, the angles formed by the ecliptic at the horizon. These too can be derived by a procedure which is simpler than that for the remaining angles [between ecliptic and altitude circles].

Now it is obvious that the angles [between ecliptic and] meridian are the same as those [between ecliptic and] horizon at sphaera recta. But, in order to calculate these angles also at sphaera obliqua, we must first prove that points on the ecliptic equidistant from the same equinox produce equal angles at the same horizon.

[See Fig. 2.14.] Let ABGD be a meridian circle, AEG the semi-circle of the equator and BED the semi-circle of the horizon. Draw two segments of the ecliptic, ZH0 and KLM, such that points Z and K both represent the autumnal equinox, and arc ZH equals arc KL.

\[ \angle EH\Theta = \angle DLK. \]

[Proof:] This is immediately obvious.

For spherical triangle EZH ≡ spherical triangle EKL.

since, from what was proven above, the corresponding sides are equal:

\[ ZH = KL \]
\[ HE = EL \text{ (arcs cut off by the intersection of the horizon with the ecliptic)} \]
\[ EZ = EK \text{ (rising-time arcs).} \]

\[ \therefore \angle EHZ = \angle ELK \]
\[ \therefore \angle EH\Theta = \angle DLK \text{ (supplements).} \]

\[ ^* \text{ecliptic'; literally 'the same inclined circle'.} \]
\[ ^* \text{ZH = KL by hypothesis; HE = EL from II 3 (p. 79); EZ = EK from II 7 (p. 91).} \]
I also say that, if two points [of the ecliptic] are diametrically opposite, the sum of the angles [between ecliptic and horizon] at the rising-point of one and the setting-point of the other is equal to two right angles.

[Proof: see Fig. 2.15.] If we draw ABGD as the circle of the horizon, and AEGZ as the circle of the ecliptic, so that they intersect at A and G, then

\[ \angle ZAD + \angle DAE = 2 \text{ right angles.} \]

But \[ \angle ZAD = \angle ZGD \]

\[ \therefore \angle ZGD + \angle DAE = 2 \text{ right angles.} \]

Q.E.D.

Since this is so, and since we have also proven that angles at the same horizon formed by points [on the ecliptic] equidistant from the same equinox are equal, a further consequence will be that, for points equidistant from the same solstice, the sum of the rising-angle at one and the setting-angle at the other will be equal to two right angles.

Hence, if we find the rising-angles from Aries to Libra [inclusive], we will simultaneously have found the rising-angles on the other semi-circle and the setting-angles on both semi-circles. We shall explain briefly how to do the calculation, again taking as example the same parallel, at which the elevation of the north pole from the horizon is 36°.

As for the angles between ecliptic and horizon at the equinoctial points, they can be calculated simply. For if [see Fig. 2.16] we draw ABGD as the meridian circle, AED as the eastern semi-circle of the horizon in question, EZ as a
quadrant of the equator, and EB and EG as two quadrants of the ecliptic such that point E is the autumnal equinox with respect to EB, and the spring equinox with respect to EG (thus B is the winter solstice and G the summer solstice), we can conclude as follows.

*Ex hypothesi*, arc $DZ = 54^\circ$ [colatitude of $36^\circ$]

and arc $BZ = arc \ ZG \approx 23;51^\circ$.

$\therefore arc \ GD = 30;9^\circ$

and arc $BD = 77;51^\circ$.
Thus, since \(E\) is the pole of meridian \(ABG\),

\[ \angle \text{DEG}, \] the angle at the beginning of Aries, is \(30;9^\circ\) where 1 right

and \(\angle \text{DEB}, \) the angle at the beginning of Libra, is \(77;51^\circ\) \(\) angle = \(90^\circ\).

In order to explain the procedure for finding the angles at other points, let us

take, for example, the problem of finding the rising-angle formed at the

beginning of Taurus and the horizon.

[See Fig. 2.17.] Let \(ABGD\) be the circle of the meridian, and \(BED\) the eastern

semi-circle of the horizon in question. Draw semi-circle \(AEG\) of the ecliptic, so

that point \(E\) represents the beginning of Taurus. Now at this latitude, when the

beginning of Taurus is rising, \(\approx 17;41^\circ\) is at lower culmination (we have shown

how such a problem can readily be solved by means of the tabulated rising-
times). Therefore arc \(EG\) is less than a quadrant. So with pole \(E\) and radius the

side of the [inscribed] square draw the great circle segment \(\Theta HZ\), and complete

the quadrants \(EGH\) and \(ED\Theta\). Both \(DGZ\) and \(ZH\Theta\) are also quadrants,
because the horizon \(BE\Theta\) goes through the poles of meridian \(ZGD\) and of the

great circle \(ZH\Theta\). Furthermore, \(\approx 17;41^\circ\) is \(22;40^\circ\) north of the equator,

measured along the great circle through the poles of the equator (we have set out

a table \([I 15]\) for that too); and the equator is \(36^\circ\) from pole \(Z\) of the horizon,

measured along the same arc, \(ZGD\). Therefore arc \(ZG = 58;40^\circ\). These

quantities being given, it then follows from the figure that

\[
\text{Crd arc } 2GD: \text{Crd arc } 2DZ = \\
(\text{Crd arc } 2GE: \text{Crd arc } 2EH). \ (\text{Crd arc } 2H\Theta: \text{Crd arc } 2Z\Theta)\]. \ [M.T. I] H159
\]

But, from the above,

\[
\text{arc } 2GD = 62;40^\circ, \text{ so } \text{Crd arc } 2GD = 62;24^\circ, \text{ arc } 2DZ = 180^\circ, \text{ so } \text{Crd arc } 2DZ = 120^\circ,
\]

\text{II 9 p. 104} (simply add \(180^\circ\) to the point of upper culmination, which is calculated for this

example in \(H.A.M.1, 42\)).
II 12. **Angles between ecliptic and altitude circle**

\[
\begin{align*}
\text{arc } \overline{GE} &= 155;22^\circ, \text{ so Crd } \text{arc } \overline{GE} = 117;14^\circ, \\
\text{arc } \overline{EH} &= 180^\circ, \text{ so Crd } \text{arc } \overline{EH} = 120^\circ.
\end{align*}
\]

\[
\therefore \text{Crd arc } \angle \overline{OH} : \text{Crd arc } \angle \overline{ZH} = (62;24 : 120)/(117;14 : 120)
\]

\[
= 63;52 : 120.
\]

And Crd arc \(\overline{OZ}\) = 120\(^\circ\).

\[
\therefore \text{Crd arc } \angle \overline{HO} = 63;52^\circ
\]

\[
\therefore \text{arc } \angle \overline{HO} = 64;20^\circ
\]

and arc \(\angle \overline{HO} = \angle \overline{HEO}\) = 32;10\(^\circ\).

Q.E.D.

To avoid lengthening the explanatory part of this treatise by continual repetition of the procedure, we will take the same method for granted for the remaining signs and latitudes.\(^{98}\)

---

**112. Angles between ecliptic and altitude circle**

It remains [to describe] the method by which we can compute the angles formed between the ecliptic and a circle through the poles of the horizon [i.e. an altitude circle] for any latitude and any position [of the ecliptic relative to the altitude circle]. As we said, this method also produces the size of the arc of the circle through the poles of the horizon cut off between the zenith and the intersection of that circle with the ecliptic. We shall again set out the preliminary theorems for this topic too: we shall prove, first, that if two points of the ecliptic are equidistant from the same solstice, and cut off an equal number of time-degrees on either side of the meridian, one to the east and the other to the west, then the great circle arcs from the zenith to those two points are equal, and the sum of the [two] angles at those points, chosen according to our [previous] definition,\(^{\text{100}}\) is equal to two right angles.

[See Fig. 2.18.] Let \(\overline{ABG}\) be a segment of the meridian, with point \(B\) on it taken as the zenith, and point \(G\) as the pole of the equator. Draw two segments of the ecliptic, \(\overline{ADE}\) and \(\overline{AZH}\), such that points \(D\) and \(Z\) are equidistant from the same solstice, and cut off, on either side of meridian \(\overline{ABG}\), equal arcs of the parallel circle which passes through them. Furthermore, draw through points \(D\) and \(Z\) the following great circle arcs: arc \(\text{GD}\) and arc \(\text{GZ}\) from the pole of the equator \(G\), and arc \(\text{BD}\) and arc \(\text{BZ}\) from the zenith \(B\).

I say that

\[
\text{arc } \overline{BD} = \text{arc } \overline{BZ}
\]

and \(\angle BDE + \angle BZA = 2\) right angles.

[Proof:] Since points \(D\) and \(Z\) cut off equal arcs of the parallel circle through them on either side of meridian \(\overline{ABG}\),

\[
\angle BGD = \angle BGZ.
\]

---

\(^{98}\)The angles between ecliptic and horizon are not explicitly tabulated by Ptolemy, but can be derived from the angles between ecliptic and altitude circle at the rising-point tabulated in Table II 13. See HAMA 47, which also tabulates them explicitly.


\(^{100}\)II 10 p. 105, with n.88.
Therefore, in the two spherical triangles BGD, BGZ
\[ GD = GZ \text{ [D, Z equidistant from solstice]} \]
\[ BG = BG \text{ (common)} \]
and \[ \angle BGD = \angle BGZ, \]
so they have two sides and the included angle equal.
\[ \therefore BD = BZ \text{ (bases)} \]
and \[ \angle BZG = \angle BDG. \]

But since we showed just above that the sum of the two angles formed by a circle through the poles of the equator at points [of the ecliptic] equidistant from the same solstice is equal to two right angles [10.2],
\[ \angle GDE + \angle GZA = 2 \text{ right angles}. \]
But we proved that \[ \angle BDG = \angle BZG. \]
\[ \therefore \angle BDE + \angle BZA = 2 \text{ right angles}. \]

Q.E.D.

Next we must prove that if we take the same point of the ecliptic at two positions equidistant from the meridian (as measured in time-degrees) on opposite sides of it, the great-circle arcs from the zenith to these two positions are equal, and the sum of the two angles [between altitude circle and ecliptic] east and west [of the meridian] is equal to twice the angle formed by the same point [of the ecliptic] at the meridian, provided that for both positions [i.e. when the point is east and west of the meridian] the points [of the ecliptic] which are [then] culminating are either both north or both south of the zenith.

Let us suppose, first, that both are south. [See Fig. 2.19.] Let ABGD be a segment of the meridian, with point G on it as the zenith, and D as the pole of the equator. Draw two segments of the ecliptic, AEZ and BHΩ, such that points E and H represent the same point, and cut off equal arcs of the parallel circle through that point on opposite sides of meridian ABGD. Again, draw through them [points E and H] the great-circle arcs GE and GH from G, and DE and

101 For \[ \angle BDE = \angle GDE + \angle BDG; \angle BZA = \angle GZA + \angle BZG. \] So, by addition (since \[ \angle BDG = \angle BZG \]), \[ \angle BDE + \angle BZA = \angle GDE + \angle GZA = 2 \text{ right angles}. \]
DH from D. By the same reasoning as before, since points E and H generate the same parallel circle and cut off equal arcs of it on either side of the meridian, spherical triangle GDE $\equiv$ spherical triangle GDH.

$\therefore$ arc $GE = arc \: GH$.

Then I say that

$\angle GEZ + \angle GHB = 2 \angle DEZ = 2 \angle DHB$.

[Proof:] Since $\angle DEZ$ is the same as $\angle DHB$ [E and H the same point]

and $\angle GED = \angle DHG$ [from congruent spherical triangles],

$\angle GED + \angle GHB = \angle DHG + \angle GHB = \angle DHB = \angle DEZ$.

Therefore, by addition $\angle GEZ + \angle GHB = 2 \angle DEZ = 2 \angle DHB$

Q.E.D.

Next, draw the same segments of the above circles again [Fig. 2.20], except that points A and B should be north of point G. I say that here too the same will apply, namely

$\angle KEZ + \angle LHB = 2 \angle DEZ$.

[Proof:] Since $\angle DEZ$ is the same as $\angle DHB$.

and $\angle DEK = \angle DHL$ [supplements of equal angles DEG, DHG],

by addition [of $\angle DHB$ to $\angle DHL$], $\angle LHB = \angle DEZ + \angle DEK$.

$\therefore \angle LHB + \angle KEZ = 2 \angle DEZ$.

Now again draw a similar figure [Fig. 2.21], except that the culminating point on the segment [of the ecliptic] east [of the meridian], namely A, should be south of the zenith G, while the culminating point on the segment west [of the meridian], namely B, should be north of the zenith.
II 12. Angles between ecliptic and altitude circle

Fig. 2.20

Fig. 2.21
I say that
\[ \angle GEZ + \angle LHB = 2 \angle DEZ \text{ plus } 2 \text{ right angles.} \]

[Proof:] Since
\[ \angle DHG = \angle DEG \]
and \[ \angle DHG + \angle DHL = 2 \text{ right angles}, \]
\[ \therefore \angle DEG + \angle DHL = 2 \text{ right angles.} \]
But \[ \angle DEZ \] is the same as \[ \angle DHB. \]
\[ \therefore \angle GEZ + \angle LHB = (\angle DEZ + \angle DEG) + (\angle DHB + \angle DHL) \]
\[ = \angle DEZ + \angle DHB \text{ plus } 2 \text{ right angles} \]
\[ = 2 \angle DEZ \text{ plus } 2 \text{ right angles.} \]
\[ \text{Q.E.D.} \]

For the remaining case, draw a similar figure [Fig. 2.22], in which point A, which is culminating on the section east [of the meridian], is north of G, while B, which is culminating on the section west [of the meridian], is south of [the zenith].

I say that
\[ \angle KEZ + \angle GHB = 2 \angle DEZ \text{ minus } 2 \text{ right angles.} \]

[Proof:] By the same reasoning as before
\[ \angle KEZ + \angle GHB = (\angle DEZ + \angle DHB) - (\angle DEK + \angle DHG) \]
\[ = 2 \angle DEZ - (\angle DEK + \angle DHG). \]
But \[ \angle DEK + \angle DHG = 2 \text{ right angles, since} \]
\[ \angle DEK + \angle DEG = 2 \text{ right angles, and } \angle DEG = \angle DHG. \]
\[ \text{Q.E.D.} \]
Of the angles and arcs formed in the way defined between the ecliptic and an altitude circle, those at the meridian and the horizon can be computed readily, as can be shown immediately in the following way.

Draw [Fig. 2.23] the meridian circle ABGD, the semi-circle of the horizon BED, and the semi-circle of the ecliptic in any position, ZEH. Then if we imagine the altitude circle through the zenith A and the culminating point of the ecliptic Z, it coincides with the meridian ABGD, and \( \angle DZE \) will immediately be given, since the point Z and the angle that [the ecliptic makes] with the meridian at point Z are given.\(^{102}\) Arc AZ will also be given, since we know the distance in degrees of point Z from the equator (measured along the meridian), and the distance of the equator from the zenith A.\(^{103}\)

Next, if we imagine the altitude circle AEG, drawn through the rising-point of the ecliptic, E, and [the zenith] A, here too it is immediately obvious that arc AE is always a quadrant, since point A is the pole of the horizon BED. For the same reason, \( \angle AED \) is always right; and since the angle which the ecliptic makes with the horizon, namely \( \angle DEH \), is given,\(^{104}\) the sum, angle AEH, will also be given.

Thus it is clear that, since the above relationships hold, if we compute, for each latitude, just the angles and arcs before [i.e. to the east of] the meridian, and just for the signs from the beginning of Cancer to the beginning of Capricorn, we will simultaneously have found the angles and arcs for the same

\(^{102}\) By II 10 (p. 109).

\(^{103}\) \( \delta \) and \( \phi \) respectively, so arc \( AZ = \phi - \delta \).

\(^{104}\) By II 11 (pp. 113-14).
signs [Cancer to Capricorn] after the meridian too, and also the angles and arcs both before and after the meridian for the remaining signs. But in order to make clear the procedure in this case too for any position [of the ecliptic], as an example we shall display the general method by means of a single solution of the problem. At the same latitude, namely where the elevation of the north pole from the horizon is 36°, we suppose that the beginning of Cancer is, e.g., one equinoctial hour to the east of the meridian. In this situation, at the above latitude, Π 16;12° is culminating, and θ 17;37° is rising.

Then let [Fig. 2.24] ABGD be the meridian circle, BED the semi-circle of the horizon, and ZHΘ the semi-circle of the ecliptic in such a position that point H is the beginning of Cancer, while Z represents Π 16;12° and Θ θ 17;37°. Draw through A, the zenith, and H, the beginning of Cancer, segment AHEG of the [altitude] great circle. Let the first problem be to find arc AH.

![Fig. 2.24](image)

Now it is clear that arc ZΘ = 91;25° [θ 17;37° − Π 16;12°] and arc HΘ = 77;37° [θ 17;37° − Π 16;12°].

Similarly, since Π 16;12° cut off 23;7° of the meridian to the north of the equator, and the equator cuts off 36° [of the meridian] from the zenith A. arc AZ = 12;53° and arc ZB = 77;7° (complement).

When these quantities are given, from the figure

\[
\text{Crd arc } 2ZB : \text{Crd arc } 2BA = (\text{Crd arc } 2ZΘ : \text{Crd arc } 2ΘH). (\text{Crd arc } 2HE : \text{Crd arc } 2EA).
\]

[M.T. I]

But arc 2ZB = 154;14°, so Crd arc 2ZB = 116;59° and arc 2BA = 180°, so Crd arc 2BA = 120°.

---

105 This example is worked through HAMA 49-50.
Furthermore arc $2Z\Theta = 182;50^\circ$, so Crd arc $2Z\Theta = 119;58^p$

and arc $2\Theta H = 155;14^\circ$, so Crd arc $2\Theta H = 117;12^p$.

$\therefore$ Crd arc $2EH$: Crd arc $2EA = (116;59 : 120)/(119;58 : 117;12)$

$\approx 114;16 : 120$.

But Crd arc $2EA = 120^p$
$\therefore$ Crd arc $2EH = 114;16^p$

$\therefore$ arc $2EH \approx 144;26^\circ$

and arc $EH = 72;13^\circ$.

$\therefore$ arc $AH = 17;47^\circ$ (complement).

Q.E.D.

Next we shall find $\angle AH\Theta$, as follows.

Draw the same figure [Fig. 2.25], and with pole $H$ and radius the side of the [inscribed] square draw the great circle segment $KLM$.

Then, since circle $AHE$ is drawn through the poles of $E\Theta M$ and $KLM$, both $EM$ and $KM$ are quadrants. Again, from the figure

![Fig. 2.25](image_url)

Crd arc $2HE$:Crd arc $2EK =$

(Crd arc $2H\Theta$:Crd arc $2\Theta L$). (Crd arc $2LM$:Crd arc $2KM$). [M.T. II]

But arc $2HE = 144;26^\circ$ [above], so Crd arc $2HE = 114;16^p$

and arc $2EK = 35;34^\circ$, so Crd arc $2EK = 36;38^p$.

Furthermore arc $2\Theta H = 155;14^\circ$, so Crd arc $2\Theta H = 117;12^p$

and arc $2OL = 24;46^\circ$, so Crd arc $2OL = 25;44^p$.

$\therefore$ Crd arc $2LM$:Crd arc $2MK = (114;16 : 36;38)/(117;12 : 25;44)$

$\approx 82;11 : 120$.

But Crd arc $2MK = 120^p$

$\therefore$ Crd arc $2LM = 82;11^p$
\[ \therefore \text{arc } 2\text{LM} = 86;28^\circ \]
\[ \text{and arc } \text{LM} = 43;14^\circ. \]
\[ \therefore \text{arc } \text{LK} = \angle \text{LHK} = 46;46^\circ \text{ (complement).} \]
\[ \therefore \angle \text{AH}\Theta = 133;14^\circ \text{ (supplement).} \]
Q.E.D.

The same method as was used for finding the above also applies to the remaining [arcs and angles]. But in order to have conveniently displayed all the other arcs and angles which it is reasonable to suppose we may need in our particular investigations, we computed these too geometrically, beginning from the parallel through Meroe, at which the longest day is 13 equinoctial hours, and going up to the parallel above Pontus [the Black Sea], through the mouths of the Borysthenes, where the longest day is 16 equinoctial hours. The intervals which we used were half an hour [of length of longest day] between parallels of latitude (as for the rising-times), one sign for the sections of the ecliptic, and one equinoctial hour for the position [of the altitude circles] to east and west of the meridian. We shall display the results in tabular form, one set of tables for each parallel of latitude, and one table for each sign. In the first column we put, first, the meridian situation, then the distance before or after the meridian, measured in equinoctial hours. In the second column we put the amount of the corresponding arc (as explained above) from the zenith to the beginning of the sign in question. In the third and fourth columns we put the amount of the angles formed by the above-mentioned intersection [between ecliptic and altitude circle], defined in the way we explained: the angles at positions to the east of the meridian in the third column, and those at positions to the west of the meridian in the fourth column. One must bear in mind that, according to our original definition, we always took the angle which lies to the rear of the intersection of the circles and to the north of the ecliptic, and expressed its magnitude in the system in which one right angle is 90 [degrees].

The layout of the tables is as follows.

### H172
The same method as was used for finding the above also applies to the remaining [arcs and angles]. But in order to have conveniently displayed all the other arcs and angles which it is reasonable to suppose we may need in our particular investigations, we computed these too geometrically, beginning from the parallel through Meroe, at which the longest day is 13 equinoctial hours, and going up to the parallel above Pontus [the Black Sea], through the mouths of the Borysthenes, where the longest day is 16 equinoctial hours. The intervals which we used were half an hour [of length of longest day] between parallels of latitude (as for the rising-times), one sign for the sections of the ecliptic, and one equinoctial hour for the position [of the altitude circles] to east and west of the meridian. We shall display the results in tabular form, one set of tables for each parallel of latitude, and one table for each sign. In the first column we put, first, the meridian situation, then the distance before or after the meridian, measured in equinoctial hours. In the second column we put the amount of the corresponding arc (as explained above) from the zenith to the beginning of the sign in question. In the third and fourth columns we put the amount of the angles formed by the above-mentioned intersection [between ecliptic and altitude circle], defined in the way we explained: the angles at positions to the east of the meridian in the third column, and those at positions to the west of the meridian in the fourth column. One must bear in mind that, according to our original definition, we always took the angle which lies to the rear of the intersection of the circles and to the north of the ecliptic, and expressed its magnitude in the system in which one right angle is 90 [degrees].

### H174—87
13. *[Layout of angles and arcs, parallel by parallel]*

[See pp. 123-9.]

### H188
Now that the treatment of the angles [between ecliptic and principal circles] has been methodically discussed, the only remaining topic in the foundations [of the rest of the treatise] is to determine the coordinates in latitude and longitude of the cities in each province which deserve note, in order to calculate

---

106 The seven parallels selected here are in fact the canonical '7 climata', for which see Introduction p. 19.

107 II 10 p. 105 with n.88.

108 The table for Clima I (Meroe) has a peculiarity. Since, alone of the parallels tabulated, its latitude is less than 6, it is possible for the point of the ecliptic which is culminating to fall north of the zenith. When this occurs at a tabulated position, the corresponding eastern or western angle is marked 'N' (for 'north'). This is a modification of the system in the ms., where BO (for βόρειος) is written above the first value in each column where the ecliptic is north of the zenith, and NO (for νότιος) above the value where it changes back to south. Since Ptolemy makes no mention of this
II 13. Table of zenith distances and ecliptic angles

**PARALLEL THROUGH MERIDIAN 13° 16:27**

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### Table of Zenith Distances and Ecliptic Angles

**Parallel Through Soene 13° 23.51°**

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### II 13. Table of zenith distances and ecliptic angles

**PARALLEL THROUGH THE HELLESPONT**

![](image)

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the [astronomical] phenomena for those cities. However, the discussion of this subject belongs to a separate, geographical treatise, so we shall expose it to view by itself [in such a treatise], in which we shall use the accounts of those who have elaborated this field to the extent which is possible. We shall [there] list for each of the cities its distance in degrees from the equator, measured along its meridian, and the distance in degrees of that meridian from the meridian through Alexandria, to the east or west, measured along the equator (for that [Alexandria] is the meridian for which we establish the times of the positions [of the heavenly bodies]).

For the time being we take the locations [of the cities] for granted, and [therefore] think it appropriate to add no more than the following. Whenever we are given the time at some standard place, and we undertake to determine what the corresponding time is at another place, then, if they lie on different meridians, we have to take the distance between the two places in degrees, measured along the equator, and determine which of them is to the east or west, and then increase or decrease the time at the standard place by the same number of time-degrees, to get the corresponding time at the required place. We increase if the required place is the further east, and decrease if the standard [place is the further east].

Because of the symmetries demonstrated in II 12 (see also H.A.M.I. 51) we have a means of checking most of the entries in these tables. The only entries which cannot be thus checked are the zenith distances (or the signs of Cancer and Capricorn. This shows that there are very few scribal errors in Heiberg's text here. However, recomputation of the data using modern formulas reveals considerable inaccuracies in Ptolemy's values. The zenith distances are generally correct to within 2', although occasional errors of up to 10' occur; but the angles regularly show errors of 10', and occasionally as much as 1° (e.g. Parallel through Middle Pontus, Gemini. 1 hour from noon, eastern angle: text 99:49'; computed 100:54').

 Corrections to Heiberg's text:

| Clima I | 2° (H175.7) μθ νη (49:58); μθ μη, with BCDL (computed: 49:49). |
| Clima V | 2° (H183.17) λβ λ (32): λβ λ. Cf. supplementary angle for Virgo: 167:30. This is simply a misprint, corrected by Manitius. |
| Clima VII | 2° (H186.17) ρλβ τ (132:10), πθ ν (89:50); ρλβ ζτ, πθ μδ. as Ger. Cf. supplementary angles at Pisces: 90:16, 47:44. Manitius noticed the discrepancy, but changed the Pisces entries. My correction is closer to the accurately computed values (132:15', 89:39'). Most of the Arabic tradition agrees with Heiberg here: L has 47:50 at Pisces, 2°, west angle. |

This promise is fulfilled in Ptolemy's Geography. However, by the time he came to write that, he decided to give distances in longitude, not from the meridian through Alexandria, but from one at the extreme west of the known world (through the Fortunate Isles), so that all longitudes could be counted in the same direction. A remnant of the original plan survives in Geography VIII, which includes a summary of time differences from Alexandria to east or west.

Excising δυοκειμενον at H189.6. Heiberg's text would mean 'and decrease if the standard place is the further west', which is the opposite of what is required. Manitius' excision of δυοκειμενον produces a good sense ('if the required place is the further west'), and the same sense is found in part of the Arabic tradition (L, Ger, P, but not T, Q). But the word order favours my correction.
In the preceding part of our treatise we have dealt with those aspects of heaven and earth which required, in outline, a preliminary mathematical discussion; also the inclination of the sun's path through the ecliptic, and the resultant particular phenomena, both at *sphaera recta* and at *sphaera obliqua* for every inhabited region. We think that we should [now] discuss, as the subject which appropriately follows the above, the theory of the sun and moon, and go through the phenomena which are a consequence of their motions. For none of the phenomena associated with the [other] heavenly bodies can be completely investigated without the previous treatment of these [two]. Furthermore, we find that the subject of the sun's motion must take first place amongst these [sun and moon], since without that it would, again, be impossible to give a complete discussion of the moon's theory from start to finish.

1. **[On the length of the year]**

The very first of the theorems concerning the sun is the determination of the length of the year. The ancients were in disagreement and confusion in their pronouncements on this topic, as can be seen from their treatises, especially those of Hipparchus, who was both industrious and a lover of truth. The main cause of the confusion on this topic which even he displayed is the fact that, when one examines the apparent returns [of the sun] to [the same] equinox or solstice, one finds that the length of the year exceeds 365 days by less than 1/4-day, but when one examines its return to [one of] the fixed stars it is greater [than 3651/4 days]. Hence Hipparchus comes to the idea that the sphere of the fixed stars too has a very slow motion, which, just like that of the planets, is towards the rear with respect to the revolution producing the first [daily] motion, which is that of a [great] circle drawn through the poles of both equator and ecliptic.

As for us, we shall show this is indeed the case, and how it takes place, in our discussion of the fixed stars (the theory of the fixed stars, too, cannot be

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1 D and part of the Arabic tradition (L, P, but not Q, T) begin chapter 1 at this point. On such variations, and the conclusion to be drawn, see Introduction p. 5.
2 See *HAMA* 54-5, Pedersen 128-34.
3 This characterisation of the daily motion by means of the rotation of a great circle through the poles of equator and ecliptic refers back to I 8 p. 47.
4 VII 2-3.
thoroughly investigated without previously establishing the theory of the sun and moon). However, for the purposes of the present investigation, it is our judgment that the only reference point we must consider when examining the length of the solar year is the return of the sun to itself, that is [the period in which it traverses] the circle of the ecliptic defined by its own motion. We must define the length of the year as the time the sun takes to travel from some fixed point on this circle back again to the same point. The only points which we can consider proper starting-points for the sun’s revolution are those defined by the equinoxes and solstices on that circle. For if we consider the subject from a mathematical viewpoint, we will find no more appropriate way to define a ‘revolution’ than that which returns the sun to the same relative position, both in place and in time, whether one relates it to the [local] horizon, to the meridian, or to the length of the day and night; and the only starting-points on the ecliptic which we can find are those which happen to be defined by the equinoxes and solstices. And if, instead, we consider what is appropriate from a physical point of view, we will not find anything which could more reasonably be considered a ‘revolution’ than that which returns the sun to a similar atmospheric condition and the same season; and the only starting-points one could find [for this revolution] are those which are the principal means of marking off the seasons from one another [i.e. solstitial and equinoctial points]. One might add that it seems unnatural to define the sun’s revolution by its return to [one of] the fixed stars, especially since the sphere of the fixed stars is observed to have a regular motion of its own towards the rear with respect to the [daily] motion of the heavens. For, this being the case, it would be equally appropriate to say that the length of the solar year is the time it takes the sun to go from one conjunction with Saturn, let us say, (or any other of the planets) to the next. In this way many different ‘years’ could be generated. For the above reasons we think it appropriate to define the solar year as the time from one equinox or solstice to the next of the same kind, as determined by observations taken at the greatest possible interval.

Now since Hipparchus is somewhat disturbed by the suspicion, derived from a series of observations which he made in close succession, that this same revolution [of the sun] is not of constant length, we shall try to show succinctly that there is nothing to be disturbed about here. We became convinced that these intervals [from solstice to solstice etc.] do not vary, from the successive solstices and equinoxes which we ourselves have observed by means of our instruments. For we find that [the times of the observed solstices etc.] do not differ by a significant amount from those derivable from the [365½-day [year]]\(^5\) (sometimes they differ by an amount roughly corresponding to the error which is explicable by the construction and positioning of the instruments). But we also guess from Hipparchus’ own calculations that his suspicion concerning the irregularity [in the length of the tropical year] is an error due mainly to the observations he used.

For, in his treatise ‘On the displacement of the solstitial and equinoctial points’, he first sets out those summer and winter solstices which he considers to

\(^5\) Literally ‘from the surplus due to the \(\frac{1}{2}\)-day’.
have been observed accurately, in succession, and himself admits that these do
not display enough discrepancies to allow one to use them to assert the existence
of any irregularity in the length of the year. He comments on them as follows:
'Now from the above observations it is clear that the differences in the year-
length are very small indeed. However, in the case of the solstices, I have to
admit that both I and Archimedes may have committed errors of up to a
quarter of a day in our observations and calculations [of the time]. But the
irregularity in the length of the year can be accurately perceived from the
[equinoxes] observed on the bronze ring situated in the place at Alexandria
called the "Square Stoa". This is supposed to indicate the equinox on the day
when the direction from which its concave surface is illuminated changes from
one side to the other'.

Then he sets out, first, the times of autumnal equinoxes which he considers to
to have been very accurately observed:

[1] In the seventeenth year of the Third Kallippic Cycle, Mesore 30 [-161 Sept.
27], about sunset.
[2] 3 years later, in the twentieth year, on the first epagomenal day [-158 Sept.
27], at dawn. This should have been at noon, so there is a ½-day discrepancy.
[3] 1 year later, in the twenty-first year, [on the first epagomenal day, -157
Sept. 27], at the sixth hour. This was in agreement with the preceding
observation.
[4] 11 years later, in the thirty-second year, at the midnight between the third
and fourth epagomenal days [-146 Sept. 26/27]. This should have been at
dawn, so again there is a ½-day discrepancy.
[5] 1 year later, in the thirty-third year, on the fourth epagomenal day [-145
Sept. 27], at dawn. This was in agreement with the previous observation.
[6] 3 years later, in the thirty-sixth year, on the fourth epagomenal day [-142
Sept. 26], in the evening. This should have been at midnight, so again there is
only a ½-day discrepancy.

Next he sets out the spring equinoxes which have been observed with a
similar accuracy:

6 Manitius claims that the reading άνισότητα τινα for άνισότητα at H194,21 is 'absolutely
necessary'. It is Halma's text, adopted from the edo pinceps. However, it is not found in any of the
principal mss., and Heiberg's text as it stands can mean the same thing.
7 For a diagram of this 'equatorial armillary' see Price, 'Precision Instruments' Fig. 343C on p.
589. It is simply a ring permanently fixed in the plane of the equator. From Ptolemy (p. 134) we
learn that there were two such rings at Alexandria in his time, in the Palaestra. Whether either was
identical with the one mentioned by Hipparchus cannot be discussed here. For what little is known
about the 'Square Stoa' and the Palaestra (presumably in the great gymnasium mentioned in Strabo
8 While there is general agreement that all the other equinox observations reported from
Hipparchus were made by him in person, there is considerable dispute whether these three were
observed by him or merely used by him. They are separated by an interval of 11 years from the next'
attested observation, which also falls into the period for which other types of observation by
Hipparchus are recorded (the lunar eclipse of -145 Apr. 21, p. 135). My own view is that this group
of three early observations was not made by Hipparchus himself, but was simply adduced by him
for comparison.
In the thirty-second year of the Third Kallippic Cycle, Mechir 27 [-145 Mar. 24], at dawn. Furthermore, he says, the ring at Alexandria was illuminated equally from both sides at about the fifth hour. Thus we can already see two different observations of the same equinox with a discrepancy of approximately 5 hours.

He says that the subsequent observations up to the thirty-seventh year [-144 to -140] were all in agreement with the times derivable from the [365] day [year].

11 years later [than 1], in the forty-third year, he says, the spring equinox occurred after midnight Mechir 29/30 [-134 Mar. 23/24]. This was in agreement with the observation [1] in the thirty-second year, and, he says, again agrees with the observations [8 to 13, -133 to -128] in the subsequent years up to the fiftieth year [14]. This took place on Phamenoth 1 [-127 Mar. 23], about sunset. This is approximately 1 1/2 days later [in the Egyptian year] than the [equinox] in the forty-third year. This also fits the 7-year interval.

Thus in these observations too there is no discrepancy worth noticing, even though it is possible for an error of up to a quarter of a day to occur not only in observations of solstices, but even in equinox observations. For suppose that the instrument, due to its positioning or graduation, is out of true by as little as 5° of the circle through the poles of the equator: then, to correct an error of that size in declination, the sun, [when it is] near the intersection [of the ecliptic] with the equator, has to move 1/10 in longitude on the ecliptic. Thus the discrepancy comes to about 1/4 of a day. The error could be even greater in the case of an instrument which, instead of being set up for the specific occasion and positioned accurately at the time of the actual observation, has been fixed once for all on a base intended to preserve it in the same position for a long period: [the error occurs when] the instrument is affected by a [gradual] displacement which is unnoticed because of the length of time over which it takes place. One can see this in the case of the bronze rings in our Palaestra, which are supposed to be fixed in the plane of the equator. When we observe with them, the distortion in their positioning is apparent, especially that of the larger and older of the two, to such an extent that sometimes the direction of illumination of the concave surface in them shifts from one side to the other twice on the same equinoctial day.12

This statement has occasionally been used (most recently by Fraser[1] I 423) as evidence that Hipparchus observed in Alexandria. On the contrary, Ptolemy's expression makes it clear that this Alexandrian observation was different (and discrepant) from Hipparchus' own. Whenever the place of an observation by Hipparchus is known, it is Rhodes (except for his weather prognostications reported in Ptolemy's Phasis, for which the place was Bithynia, presumably Hipparchus' native Nicea).

Reading ἄξολοθῳν at H196,15 for the misprint ἄξολοθῳν.

Ptolemy says that an observational error of 6' in declination corresponds, near equinox, to an ecliptic motion of 1° or (since the sun moves about 1° per day in the ecliptic) to an error of 1 day in the time of observation. This is easily verified by linear interpolation in the declination table I 15, where the declination for 1° is 0;24,16'°.

For the ring see p. 133 n. 7. If the instrument was correctly set up, at the moment of equinox the direction of illumination would shift from below the shadowing part to above it in spring (and vice
III 1. Why Hipparchus suspected length of year to vary

However, Hipparchus himself does not think that there is anything in the above observations which provides convincing support for his suspicion that there is an irregularity in the length of the year. Instead he makes computations on the basis of certain lunar eclipses, and declares that he finds that the variation in the length of the year, with respect to the mean value, is no more than ½ of a day. This would be sufficiently great to take some account of, if it were indeed so; but it can be seen to be false from the very considerations which he adduces [to support it]. For he uses certain lunar eclipses which were observed to take place near [specific] fixed stars to compare the distance of the star called Spica in advance of the autumnal equinox at each [eclipse]. By this means he thinks he finds, on one occasion, a distance of 6°, the maximum in his time, and on another a distance of 5½°, the minimum [in his time]. Thence he concludes that, since it is impossible for Spica [itself] to move so much in such a short time, it is plausible to suppose that the sun, which Hipparchus uses to determine the positions of the fixed stars, does not have a constant period of revolution. But this kind of computation cannot be made without using the sun’s position at the eclipse as a basis. Thus, though he does not realise it, at each eclipse he is applying for this purpose [determination of the sun’s position] the accurate observations of solstices and equinoxes which he himself has made in these same years. By the very act of doing this he shows that, when one compares the length of those years, there is no discrepancy from the [365½-day interval.

To take a single example: from the eclipse observation in the thirty-second year of the Third Kallippic Cycle which he adduces, he claims to find that Spica is 6° in advance of the autumnal equinox, whereas from the eclipse observation in the forty-third year of that cycle he claims to find that it is 5½° in advance. In order to carry out the computations for the above, he adduces the spring equinoxes which he had accurately observed in those years. This was in order that from the latter he could find the position of the sun at the middle of each eclipse, from these the positions of the moon, and from the positions of the moon those of the stars. He says that the spring equinox in the thirty-second year took place on Mechir 27 [-145 March 24] at dawn, and the one in the forty-third year on [Mechir] 29/30 [-134 March 23/24] after midnight, later [in the Egyptian year] than that in the thirty-second by approximately 2½ days, which is the same amount as is produced by the addition of precisely 4-day in each of

-versa in autumn). Maniúus (I 427 n.21) explains the phenomenon reported here by Ptolemy as due to the effect of refraction on a correctly placed ring. His argument is dismissed by Rome[5] I 230-5 and [1] II p. 818 n., on the grounds that the true one of the two ‘equinoxes’ could easily be determined by the direction of shift. This does not of course invalidate Maniúus’ explanation. The only good detailed discussion is Britton[1] 29-42, correcting both Maniúus and Rome, and concluding (p. 34) that multiple “equinoxes” on a well-aligned ring would be normal.

Reading ἔτοι ταύτου (with D, Ar) at H198.24 for ἔτοι ταύτου (‘which were made in his time’).

The eclipses in question are those of -145 Apr. 21 and -134 Mar. 21 (misprinted March 31 in Pedersen Appendix A, 414). We have no further data on Hipparchus’ observations of these eclipses. For a detailed discussion of the procedures involved see Rome[5] II. From VII 2 (p. 327) it seems that Hipparchus eventually settled on a compromise figure of 6° from the autumnal equinox in his own time.

Meaning ‘as in the other similar calculations’. D’s reading is ὅμως, ‘however’, which makes good sense, but is not supported by the Arabic tradition.
the intervening 11 years. Since, then, the sun has been shown to complete its revolution (as measured with respect to those equinoxes) in a time neither greater nor less than the [365]-day interval, and since it is impossible for Spica to move $1^\circ$ in such a small number of years, surely it is perverse to use calculations based on the above foundations to impugn the very foundations on which they were based. It is perverse to ascribe the reason for such an impossibly large motion of Spica solely to the equinoxes on which the calculations are based (which entails the simultaneous assumptions, both that they are accurately observed, and that they have been inaccurately observed), when there are several possible causes for so great an error. It is more plausible to suppose, either that the distances of the moon from the nearest stars at the eclipses have been too crudely estimated, or that there has been an error or inaccuracy in the determinations of the moon's parallax with respect to its apparent position, or of the motion of the sun from the equinox to the time of mid-eclipse.

However, it is my opinion that Hipparchus himself realised that this kind of argumentation provides no persuasive evidence for the attribution of a second anomaly to the sun, but his love of truth led him not to suppress anything which might in any way lead some people to suspect [such an anomaly]. At any rate, he himself, in his theories of the sun and moon, assumes that the sun has a single and invariable anomaly, the period of which is the length of the year as defined by [return to] solstices and equinoxes. Furthermore, when we assume that the period of these revolutions of the sun is constant, we see that there is never any significant difference between the phenomena observed at eclipses and those calculated on the above assumption. Yet there would be a very perceptible difference if there were some correction due to a variation in the length of the year which we failed to take into account, even if that correction were as little as a single degree, which corresponds to approximately two equinoctial hours.\(^{16}\)

From all the above considerations, and from our own determination of the period of the [solar] revolution, by means of a series of observations of the sun’s position, we conclude that the length of the year is constant, provided that it is always defined with respect to the same criterion, and not with respect to the solsticial and equinoctial points at one time and to the fixed stars at another. We also conclude that the most natural definition of the revolution is that in which the sun, starting from one solstice or equinox or any point on the ecliptic, returns to the same point again. And in general, we consider it a good principle to explain the phenomena by the simplest hypotheses possible, in so far as there is nothing in the observations to provide a significant objection to such a procedure.\(^{17}\)

Now it was already clear to us from Hipparchus’ demonstrations that the length of the year, defined with respect to the solstices and equinoxes, is less than $\frac{1}{2}$-day in excess of 365 days. The amount by which it falls short [of 1-day] cannot

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\(^{16}\) The time of an eclipse depends on the speeds of sun and moon. Assuming, with Ptolemy, round figures of $1^\circ$° for the sun’s motion and $13^\circ$° for the moon’s, we get a relative motion of $12^\circ$°, or $\frac{1}{3}$ per hour. Thus a shift of $1^\circ$° in the position of the sun at an eclipse leads to a shift of 2 hours in the time.

\(^{17}\) This general principle of the desirability of simplicity in the hypotheses is repeated, but modified, at XIII 2 p. 600. Cf. also III 4 p. 153.
be determined with absolute certainty, since the difference is so small that for many years in succession the increment [over 365 days] remains sensibly the same as a constant 1/4-day increment. Hence it is possible, when comparing observations taken over quite a long period, that the surplus days [over 365], which have to be obtained by distributing [the total surplus] over the years of the interval [between the observations], may appear to be the same whether one takes [observations over] a greater or lesser number of years. However, the longer the time between the observations compared, the greater the accuracy of the determination of the period of revolution. This rule holds good not only in this case, but for all periodic revolutions. For the error due to the inaccuracy inherent in even carefully performed observations is, to the senses of the observer, small and approximately the same at any [two] observations, whether these are taken at a large or a small interval. However, this same error, when distributed over a smaller number of years, makes the inaccuracy in the yearly motion [comparatively] greater (and [hence increases] the error accumulated over a longer period of time), but when distributed over a larger number of years makes the inaccuracy [comparatively] less. Hence we must consider it sufficient if we endeavour to take into account only that increase in the accuracy of our hypotheses concerning periodic motions which can be derived from the length of time between us and those observations we have which are both ancient and accurate. We must not, if we can avoid it, neglect the proper examination [of such records]; but as for assertions of validity 'for eternity', or even for a length of time which is many times that over which the observations have been taken, we must consider such as alien to a love of science and truth.\(^{18}\)

Now, as far as concerns antiquity [of the observations], the summer solstices observed by the school of Meton and Euktemon, and, later, the school of Aristarchus, deserve to be compared with those of our own time.\(^{19}\) However, since observations of solstices are, in general, hard to determine accurately, and since, furthermore, the observations handed down by the above-mentioned people were conducted rather crudely (as Hipparchus too seems to think), we abandoned those, and have used instead, for the comparison we propose, equinox observations, choosing amongst them, for the sake of accuracy, those which Hipparchus especially noted as very securely determined by him, and those which we ourselves have made with the greatest accuracy using the instruments for such purposes described at the beginning of our treatise [I 12]. For these we find that the solstices and equinoxes occur earlier than [one would expect from a year of 365] 1/4 days by one day in approximately 300 years.

For Hipparchus noted that in the thirty-second year of the Third Kallippic

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\(^{18}\) This remarkably sensible attitude towards the validity of mean motions derived from observations was not imitated by most of Ptolemy's successors throughout antiquity and the middle ages. The contemptuous remark about 'eternity' may be a glance at the ἀείωνας καψόνες mentioned at IX 2 p. 422 (see n. 12 there).

\(^{19}\) The only solstices known to have been observed by these men are that of -431 June 27, ascribed below (p. 138) to 'the school of Meton and Euktemon', and that of -279 (no further details known) ascribed below (p. 138) to 'the school of Aristarchus'. The latter is Aristarchus of Samos, now famous mainly for his 'heliocentric hypothesis'. See Heath, *Aristarchus*. On Meton see Toomer[7]. By 'the school of... ' I translate ὣν πέπο... The precise way to interpret the phrase here and elsewhere in the Almagest remains obscure.
CycJe he had made a very accurate observation of the autumnal equinox, and says that he calculated that it occurred at midnight, third-fourth epagomenal day \([-146 \text{ Sept. 26}/27]\). The year is the 178th from the death of Alexander.\(^{20}\)

285 years later, in the third year of Antoninus, which is the 463rd from the death of Alexander, we observed, again very securely, that the autumnal equinox occurred on Athyr 9 \([139 \text{ Sept. 26}]\), approximately one hour after sunrise.\(^{21}\)

Therefore the period of return comprised, over 285 complete Egyptian years (that is years of 365 days), \(70\frac{1}{2}\) days plus approximately \(\frac{8}{285}\) of a day, instead of the \(71\frac{1}{2}\) days corresponding to the \(\frac{1}{7}\)-day surplus for the above \([285]\) years. Thus the return took place earlier than it would have with the \([365]\)-day year by one day less about \(\frac{1}{285}\) day.

Similarly, Hipparchus says that the spring equinox in the same thirty-second year of the Third Kallippic Cycle, which he observed most accurately, took place on Mechir 27 \([-145 \text{ Mar. 24}]\) at dawn. The year is the 178th from the death of Alexander. We find that the corresponding spring equinox 285 years later, in the 463rd year from the death of Alexander, took place on Pachon 7 \([140 \text{ Mar. 22}]\), approximately 1 hour after noon. Thus this period too comprised an increment \([over 285\] Egyptian years\) of the same amount, \(70\frac{1}{2} + \approx \frac{8}{285}\) days, instead of the \(71\frac{1}{2}\) days corresponding to the \(\frac{1}{7}\)-day surplus for the 285 years. Here too, then, the return of the spring equinox took place earlier than it would have with the \([365]\)-day year by \(\frac{8}{285}\) of a day. Hence, since

\[1 \text{ day} : \frac{8}{285} \text{ day} = 300 : 285,\]

we conclude that the return of the sun to the equinoctial points takes place earlier than it would for a \([365]\)-day year by approximately one day in 300 years.

Furthermore if, because of its antiquity, we compare the summer solstice observed by the school of Melon and Euklemon (though somewhat crudely recorded) with the solstice which we determined as accurately as possible, we will get the same result. For that \([solstice]\) is recorded as occurring in the year when Apsedes was archon at Athens, on Phamenoth 21 in the Egyptian calendar \([-431 \text{ June 27}]\), at dawn.\(^{22}\)

We determined securely that the \([summer solstice]\) in the above-mentioned 463rd year from the death of Alexander occurred on Mesore 11/12 \([140 \text{ June 24}/25]\) about 2 hours after midnight. Now there are 152 years (as Hipparchus too reckons) from the summer solstice recorded in the archonship of Apsedes to the solstice observed by the school of Aristarchus in the fiftieth year of the First Kallippic Cycle \([-279]\), and from that fiftieth year, which corresponds to the 44th year from the death of Alexander, to the 463rd year, in which our observation was made, is 419 years. Therefore in

\(^{20}\) On this \((-323,\ not -322, the actual year of Alexander's death) see Introduction p. 10 n.16. '178th' is inclusive reckoning.

\(^{21}\) Notoriously, like Ptolemy's spring equinox and summer solstice observations below, this is about 1 day later than the actual event. This is the strongest argument of those modern critics who have maintained that Ptolemy 'faked' observations. See Toomer[5] 189. The best discussion of this difficult problem is Britton[1] Chapter II.

\(^{22}\) The Egyptian date of this observation was not given by Meton himself, who dated it to Skirophorion 13 in his calendar, but is a later conversion (found in the Milesian parapgebra of the late second century B.C., see Samuel, Greek and Roman Chronology 44 or Toomer[7] 338, but no doubt already made by Hipparchus).
III 1. Length of tropical year according to Hipparchus

the whole interval of 571 years, if the summer solstice observed by the school of Euktemon took place around the dawning of Phamenoth 21, there is an increment of approximately 1401/2 days over complete Egyptian years,23 instead of the 1421/2 days corresponding to the 1-day surplus for 571 years. Thus the return in question took place earlier than it would have with the [365]1/4-day year by 11/2 days. Here too, then, it is clear that in a round 600 years the [true] year-length accumulates a decrement of approximately 2 complete days against the [365]1/4-day year.

We find the same result from a number of other observations of our own, and we see that Hipparchus agrees with it on more than one occasion. For in his work 'On the length of the year' he compares the summer solstice observed by Aristarchus at the end of the fiftieth year of the First Kallippic Cycle [-279] with the one which he himself had determined, again with accuracy, at the end of the forty-third year of the Third Kallippic Cycle [-134], and then says: 'It is clear: then, that over 145 years the solstice occurs sooner than it would have with a [365]1/4-day year by half the sum of the length of day and night'. Again, in 'On intercalary months and days' also, after remarking that according to the school of Meton and Euktemon the length of the year comprises 3651/4 + 1/4 days, but according to Kallippos only 3651/4 days,24 he comments, in his own words, as follows: 'As for us, we find the number of whole months comprised in 19 years to be the same as they found, but we find the year to be even less than 1-[day beyond 365], by approximately 1/30th of a day. Thus in 300 years its [accumulated] deficit is 5 days compared with Meton's figure, and 1 day compared with Kallippos.' And when he more or less sums up his opinions in his list of his own writings,25 he says: 'I have also composed a work on the length of the year in one book, in which I show that the solar year (by which I mean the time in which the sun goes from a solstice back to the same solstice, or from an equinox back to the same equinox) contains 365 days, plus a fraction which is less than 1 by about 1/30th of the sum of one day and night, and not, as the mathematicians26 suppose, exactly 1-day beyond the above-mentioned number [365] of days.'

Thus I think it appears plainly from the agreement of present-day [observations] with earlier ones, that all phenomena observed up to the present

23 Ptolemy apparently reckons 'dawn' (πρωιός) in the earlier observation as 6 a.m. in equinoctial hours (despite the fact that at Athens sunrise at summer solstice occurs at about 4:45 a.m.), and means '2 hours after midnight' in his own observation to be 2 a.m., i.e. equinoctial hours. Then the increment over whole days between the observations is 20 equinoctial hours = 3/4 day. If we were to take the times as 'precisely sunrise' and '2 seasonal hours', the interval would be closer to 21 hours, or 1 day.


25 This phrase, which appears to have been misunderstood by all earlier translators, but is correctly interpreted by Rehm, 'Hipparchos' col. 1666, shows that Hipparchus published a catalogue of his own works with a summary of the contents of each. An example of this kind of publication which has come down to us is Galen's 'On his own Books' (περὶ τῶν ἰδιῶν βιβλίων), Scripta Minora II 91 ff. From Galen's work it is apparent that for a prolific writer of monographs, like Hipparchus, such a catalogue was necessary as a check on the ascription of his works (perhaps circulating in unauthorised versions) to others.

26 οἱ μαθηματικοὶ, which includes astronomers. One might almost think from Hipparchus' tone that he means 'astrologers' (this is a standard meaning in later Greek). Ptolemy, however, does not use the word in this sense (cf. pp. 175 and 421, where I have translated it 'astronomers').
time having to do with the length of the solar year accord with the above-
mentioned figure for the return to solstices or equinoxes. This being so, if we 
distribute the one day over the 300 years, every year gets 12 seconds of a day. 
Subtracting these from the 365;15\textsuperscript{d} of the 1-day increment, we get the required 
length of the year as 365;14,48\textsuperscript{d}. Such, then, is the closest possible approxima-
tion which we can derive from the available data.

Now, with regard to the determination of the positions of the sun and the 
other [heavenly bodies] for any given time, which the construction of individual 
tables is designed to provide in a handy and as it were readymade form: we 
think that the mathematician's task and goal ought to be to show all the 
heavenly phenomena being reproduced by uniform circular motions, and that 
the tabular form most appropriate and suited to this task is one which separates 
the individual uniform motions from the non-uniform [anomalistic] motion 
which [only] seems to take place, and is [in fact] due to the circular models; the 
apparent places of the bodies are then displayed by the combination of these 
two motions into one.\textsuperscript{27} In order to have this type of table in a form which shall 
be usable and ready to hand for the actual proofs [which are to come], we shall 
now set out the individual uniform motions of the sun in the following manner.

Since we have shown that one revolution contains 365;14,48\textsuperscript{d}, dividing the 
latter into the 360\textdegree of the circle, we find the mean daily motion of the sun as 
approximately 0;59,8,17,13,12,31\textdegree (it will be sufficient to carry out divisions to 
this number [i.e. 6] of sexagesimal places).

Next, taking \frac{1}{48th} of the daily motion, we find the hourly motion as 
approximately 0;2.27,50,43,3.1\textdegree.

Similarly, we multiply the daily motion by 30, the number of days in one 
month, and get as the mean monthly motion 29;34,8,36,36,15,30\textdegree;

and, multiplying it by 365, the number of days in one Egyptian year, we get 
the mean annual motion as 359;45,24,45,21,8,35\textdegree.

Then we multiply the yearly motion by 18 years, since this number will 
produce symmetry in the layout of the tables,\textsuperscript{28} and, after reduction of complete 
circles, we find the increment over 18 years to be 355;37,25,36,20,34,30\textdegree.

So we have set out three tables for the uniform motion of the sun, each again 
containing 45 lines, and each having two [vertical] sections. The first table will 
contain the mean motions of the 18-year periods, the second will contain the

\textsuperscript{27} This is an implicit polemic against the ephemeris kind of astronomical table which gives the 
true positions of the planets (their 'apparent places'). To judge from the surviving papyri, the most 
common kind of planetary table was that giving the entries of the heavenly bodies into the zodiacal 
signs for a period of years (see HAMA II 785 ff.). Ptolemy was perhaps thinking of a kind of 
'perpetual almanac' which gives the true positions of the planets at regular intervals over a whole 
planetary period. His argument is that his approach (mean motion tables modified by equation 
tables) gives a truer picture of the actual motions, which are uniform and circular.

\textsuperscript{28} Despite Ptolemy's clear statement here of his motivation for choosing the 18-year period, it has 
been the subject of much fruitless debate. Starting from a standard height of 45 lines (see I 10 p. 56 
n.67), and allowing some space for headings, he is led by the combination of single years on the same 
sheet with hours to 18 lines for that table (18 + 24 = 42 = 12 + 30 [months and days]). That is also 
the reason why the table for 18-year periods goes up to only 810 years (45 x 18), even though this 
does not reach Ptolemy's own time from his epoch. By the time he came to compose the Handy 
Tables, he had realised the inconvenience of this arrangement, and switched to 25-year periods and 
an epoch closer to his own time (Era Philip. \textsuperscript{-323} Nov. 12).
yearly motions above and the hourly motions below, and the third will contain
the monthly motions above and the daily motions below. The numbers
representing time will be in the first [i.e. left-hand] section, and the
corresponding degrees, obtained by successive addition of the appropriate
amount for each [time-unit], in the second [i.e. right-hand] section. The tables
are as follows.

2. `{Table of the mean motion of the sun}`

[See pp. 142-3.]

3. `{On the hypotheses for uniform circular motion}`

Our next task is to demonstrate the apparent anomaly of the sun. But first we
must make the general point that the rearward displacements of the planets
with respect to the heavens are, in every case, just like the motion of the universe
in advance, by nature uniform and circular. That is to say, if we imagine the
bodies or their circles being carried around by straight lines, in absolutely every
case the straight line in question describes equal angles at the centre of its
revolution in equal times. The apparent irregularity [anomaly] in their motions
is the result of the position and order of those circles in the sphere of each by
means of which they carry out their movements, and in reality there is in essence
nothing alien to their eternal nature in the 'disorder' which the phenomena are
supposed to exhibit. The reason for the appearance of irregularity can be
explained by two hypotheses, which are the most basic and simple. When their
motion is viewed with respect to a circle imagined to be in the plane of the
ecliptic, the centre of which coincides with the centre of the universe (thus its
centre can be considered to coincide with our point of view), then we can
suppose, either that the uniform motion of each [body] takes place on a circle
which is not concentric with the universe, or that they have such a concentric
circle, but their uniform motion takes place, not actually on that circle, but on
another circle, which is carried by the first circle, and [hence] is known as the
'epicycle'. It will be shown that either of these hypotheses will enable [the
planets] to appear, to our eyes, to traverse unequal arcs of the ecliptic (which is
concentric to the universe) in equal times.

In the eccentric hypothesis: [see Fig. 3.1] we imagine the eccentric circle, on
which the body travels with uniform motion, to be ABGD on centre E, with
diameter AED, on which point Z represents the observer. Thus A is the
apogee, and D the perigee. We cut off equal arcs AB and DG, and join BE, BZ,
GE and GZ. Then it is immediately obvious that the body will traverse the arcs

\[^{29}\text{Corrections to Heiberg's text: H210. 23-5, column of fourths (for arguments 342, 360 and 378).}
\text{A misprint has disrupted the order, which should be }\lambda, \upsilon, \nu, \iota, \beta, \lambda \text{, but has become }\nu, \iota, \beta, \lambda \text{.}\]
\[^{30}\text{See } \text{HAMA 55-7, Pedersen 134-44.}\]
\[^{31}\text{the observer'; literally 'our point of view'.}\]
### TABLE OF THE SUN'S MEAN MOTION

Distance [in Anomaly] from the Sun's Apogee in $\Box 5:30^\circ$ to its Mean Longitude in the 1st Year of Nabonassar, $X 0:45^\circ : 265:15^\circ$

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<td>38</td>
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</table>

| 126             | 329 | 21 | 59 | 14 | 24 | 1  | 30 |
| 144             | 324 | 59 | 24 | 30 | 44 | 36 | 0  |
| 162             | 320 | 36 | 50 | 27 | 5  | 10 | 30 |
| 180             | 316 | 14 | 16 | 3  | 25 | 45 | 0  |
| 198             | 311 | 51 | 41 | 39 | 46 | 19 | 30 |
| 216             | 307 | 29 | 7  | 16 | 6  | 54 | 0  |

| 234             | 303 | 6  | 32 | 52 | 27 | 28 | 30 |
| 252             | 298 | 43 | 58 | 28 | 48 | 3  | 0  |
| 270             | 294 | 21 | 24 | 5  | 8  | 37 | 30 |

| 288             | 289 | 58 | 49 | 41 | 29 | 12 | 0  |
| 306             | 285 | 36 | 15 | 17 | 49 | 46 | 30 |
| 324             | 281 | 13 | 40 | 54 | 10 | 21 | 0  |
| 342             | 276 | 51 | 6  | 30 | 30 | 55 | 30 |
| 360             | 272 | 28 | 32 | 6  | 51 | 30 | 0  |
| 378             | 268 | 5  | 57 | 43 | 12 | 4  | 30 |
| 396             | 263 | 43 | 23 | 19 | 32 | 39 | 0  |
| 414             | 259 | 20 | 48 | 55 | 53 | 13 | 30 |
| 432             | 254 | 58 | 14 | 32 | 13 | 48 | 0  |
| 450             | 250 | 35 | 40 | 8  | 34 | 22 | 30 |
| 468             | 246 | 13 | 5  | 44 | 54 | 57 | 0  |
| 486             | 241 | 50 | 31 | 21 | 15 | 31 | 30 |
| 504             | 237 | 27 | 56 | 57 | 36 | 6  | 0  |
| 522             | 233 | 5  | 22 | 33 | 56 | 40 | 30 |
| 540             | 228 | 42 | 48 | 10 | 17 | 15 | 0  |
| 558             | 224 | 20 | 13 | 46 | 37 | 49 | 30 |
| 576             | 219 | 37 | 39 | 22 | 58 | 24 | 0  |
| 594             | 215 | 35 | 4  | 59 | 18 | 58 | 30 |
| 612             | 211 | 12 | 30 | 35 | 39 | 33 | 0  |
| 630             | 206 | 49 | 56 | 12 | 0  | 7  | 30 |
| 648             | 202 | 27 | 21 | 48 | 20 | 42 | 0  |

| 666             | 198 | 4  | 47 | 24 | 41 | 16 | 30 |
| 684             | 193 | 42 | 13 | 1  | 1  | 51 | 0  |
| 702             | 189 | 19 | 38 | 37 | 22 | 25 | 30 |
| 720             | 184 | 57 | 4  | 13 | 43 | 0  | 0  |
| 738             | 180 | 34 | 29 | 30 | 3  | 34 | 30 |
| 756             | 176 | 11 | 55 | 26 | 24 | 9  | 0  |
| 774             | 171 | 49 | 21 | 2  | 44 | 43 | 30 |
| 792             | 167 | 26 | 46 | 39 | 5  | 18 | 0  |
| 810             | 163 | 4  | 12 | 15 | 25 | 52 | 30 |
### III 2. Solar mean motion table

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AB and GD in equal times, but will [in so doing] appear to have traversed unequal arcs of a circle drawn on centre Z. For
\[ \angle BEA = \angle GED. \]
But \[ \angle BZA < \angle BEA \] (or \[ \angle GED \]),
and \[ \angle GZD > \angle GED \] (or \[ \angle BEA \]).

In the epicyclic hypothesis: we imagine [see Fig. 3.2] the circle concentric with the ecliptic as ABGD on centre E, with diameter AEG, and the epicycle carried by it, on which the body moves, as ZH0K on centre A.

Then here too it is immediately obvious that, as the epicycle traverses circle ABGD with uniform motion, say from A towards B, and as the body traverses the epicycle with uniform motion, then when the body is at points Z and Θ, it will appear to coincide with A, the centre of the epicycle, but when it is at other points it will not. Thus when it is, e.g., at H, its motion will appear greater than the uniform motion [of the epicycle] by arc AH, and similarly when it is at K its motion will appear less than the uniform by arc AK.

Now in this kind of eccentric hypothesis the least speed always occurs at the apogee and the greatest at the perigee, since \[ \angle AZB \] [in Fig. 3.1] is always less than \[ \angle DZG \]. But in the epicyclic hypothesis both this and the reverse are possible. For the motion of the epicycle is towards the rear with respect to the heavens, say from A towards B [in Fig. 3.2]. Now if the motion of the body on the epicycle is such that it too moves rearwards from the apogee, that is from Z towards H, the greatest speed will occur at the apogee, since at that point both

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12 Ptolemy is hinting at the existence of another kind of eccentric hypothesis, one which is geometrically equivalent to that epicyclic hypothesis in which the sense of rotation is the same for both planet and epicycle. But he does not discuss this until XII 1 (p. 555), where we learn that the equivalence was already known to Apollonius of Perge (c. 200 B.C.). See HAMA 149-50.
epicycle and body are moving in the same direction. But if the motion of the body from the apogee is in advance on the epicycle, that is from Z towards K, then the reverse will occur: the least speed will occur at the apogee, since at that point the body is moving in the opposite direction to the epicycle.

Having established that, we must next make the additional preliminary point that for bodies which exhibit a double anomaly both the above hypotheses may be combined, as we shall prove in our discussions of such bodies, but for a body which displays a single invariant anomaly, a single one of the above hypotheses will suffice; and [in this case] all the phenomena will be represented, with no difference, by either hypothesis, provided that the same ratios are preserved in both. By this I mean that the ratio, in the eccentric hypothesis, of the distance between the centre of vision and the centre of the eccentre to the radius of the eccentre, must be the same as the ratio, in the epicyclic hypothesis, of the radius of the epicycle to the radius of the deferent; and furthermore that the time taken by the body, travelling towards the rear, to traverse the immovable eccentre, must be the same as the time taken by the epicycle, also travelling towards the rear, to traverse the circle with the observer as centre [the deferent], while the body moves with equal [angular] speed about the epicycle, but so that its motion at the apogee [of the epicycle] is in advance.

If these conditions are fulfilled, the identical phenomena will result from either hypothesis. We shall briefly show this [now] by comparing the ratios in abstract, and later by means of the actual numbers we shall assign to them for

\[\text{'deferent': see Introduction p. 21.}\]
the sun's anomaly.\textsuperscript{34} I say then, first, that in both hypotheses, the greatest difference between the uniform motion and the apparent, non-uniform motion (which is also the notional position of the mean speed for the bodies)\textsuperscript{35} occurs when the apparent distance from the apogee comprises a quadrant, and that the time between apogee [position] and the above-mentioned mean speed [position] is greater than the time between mean speed and perigee. Hence, for the eccentric hypothesis always, and for the epicyclic hypothesis when the motion at apogee is in advance, the time from least speed to mean is greater than the time from mean speed to greatest; for in both hypotheses the slowest motion takes place at the apogee. But [for the epicyclic hypothesis] when the sense of revolution of the body is rearwards from the apogee on the epicycle, the reverse is true: the time from greatest speed to mean is greater than the time from mean to least, since in this case the greatest speed occurs at the apogee.

First, then, [see Fig. 3.3] let the body's eccenter be ABGD on centre E, with diameter AEG. On this diameter take the centre of the ecliptic, that is, the position of the observer, at Z, and draw BZD through Z at right angles to AEG. Let the positions of the body be B and D, so that, obviously, its apparent distance from apogee A is a quadrant on either side. We have to prove that the greatest difference between mean and anomalistic motion takes place at points B and D.

Join EB and ED.

It is immediately obvious that the ratio of $\angle EBZ$ to 4 right angles equals the

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\textsuperscript{34} Reference to III 4 p. 157.

\textsuperscript{35} Ptolemy never attempts to prove this statement about the position where the apparent motion equals the mean motion, but it is intuitively seen to be true from the epicyclic model. See HAM.157, Pedersen 143.
ratio of the arc of the difference due to the anomaly\textsuperscript{36} to the whole circle; for \(\angle AEB\) subtends the arc of the uniform motion, and \(\angle AZB\) subtends the arc of the apparent, non-uniform motion, and the difference between them is \(\angle EBZ\).

I say, then, that no angle greater than these two [\(\angle EBZ\) and \(\angle EDZ\)] can be constructed on line \(EZ\) at the circumference of circle \(ABGD\).

[Proof:] Construct at points \(\Theta\) and \(K\) angles \(E\Theta Z\) and \(EKZ\), and join \(\Theta D\), \(KD\). Then since, in any triangle, the greater side subtends the greater angle,\textsuperscript{37} and \(\Theta Z > \Theta D\),

\[\therefore \angle \Theta DZ > \angle D\Theta Z.\]

But \(\angle ED\Theta = \angle E\Theta D\), since \(E\Theta = ED\) [radii].

Therefore, by addition, \(\angle EDZ = \angle EBD\) > \(\angle E\Theta Z\).

Again, since \(DZ > KZ\),

\[\angle ZKD > \angle ZDK.\]

But \(\angle EKD = \angle EDK\), since \(EK = ED\).

Therefore, by subtraction, \(\angle EDZ = \angle EBZ\) > \(\angle EKZ\).

Therefore it is impossible for any other angle to be constructed in the way defined greater than those at points \(B\) and \(D\).

Simultaneously it is proven that arc \(AB\), which represents the time from least speed to mean, exceeds \(BG\), which represents the time from mean speed to greatest, by twice the arc comprising the equation of anomaly. For \(\angle AEB\) exceeds a right angle (\(\angle EZB\)) by \(\angle EBZ\), and \(\angle BEG\) falls short of a right angle by the same amount.

To prove the same theorem again for the other hypothesis, let [Fig. 3.4] the circle concentric with the universe be \(ABG\) on centre \(D\) and diameter \(ADB\), and let the epicycle which is carried around it in the same plane be \(EZH\) on centre \(A\). Let us suppose the body to be at \(H\) when its apparent distance from the apogee is a quadrant. Join \(AH\) and \(DHG\).

I say that \(DHG\) is tangent to the epicycle; for that is the position in which the difference between uniform and anomalistic motion is greatest.

[Proof:] The mean motion, counted from the apogee, is represented by \(\angle EAH\); for the body traverses the epicycle with the same [angular] speed as the epicycle traverses circle \(ABG\). Furthermore the difference between mean and apparent motion is represented by \(\angle ADH\). Therefore it is clear that the amount by which \(\angle EAH\) exceeds \(\angle ADH\) (namely \(\angle AHD\)) represents the apparent distance of the body from the apogee. But this distance is, by hypothesis, a quadrant. Therefore \(\angle AHD\) is a right angle, and hence line \(DHG\) is tangent to epicycle \(EZH\). Therefore arc \(AG\), since it comprises the distance between the centre \(A\) and the tangent, is the greatest possible difference due to the anomaly.

By the same reasoning, arc \(EH\), which according to the sense of rotation on

\textsuperscript{36}This expression is later used as a technical term for the angle corresponding to \(\angle EBZ\) here, and is usually translated 'equation of anomaly'. See Introduction pp. 21–2.

\textsuperscript{37}Precisely this statement, that the greater angle is subtended by the greater side, is the enunciation of Euclid I 19 (which Heiberg refers to ad loc.). But in fact what underlies Ptolemy’s statement is that, if side \(a\) is greater than side \(b\), angle \(A\) is greater than angle \(B\), which is Euclid I 18. Perhaps we should adopt the reading of D, Ὠτὸ τὸν μείζονα πλευράν ἢ μείζον γωνία ὑποστίναι ('the greater angle subtends the greater side'), and assume that the text has been assimilated to the (wrong) Euclidean wording.
the epicycle assumed here, represents the time from least speed to mean, exceeds arc HZ, which represents the time from mean speed to greatest, by twice arc AG. For if we produce DH to Θ and draw AKΘ at right angles to EZ,

\[ \angle KAH = \angle ADG, \]

and arc KH = arc AC. And arc EKH is greater than a quadrant by arc KH, while arc ZH is less than a quadrant by arc KH.

Q.E.D.

It is also true that the same effects will be produced by both hypotheses if one takes a partial motion over the same stretch of time for both, whether one considers the mean motion or the apparent, or the difference between them, that is the equation of anomaly. The best way to see that is as follows.

[See Fig. 3.5.] Let the circle concentric with the ecliptic be ABG on centre D, and let the circle which is eccentric but equal to the concentre ABG be EZH on centre Θ. Let the common diameter through their centres D, Θ and the
III 3. Equivalence of eccentric and epicyclic hypotheses

I ll 3. Equivalence of eccentric and epicyclic hypotheses

apogee E be $EA\Theta D$. Cut off at random an arc $AB$ on the concentre, and with centre $B$ and radius $D\Theta$ draw the epicycle $KZ$. Join $KBD$.

I say that the body will be carried by both kinds of motion [i.e. according to both hypotheses] to point $Z$, the intersection of the eccentric and the epicycle, in the same time in all cases (that is, the three arcs, $EZ$ on the eccentric, $AB$ on the concentre, and $KZ$ on the epicycle, are all similar), and that the difference between uniform and anomalistic motion, and the apparent positions of the body, will turn out to be one and the same according to both hypotheses.

[Proof:] Join $Z\Theta$, $BZ$ and $DZ$.

Since, in the quadrilateral $BD\Theta Z$, the opposite sides are equal, $Z\Theta$ to $BD$ and $BZ$ to $D\Theta$, $BD\Theta Z$ is a parallelogram.

Therefore $\angle E\Theta Z = \angle ADB = \angle ZBK$.

Therefore, since they are angles at the centre [of circles], the arcs subtended by them are also similar, i.e.

Arc $EZ$ of the eccentric $||$ arc $AB$ of the concentre $||$ arc $KZ$ of the epicycle.

Therefore the body will be carried by both kinds of motions in the same time to the same point, $Z$, and will appear to have traversed the same arc $AL$ of the ecliptic from the apogee, and accordingly the equation of anomaly will be the same in both hypotheses; for we showed that that equation is represented by $\angle DZ\Theta$ in the eccentric hypothesis and by $\angle BDZ$ in the epicyclic hypothesis, and these two angles are alternate and equal, since, as we have shown, $Z\Theta$ is parallel to $BD$.

It is obvious that the same results will hold good for all distances [of the body from the apogee]. For quadrilateral $\Theta DZB$ will always be a parallelogram, and [hence] the motion of the body on the epicycle will actually describe the
eccentric circle, provided the ratios\(^{11}\) are similar and their members equal in both hypotheses.

Moreover, even if the members are unequal in size, provided their ratios are similar, the same phenomena will result. This can be shown as follows.

As before [see Fig. 3.6] let the circle concentric with the universe be ABG on centre D and the diameter, on which the body reaches apogee and perigee positions, ADG. Let the epicycle be drawn on point B, at an arbitrary distance, arc AB, from apogee A. Let the arc traversed by the body [on the epicycle] be EZ, which is, obviously, similar to AB, since the revolutions on [both] circles have the same period. Join DBE, BZ, DZ.

![Fig. 3.6](image)

Now it is immediately obvious that, according to this [epicyclic] hypothesis, \(\angle ADE\) will always equal \(\angle ZBE\), and the body will appear to lie on line DZ.

But I say that the body will also appear to lie on the same line DZ according to the eccentric hypothesis, whether the eccentre is greater or smaller than the concentre ABG, provided only that one assumes that the ratios are similar and that the periods of revolution are the same.

[Proof:] Let the eccentre be drawn under the conditions we have described, greater [than the concentre] as H© on centre K ([which must lie] on AG), and

\(^{11}\) The ratios are \(c:R\) and \(e:R\).
III 3. Symmetries in eccentric hypothesis

smaller [than the concentric] as LM on centre N (this too [must lie on AG]).
Produce DZ as DMZΘ, and DA as DLAH, and join ΘK, MN.

Then since

\[ DB : BZ = \Theta K : KD = MN : ND \] [by hypothesis],

and \( \angle BZD = \angle MDN \) (since DA is parallel to BZ);

the three triangles \( \{ZDB, D\Theta K, DMN\} \) are equiangular,

and \( \angle BDZ = \angle D\Theta K = \angle DMN \) (angles subtended by corresponding sides).

Therefore \( DB, \Theta K \) and \( MN \) are parallel.

\[ \therefore \angle ADB = \angle AK\Theta = \angle ANM. \]

Since these angles are at the centres of their circles, the arcs on them, \( AB, H\Theta \) and \( LM \), will also be similar.

So it is true, not only that the epicycle has traversed arc \( AB \) in the same time as the body has traversed arc \( EZ \), but also that the body will have traversed arcs \( H\Theta \) and \( LM \) on the eccentres in that same time; hence in every case it will be seen along the same line \( DMZ\Theta \), according to the epicyclic [hypothesis] at point \( Z \), according to the greater eccentre at point \( \Theta \), and according to the smaller eccentre at point \( M \). The same will hold true in all positions.

A further consequence is that where the apparent distance of the body from apogee [at one moment] equals its apparent distance from perigee [at another], the equation of anomaly will be the same at both positions.

[Proof:] In the eccentric hypothesis [see Fig. 3.7], we draw the eccentric circle

\[ \text{ABGD on centre } E \text{ and diameter AEG through apogee } A. \text{ We suppose the observer to be located at } Z, \text{ and draw an arbitrary [chord] } BZD \text{ through } Z, \text{ and join } EB \text{ and } ED. \text{ Then the apparent positions [of the body at } B \text{ and } D] \text{ will be equal and opposite, that is the angle } AZB \text{ from the apogee will be equal and} \]

Fig. 3.7

ABGD on centre E and diameter AEG through apogee A. We suppose the observer to be located at Z, and draw an arbitrary [chord] BZD through Z, and join EB and ED. Then the apparent positions [of the body at B and D] will be equal and opposite, that is the angle AZB from the apogee will be equal and
opposite to angle GZD from the perigee; and the equation of anomaly will be the same [in both cases], since

$$BE = ED, \text{ and } \angle EBZ = \angle EDZ.$$ 

So the arc $[AB]$ of mean motion counted from the apogee $A$ will exceed the arc of apparent motion (i.e. the arc subtended by angle $AZB$) by the same equation [equal to $\angle EBZ$] as the arc of mean motion counted from the perigee $G$ is exceeded by the arc of apparent motion (i.e. the [equal] arc subtended by $\angle GZD$). For

$$\angle AEB > \angle AZB, \text{ and } \angle GED < \angle GZD.$$ 

In the epicyclic hypothesis [see Fig. 3.8] if, as before, we draw the concentre $ABG$ on centre $D$ and diameter $ADG$, and the epicycle $EZH$ on centre $A$, draw an arbitrary line $DHBZ$, and join $AZ$ and $AH$, then the arc $AB$ representing the equation of anomaly will be the same at both positions, i.e. whether the body is at $Z$ or at $H$. And the distance of the body from the point on the ecliptic corresponding to the apogee when it is at $Z$ will be equal to its distance from the point corresponding to the perigee when it is at $H$. For the arc of its apparent distance from the apogee is represented by $\angle DZA$, since, as we showed, this is the difference between the mean motion and the equation of anomaly.$^{42}$ And the arc of its apparent distance from the perigee is represented by $\angle ZHA$ (for this, too, is equal to the mean motion from the perigee plus the equation of anomaly).

But $\angle DZA = \angle ZHA$, since $AZ = AH$. 

$^{42} \angle DZA = \angle EAZ - \angle ADZ$. Shown p. 147.
Thus here too we conclude that the mean motion exceeds the apparent near the apogee (i.e. $\angle EAZ$ exceeds $\angle AZD$) by the same equation (namely $\angle ADH$) as the mean motion is exceeded by the (same) apparent motion (i.e. $\angle HAD$ by $\angle AHZ$) near the perigee. Q.E.D.

4. [On the apparent anomaly of the sun]^{43}

Having set out the above preliminary theorems, we must add a further preliminary thesis concerning the apparent anomaly of the sun. This has to be a single anomaly, of such a kind that the time taken from least speed to mean shall always be greater than the time from mean speed to greatest, for we find that this accords with the phenomena. Now this could be represented by either of the hypotheses described above, though in case of the epicyclic hypothesis the motion of the sun on the apogee arc of the epicycle would have to be in advance. However, it would seem more reasonable to associate it with the eccentric hypothesis, since that is simpler and is performed by means of one motion instead of two.^{44}

Our first task is to find the ratio of the eccentricity of the sun's circle, that is, the ratio which the distance between the centre of the eccentric and the centre of the ecliptic (located at the observer) bears to the radius of the eccentric. We must also find the degree of the ecliptic in which the apogee of the eccentric is located. These problems have been solved by Hipparchus with great care.^{45} He assumes that the interval from spring equinox to summer solstice is 94\(^{1}\) days, and that the interval from summer solstice to autumnal equinox is 92\(^{1}\) days, and then, with these observations as his sole data, shows that the line segment between the above-mentioned centres [of eccentric and ecliptic] is approximately $\frac{53}{10}$th of the radius of the eccentric, and that the apogee is approximately $24^{10}$ (where the ecliptic is divided into 360°) in advance of the summer solstice. We too, for our own time, find approximately the same values for the times [taken by the sun to traverse] the above-mentioned quadrants, and for those ratios. Hence it is clear to us that the sun's eccentric always maintains the same position relative to the solstitial and equinoctial points.\(^{46}\)

In order not to neglect this topic, but rather to display the theorem worked out according to our own numerical solution, we too shall solve the problem, for the eccentric, using the same observed data, namely, as already stated, that the interval from spring equinox to summer solstice comprises 94\(^{1}\) days, and that...

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^{43} See HAMA 57-8, Pedersen 144-9.

^{44} On the desirability of simplicity in hypotheses see III 1 p. 136 with n.17.

^{45} Reading $\mu\nu\mu\vartheta\mu\nu\vartheta$ (with D, Ar) at H233,1-2 for $\mu\vartheta \vartheta\mu\vartheta$ ("with care").

^{46} According to Ptolemy the sun's apogee (unlike those of the five planets, as it later turns out, IX 7) does not share in the motion of precession. The reproaches that have been cast on Ptolemy (e.g. by Manitius 1 428-9) for failing to discover that the sun's apogee too has a motion through the ecliptic are unjustified. To do that he would have needed observations of the time of equinox and solstice far more accurate than those available (to the nearest 1-day), and not only for his own time but also for an earlier time. See the papers by Rome[3] and Petersen and Schmidt for a mathematical demonstration of this.
from summer solstice to autumnal equinox $92\frac{1}{2}$ days. For our own very precise observations of the equinoxes and the summer solstice in the 463rd year from the death of Alexander confirm the day-totals in these intervals: as we said, [III 1, p. 138], the autumnal equinox occurred on Athyr [III] 9, [139 Sept. 26], after sunrise, the spring equinox on Pachon [IX] 7 [140 March 22], after noon (thus the interval [between them] is $178\frac{1}{2}$ days), and the summer solstice on Mesore [XII] 11/12, [140 June 24/25], after midnight. Thus this interval, from spring equinox to summer solstice, comprises $94\frac{1}{2}$ days, which leaves approximately $92\frac{1}{2}$ days to complete the year; this number represents the interval from the summer solstice to the following autumnal equinox.

[See Fig. 3.9.] Let the ecliptic be $ABGD$ on centre $E$. In it draw two diameters, $AG$ and $BD$, at right angles to each other, through the solsticial and equinoctial points. Let $A$ represent the spring [equinox], $B$ the summer [solstice], and so on in order.

Fig. 3.9

$^4$In III 1 the precise times of day given are '1 hour after sunrise', '1 hour after noon' and '2 hours after midnight'. Thus the precise intervals are $178\frac{1}{2}$ days and $94\frac{1}{2} 13^b$, leading to corrected figures of $94^a 13^b$ and $92^c 11^a$ for the intervals used in the computation. But see p. 139 n.23 for the possibility that the time of solstice is '2 seasonal hours' (i.e. equinoctial hours). Even as small a change as 1 hour in an interval has an effect of about 1° in the location of the apogee (cf. Petersen and Schmidt 80-3 and Rome[3] 13-15).
Now it is clear that the centre of the eccentre will be located between lines EA and EB. For semi-circle ABG comprises more than half of the length of the year, and hence cuts off more than a semi-circle of the eccentre; and quadrant AB too comprises a longer time and cuts off a greater arc of the eccentre than quadrant BG. This being so, let point Z represent the centre of the eccentre, and draw the diameter through both centres and the apogee, EZH. With centre Z and arbitrary radius draw the sun’s eccentre ΘKLM, and draw through Z lines NXO parallel to AG and PRS parallel to BD. Draw perpendicular ΘTY from Θ to NXO and perpendicular KFQ from K to PRS.

Now since the sun traverses circle ΘKLM with uniform motion, it will traverse arc ΘK in 94$rac{1}{2}$ days, and arc KL in 92$rac{1}{2}$ days. In 94$rac{1}{2}$ days its mean motion is approximately 93;9°, and in 92$rac{1}{2}$ days 91;11°. Therefore

\[
\text{arc } \Theta KL = 184;20^\circ
\]

and, by subtraction of the semi-circle NPO [from arc ΘKL],

\[
\text{arc } N\Theta + \text{arc } LO \approx 184;20^\circ - 180^\circ = 4;20^\circ
\]

So arc ΘNY = 2 arc ΘN = 4;20^\circ also,

\[
\therefore \Theta Y = \text{Crd arc } \Theta NY = 4;32^\circ,
\]

where the diameter of the eccentre is 120°.

Now since arc ΘNPK = 93;9°,

and arc ΘN = 2;10^\circ and quadrant NP = 90°,

by subtraction, arc PK = 0;59^\circ,

and arc KPQ = 2 arc PK = 1;58^\circ.

\[
\therefore KFQ = \text{Crd arc } KPQ = 2;4^\circ,
\]

where the diameter of the eccentre is 120°.

And we have shown that EX = 2;16^\circ in the same units.

Now since EZ$^2$ = ZX$^2$ + EX$^2$,

EZ =$^2$291 where the radius of the eccentre is 60°.

Therefore the radius of the eccentre is approximately 24 times the distance between the centres of the eccentre and the ecliptic.

Now, since EZ:ZX = 2;29:1:2,

ZX will be about 49;46° where hypotenuse EZ = 120°.

Therefore, in the circle about right-angled triangle EZX,

\[
\text{arc } ZX = 49^\circ
\]

\[
\therefore \angle ZEX = \left\{ \begin{array}{l} 49^\circ \text{ where 2 right angles } = 360^\circ \\ 24;30^\circ \text{ where 4 right angles } = 360^\circ \end{array} \right.
\]

So, since \angle ZEX is an angle at the centre of the ecliptic, arc BH, which is the amount by which the apogee at H is in advance of the summer solstice at B, is also 24;30°.

Furthermore, since quadrants OS and SN are each 90°,

and arc OL = arc ΘN =2;10°,

and arc MS = 0;59°,

\[
\therefore \text{arc } LM = 86;51^\circ,
\]

and arc MΘ = 88;49°.

But the sun in its uniform motion travels

86;51° in about 88$\frac{1}{2}$ days,

and 88;49° in about 90$\frac{1}{2}$ days.

Hence it is clear that the sun will traverse arc GD, which extends from the
autumnal equinox to the winter solstice, in about 88\(\frac{1}{2}\) days, and arc DA, which extends from the winter solstice to the spring equinox, in about 90\(\frac{1}{2}\) days. The above conclusions are in agreement with what Hipparchus says.

Using these quantities, then, let us first see what the greatest difference between mean and anomalistic motions is, and at what points it will occur.

[See Fig. 3.10.] Let the eccentric circle be ABG on centre D and diameter ADG through the apogee A, on which E represents the centre of the ecliptic. Draw EB at right angles to AG, and join DB.

Now since, where BD, the radius, equals 60°, DE, the eccentricity, equals 2;30° (according to the ratio 24:1),

in the circle about right-angled triangle BDE,

\[ DE = 5° \text{ where hypotenuse } BD = 120°, \]

and arc DE \(\approx 4;46°\).

Therefore \(\angle DBE\), which represents the greatest equation of anomaly,

\[ \frac{4;46°}{2 \text{ right angles} = 360°} = \frac{2;23°}{4 \text{ right angles} = 360°}. \]

In the same units, right angle BED = 90°,

and \(\angle BDA = \angle DBE + \angle BED = 92;23°\).

Thus, since \(\angle BDA\) is at the centre of the eccentric and \(\angle BED\) is at the centre of the ecliptic, we conclude that the greatest equation of anomaly is 2;23°, and the position where it occurs is 92;23° from the apogee, measured along the eccentric in uniform motion, and (as we proved earlier) a quadrant, or 90° [from the apogee], measured along the ecliptic in anomalistic motion. It is obvious from our previous results that in the opposite semi-circle\(^{18}\) the mean speed and the greatest equation of anomaly will occur at 270° of apparent motion, and at 267;37° of mean motion on the eccentric.

\(^{18}\) Reading ἴμικύκλιον (with D.Ar) for τμήμα ('segment') at H239.12.
We now want to use numerical computation, as we promised [pp. 145-6], to show that one derives the same quantities from the epicyclic hypothesis too, provided the same ratios are preserved in the way we explained.

[See Fig. 3.11.] Let the circle concentric to the ecliptic be ABG on centre D and diameter ADG, and the epicycle circle EZH on centre A. From D draw a tangent to the epicycle, DZB, and join AZ. Then, as before, in the right-angled triangle ADZ, AD is 24 times AZ, so that, in the circle about right-angled triangle ADZ, AZ is, again, 5° where hypotenuse AD is 120°, and the arc on AZ is 4;46°.

\[ \angle ADZ = \begin{cases} 4;46° & \text{where 2 right angles} = 360° \\ 2;23° & \text{where 4 right angles} = 360°. \end{cases} \]

Therefore the greatest equation of anomaly, namely arc AB, has been found to be 2;23° here too, in agreement with [the previous result], and the arc of anomalistic motion is 90°, since it is represented by the right angle AZD, while the arc of mean motion, which is represented by \( \angle EAZ \), is again 92;23°.

5. {On the construction of a table for individual subdivisions of the anomaly}^{49}

In order to enable one to determine the anomalistic motion over any

\footnote{Reading τῶν ἀνωμαλίων κανονοποιίας at H240.16-17, with D (cf. all Greek mss. in the table of contents, H190.9-10) for τῆς ἀνωμαλίας ἐπισκέψεως ('investigation of the anomaly for partial stretches', which is the reading of Ar in both places).
On chs. 5 and 6 see H.A.M.1 58-60, Pedersen 149-51.}
subdivision [of the circle], we shall show, again for both hypotheses, how, given one of the arcs in question, we can compute the others.

H241 [See Fig. 3.12.] First, let the circle concentric to the ecliptic be \( ABG \) on centre \( D \), the eccentre \( EZH \) on centre \( O \), and let the diameter through both centres and the apogee \( E \) be \( EA\Theta DH \). Cut off arc \( EZ \), and join \( ZD, Z\Theta \). First, let arc \( EZ \) be given, e.g. as \( 30^\circ \).

![Fig. 3.12](image_url)

Produce \( Z\Theta \) and drop the perpendicular to it from \( D, DK \).

Then, since arc \( EZ \) is, by hypothesis, \( 30^\circ \),

\[
\angle E\Theta Z = \angle D\Theta K = \begin{cases} 
30^\circ & \text{where 4 right angles } = 360^\circ \\
60^\circ & \text{where 2 right angles } = 360^\circ.
\end{cases}
\]

Therefore, in the circle about right-angled triangle \( D\Theta K \),

\[
\begin{align*}
dk &= 60^\circ \\
and \ k\Theta &= 120^\circ \text{ (supplement).}
\end{align*}
\]

Therefore the corresponding chords

\[
\begin{align*}
dk &= 60^\circ \\
and \ k\Theta &= 103;55^\circ
\end{align*}
\]

Therefore, where \( D\Theta = 2;30^\circ \) and radius \( Z\Theta = 60^\circ \),

\[
\begin{align*}
dk &= 1;15^\circ \text{ and } \Theta K = 2;10^\circ.
\end{align*}
\]

H242 Therefore, by addition [of \( \Theta K \) to radius \( Z\Theta \)], \( K\Theta Z = 62;10^\circ \)

Now since \( DK^2 + K\Theta Z^2 = ZD^2 \),

the hypotenuse \( ZD \approx 62;11^\circ \).

Therefore, where \( ZD = 120^\circ \), \( DK = 2;25^\circ \),

and, in the circle about right-angled triangle \( ZDK \),

\[
\begin{align*}
dk &= 2;18^\circ.
\end{align*}
\]
III 5. Derivation of mean motion from anomalistic motion

\[ \angle DZK = \begin{cases} 2;18^\circ & \text{where 2 right angles} = 360^\circ \\ 1;9^\circ & \text{where 4 right angles} = 360^\circ \end{cases} \]

That [1;9°] will be the amount of the equation of anomaly at this position.

And \( \angle E\Theta Z \) was taken as 30°.

Therefore, by subtraction, \( \angle ADB \) (which equals arc AB of the ecliptic) equals 28;51°.

Furthermore, if any other of the [relevant] angles be given [instead of \( \angle E\Theta Z \)], the remaining angles will be given, as is immediately obvious if, in the same figure [see Fig. 3.13] we drop perpendicular \( \Theta L \) from \( \Theta \) on to \( ZD \).

For suppose first that arc AB of the ecliptic, i.e. \( \angle \Theta DL \), is given. Then the ratio \( D\Theta:\Theta L \) will be given. And since \( D\Theta:\Theta Z \) is also given, \( \Theta Z:\Theta L \) will be given. Hence \( \angle \Theta ZL \), the equation of anomaly, will be given. and so will \( \angle E\Theta Z \), i.e. arc EZ of the eccentric.

Or suppose, secondly, that the equation of anomaly, i.e. \( \angle \Theta ZD \), is given: we will get the same results in reverse order. For from \( \angle \Theta ZD \) the ratio \( \Theta Z:\Theta L \) will be given, and \( \Theta Z:\Theta D \) is given from the beginning. Hence \( D\Theta:\Theta L \) will be given, and hence \( \angle \Theta DL \), i.e. arc AB of the ecliptic, and [hence] \( \angle E\Theta Z \), i.e. arc EZ of the eccentric.

---

50 Euclid Data 40: if the angles of a triangle are given, its sides are given in form (i.e. the ratio of the sides is given, cf. Data 3).

51 Euclid Data 8: magnitudes having a given ratio to the same magnitude have a given ratio to each other. \( D\Theta:\Theta Z \) is given as the ratio of eccentricity.

52 Euclid Data 43: if, in a right-angled triangle, the sides about one of the acute angles have a given ratio, the triangle is given in form (cf. n.50).
Next [see Fig. 3.14] let the circle concentric with the ecliptic be ABG on centre D and diameter ADG, and let the epicycle (in the same ratio to circle ABG as the eccentricity to the eccentre) be EZHΘ on centre A. Cut off arc EZ and join ZBD and ZA. Let arc EZ again be taken in the same amount, 30°. Drop perpendicular ZK from Z on to AE.

Fig. 3.14

Since arc EZ = 30°,
\[ \angle EAZ = \begin{cases} 30° & \text{where 4 right angles = 360°} \\ 60° & \text{where 2 right angles = 360°} \end{cases} \]

Therefore in the circle about right-angled triangle AZK,
arc ZK = 60°
and arc AK = 120° (supplement).
Therefore the corresponding chords
\[ ZK = 60° \]
and \[ KA = 103;55° \] where the diameter AZ = 120°.
Therefore where hypotenuse AZ = 2;30° and radius AD = 60°,
\[ ZK = 1;15°, KA = 2;10°, \]
and, by addition, KAD = 62;10°.
And since \[ ZK^2 + KD^2 = ZBD^2, \]
\[ ZD = 62;11°, \] where \[ ZK = 1;15°. \]
So where hypotenuse DZ = 120°, ZK = 2;25°,
and, in the circle about right-angled triangle DZK,
arc ZK = 2;18°.
\[ \therefore \angle ZDK = \begin{cases} 2;18° & \text{where 4 right angles = 360°} \\ 1;9° & \text{where 2 right angles = 360°} \end{cases} \]
This is, again, the amount of the equation of anomaly, which is represented by arc AB.

And $\angle EAZ$ was taken as $30^\circ$.

Therefore, by subtraction, $\angle AZD$, which represents the arc of apparent motion on the ecliptic, is $28.51^\circ$.

These amounts are in agreement with what we found for the eccentric hypothesis.

Here too, if any other angle be given [instead of $\angle EAZ$], the remaining angles will be given, [as can be seen] on the same figure [see Fig. 3.15] if the perpendicular $AL$ is dropped from $A$ on to $DZ$.

For if, as before, we first take the arc of apparent motion on the ecliptic, i.e. $\angle AZD$, as given, from this the ratio $ZA:AL$ will be given. And since $ZA:AD$ was given from the beginning, $DA:AL$ will be given. Hence $\angle ADB$ will be given, i.e. arc $AB$, the arc of the equation of anomaly, and so will $\angle EAZ$, i.e. arc $EZ$ of the epicycle.

Of if, secondly, we take the equation of anomaly, i.e. $\angle ADB$, as given, then, in the same way but in reverse order, from this $AD:AL$ will be given; and since $DA:AZ$ was given from the beginning, $ZA:AL$ will also be given; and hence $\angle AZD$ will be given, which corresponds to the arc of apparent motion on the ecliptic, and so will $\angle EAZ$, i.e. arc $EZ$ of the epicycle.

Let us again take the previous figure for the eccentric [see Fig. 3.16], and cut off from $H$, the perigee of the eccentric, arc $HZ$, which we again take as $30^\circ$. Join $DZB$ and $Z\Theta$, and drop perpendicular $DK$ from $D$ on to $\Theta Z$. 

Fig. 3.15
Then since arc $ZH = 30^\circ$,
\[ \angle Z\Theta H = \begin{cases} 30^\circ & \text{where 4 right angles} = 360^\circ \\ 60^\circ & \text{where 2 right angles} = 360^\circ. \end{cases} \]
Therefore in the circle about right-angled triangle $D\Theta K$,
arc $DK = 60^\circ$
and arc $K\Theta = 120^\circ$ (supplement).
Therefore the corresponding chords
\[ \begin{align*} 
DK &= 60^\circ \\
and K\Theta &= 103;55^\circ \end{align*} \]
where diameter $D\Theta = 120^\circ$.
Therefore where hypotenuse $D\Theta = 2;30^\circ$ and radius $\Theta Z = 60^\circ$,
\[ DK = 1;15^\circ \text{ and } \Theta K = 2;10^\circ, \]
and $KZ = 57;50^\circ$ by subtraction [of $\Theta K$ from $\Theta Z$]
And since $DZ^2 = DK^2 + KZ^2$,
$DZ \approx 57;51^\circ$ where $DK = 1;15^\circ$.
Therefore where hypotenuse $DZ = 120^\circ$, $DK = 2;34^\circ$.\(^{53}\)
And, in the circle about right-angled triangle $DZK$,
arc $DK = 2;27^\circ$.
\[ \therefore \angle DZK = \begin{cases} 2;27^\circ & \text{where 2 right angles} = 360^\circ \\ 1;14^\circ & \text{(approximately) where 4 right angles} = 360^\circ. \end{cases} \]

\(^{53}\) Reading $\overline{\beta \lambda \delta}$ for $\overline{\beta \lambda \delta \lambda \gamma}$ (2;34,36) at H247,6, with Ar. Accurate computation gives 2;35,34 (cf. reading of $D^3$), but Ptolemy gives his results here only to minutes, and 2;34 is correct, since $\text{Crdf } 2;27^\circ = 2;33;55^\circ = 2;34^\circ$. The 36 was presumably a marginal correction to the 34 (cf. reading of D at H249,20), which was later mistakenly incorporated as an extra place. The same correction has to be made at H249,20 (both made by Manitius).
This \( [1;14^\circ] \), then, is the equation of anomaly.

And since \( \angle Z\Theta H \) was taken as \( 30^\circ \), by addition, \( \angle BDG \), i.e. arc GB of the ecliptic, equals \( 31;14^\circ \).

Here too, in the same way [as before], [see Fig. 3.17], we produce BD and drop perpendicular \( \Theta L \) on to it.

\[ \text{Fig. 3.17} \]

Then if, first, we take arc GB of the ecliptic, i.e. \( \angle \Theta DL \), as given, from this the ratio \( D\Theta:\Theta L \) will be given. And since \( \Theta D:\Theta Z \) was also given from the beginning, \( Z\Theta:\Theta L \) will be given. Hence we will have as given angles

\( \angle \Theta ZD \), i.e. the equation of anomaly

and \( \angle Z\Theta D \), i.e. arc HZ of the eccentric.

Or if, secondly, we take the equation of anomaly, i.e. \( \angle \Theta ZD \), as given, then conversely, from this \( Z\Theta:\Theta L \) will be given. And since \( Z\Theta:\Theta D \) was also given from the beginning, \( D\Theta:\Theta L \) will be given. Hence we will have, as given angles.

\( \angle \Theta DL \), which corresponds to arc GB of the ecliptic

and \( \angle Z\Theta H \), i.e. arc HZ of the eccentric.

Similarly, on the previous figure of concentre and epicycle [see Fig. 3.18], we cut off arc \( \Theta H \) from the perigee, in the same amount of \( 30^\circ \), join AH and DHB, and drop perpendicular HK from H on to AD.

Then since arc \( \Theta H \) is again \( 30^\circ \),

\[
\angle \Theta AH = \begin{cases} 30^\circ & \text{where 4 right angles} = 360^\circ \\ 60^\circ & \text{where 2 right angles} = 360^\circ \end{cases}
\]

Therefore in the circle about right-angled triangle HKA,

\[ \text{arc HK} = 60^\circ \]

and \( \text{arc AK} = 120^\circ \) (supplement).
Therefore the corresponding chords
\[ \text{HK} = 60^\circ \]
and \[ \text{AK} = 103;55^\circ \]
where hypotenuse \( \text{AH} = 120^\circ \).

Therefore where \( \text{AH} = 2;30^\circ \) and radius \( \text{AD} = 60^\circ \),
\[ \text{HK} = 1;15^\circ, \quad \text{AK} = 2;10^\circ \]
and \( \text{KD} = 57;50^\circ \), by subtraction.
and since \( \text{HK}^2 + \text{KD}^2 = \text{DH}^2 \),
\[ \text{DH} \approx 57;51^\circ \text{ where } \text{KH} = 1;15^\circ. \]
Therefore where hypotenuse \( \text{DH} = 120^\circ \)
\[ \text{HK} = 2;34^\circ, \]
and, in the circle about \( \text{DHK} \), arc \( \text{HK} = 2;27^\circ. \)

\[ \therefore \angle \text{HDK} = \begin{cases} 2;27^\circ & \text{where 2 right angles } = 360^\circ \\ 1;14^\circ (\text{approximately}) & \text{where 4 right angles } = 360^\circ \end{cases} \]

Here too, then, that is the size of the equation of anomaly, i.e. arc \( \text{AB} \).

And since \( \angle \text{KAH} \) was taken as \( 30^\circ \), by addition, \( \angle \text{BHA} \), which represents the apparent motion on the ecliptic [counted from perigee], is \( 31;14^\circ \). These amounts agree with those found for the eccentric [hypothesis].

Here too, in the same way [as before], we drop perpendicular \( \text{AL} \) on to \( \text{DB} \) [see Fig. 3.19].

Then if, first, we take the arc of the ecliptic, i.e. \( \angle \text{AHL} \), as given, from this the ratio \( \text{HA}:\text{AL} \) will be given. And since \( \text{HA}:\text{AD} \) was given from the beginning, \( \text{DA}:\text{AL} \) will be given. Thence we will have as given angles
\( \angle \text{ADB} \), i.e. arc \( \text{AB} \), representing the equation of anomaly
and \( \angle \Theta \text{AH} \), i.e. arc \( \Theta \text{H} \) of the epicycle.

Or if, secondly, we take as given arc \( \text{AB} \), representing the equation of
anomaly, i.e. $\angle ADB$, then, in the same way but in reverse order, from this the ratio $DA:AL$ will be given. And since $DA:AH$ is given from the beginning, $HA:AL$ will also be given. Hence we will have as given angles

$\angle AHL$, i.e. the arc of the ecliptic
and $\angle \Theta AH$, i.e. arc $\Theta H$ of the epicycle.
Thus we have proved what we set out to do.

In order to have conveniently available the amount of the correction for any given position, [we want] to establish a table, subdivided into [appropriate] sections, for the computation of the apparent positions from the anomaly. The above theorems would allow a wide variety in the form of such a table, but we prefer that form in which the argument is the mean motion and the function is the equation of anomaly. For this form accords well with the actual theories, and it also provides a simple but highly practical way of computing any desired result. So using the first set of theorems [i.e. with the eccentric hypothesis] which we used in the numerical examples above, we computed geometrically, in the way described, for the individual subdivisions [of the circle], the equation of anomaly corresponding to the arc of mean motion. In general, both for the sun and for the other bodies, we divided the quadrants near the apogee into 15 subdivisions (thus in these quadrants the interval of tabulation is $6^\circ$), and the
quadrants near the perigee into 30 subdivisions (thus in these the interval of tabulation is $3^\circ$). The reason is that the differences between [successive] equations of anomaly, for equal subdivisions [of the argument], are greater near the perigee than near the apogee.

We shall set out the table of the sun's anomaly, then, in 45 lines, as before, and 3 columns. The first two columns will contain the numbers of the mean motion through $360^\circ$: the first 15 lines will comprise the two quadrants near the apogee, the next 30 the two quadrants near the perigee. The third column will contain the degrees of equation of anomaly to be added or subtracted, corresponding to the appropriate mean motion. The table is as follows.

### 6. Table of the Sun's Anomaly

[See p. 167.]

### 7. On the Epoch of the Sun's Mean Motion

It remains to establish the epoch of the sun's mean motion, in order to be able to compute the particular position for any given time. In making our exposition of that matter, we shall again use those positions of the body which we ourselves have observed most accurately (this is our general rule both for the sun and for the other planets), but we use the mean motions we have derived to compute back to the beginning of the reign of Nabonassar for the epochs we establish. For that is the era beginning from which the ancient observations are, on the whole, preserved down to our own time.

[See Fig. 3.20.] Let the circle concentric with the ecliptic be ABG on centre D, and the sun's eccentric EZH on centre $\Theta$, and let the diameter through both centres and the apogee E be EAHG. Let B represent the autumnal equinox on the ecliptic. Join BZD and Z$\Theta$, and drop perpendicular $\Theta K$ from $\Theta$ on to ZD produced.

Then since B, the autumnal equinox, is located at the beginning of Libra, and G, the perigee, at $\chi$ $51^\circ$,

\[
\text{arc } BG = 65;30^\circ.
\]

\[
\therefore \angle BDG = \angle \Theta DK = \begin{cases} 65;30^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\ 131^\circ \text{ where } 2 \text{ right angles } = 360^\circ. \end{cases}
\]

Therefore in the circle about right-angled triangle $D\Theta K$,

\[
\text{arc } \Theta K = 131^\circ,
\]

and its chord $\Theta K = 109;12^\circ$ where the diameter $D\Theta = 120^\circ$.

---

57 See HAMA 58-60, Pedersen 151-3.
58 Reading ποιησόμεθα (with D) for ἐποιησόμεθα ('we used') at H254.5. It is unclear what reading(s) lie behind the Arabic translations.
59 This statement is borne out not only by the Babylonian observations preserved in the Almagest (the earliest of which is the lunar eclipse of -720 Mar. 19, in the 1st year of Mardokempad, or the 27th year of the era Nabonassar, IV 6 p. 191, but also by the extant cuneiform records: the earliest surviving astronomical observations (apart from the special case of the Venus tablets of Ammisaduqa) are from -651 (Sachs[1] 44).
### Table of the Sun's Anomaly

<table>
<thead>
<tr>
<th>Common Numbers</th>
<th>Equation</th>
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Therefore where $D\Theta = 5^\circ$ and the hypotenuse $Z\Theta = 120^\circ$, 
$\Theta K = 4;33^\circ$.

And, in the circle about right-angled triangle $\Theta ZK$,

$\text{arc } \Theta K = 4;20^\circ$.

$\therefore \angle \Theta ZK = \begin{cases} 
4;20^\circ & \text{where 2 right angles} = 360^\circ \\
2;10^\circ & \text{where 4 right angles} = 360^\circ.
\end{cases}$

And we found $\angle BDG = 65;30^\circ$.

Therefore, by subtraction, $\angle ZOH$ (i.e. arc $ZH$ of the eccentric) = 63;20°. Therefore, when the sun is at the autumnal equinox, it is 63;20° in mean motion in advance of the perigee (i.e. $\varphi$ 5½°), and 116;40° in mean motion to the rear of the apogee (i.e. $\varpi$ 5;30°).

Now that we have established that, among the first of the equinoxes observed by us, one of the most accurately determined was the autumnal equinox which occurred in the seventeenth year of Hadrian, on Athyr [III] 7 in the Egyptian calendar [132 Sept. 25], about 2 equinoctial hours after noon. [From the above computation] it is clear that at that time the sun, in its mean motion, was 116;40° to the rear of the apogee on the eccentric. Now from [the beginning of] the reign of Nabonassar [−746 Feb. 26] to the death of Alexander [−323 Nov. 12] is a total of 424 Egyptian years, and from the death of Alexander to [the beginning of] the reign of Augustus [−29 Aug. 31] 294 years, and from the first year of Augustus, Thoth 1 in the Egyptian calendar, noon (for we establish all epochs at noon), to the seventeenth year of Hadrian, Athyr 7, 2 equinoctial hours after noon, is 161 years 66 days 2 equinoctial hours. Therefore the sum total from the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, up to the time of the above autumnal equinox, is 879 Egyptian years 66 days and 2 equinoctial hours. In that interval the mean motion of sun is approximately 211;25° beyond
complete revolutions. Therefore, to the 116;40°, which is the [sun's] distance from the apogee of the eccentric at the above autumnal equinox, we add the 360° of one revolution, and subtract from the result the 211;25° of the increment in mean motion over the interval [in question], we find for the epoch in mean motion in the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, that the sun's distance in mean motion is 265;15° to the rear of the apogee. Thus its mean position is X 0;45°.

8. {On the calculation of the solar position}^61

So whenever we want to know the sun's position for any required time, we take the time from epoch to the given moment (reckoned with respect to the local time at Alexandria), and enter with it into the table of mean motion. We add up the degrees [and their subdivisions] corresponding to the various arguments [18-year periods, years, months, etc.], add to this the elongation [from apogee at epoch],^62 265;15°, subtract complete revolutions from the total, and count the result from Π 5;30° rearwards through [i.e. in the order of] the signs. The point we come to will be the mean position of the sun. Next we enter with the same number, that is the distance from apogee to the sun's mean position, into the table of anomaly, and take the corresponding amount in the third column. If the argument falls in the first column, that is if it is less than 180°, we subtract the [equation] from the mean position; but if the argument falls in the second column, i.e. is greater than 180°, we add it to the mean position. Thus we obtain the true or apparent [position of the] sun.

9. {On the inequality in the [solar] days}^63

Such, then, we may say, are the theories concerning the sun alone. Following this it seems appropriate to add a brief discussion of the subject of the inequality of the solar day. A grasp of this topic is a necessary prerequisite, since the mean motions which we tabulate for each body are all arranged on the simple system of equal increments, as if all solar days were of equal length. However, it can be seen that this is not so. The revolution of the universe takes place uniformly about the poles of the equator. The more prominent ways of marking that revolution are by its return to the horizon, or to the meridian. Thus one revolution of the universe is, clearly, the return of a given point on the equator from some place on either the horizon or the meridian to the same place; and a solar day, simply defined, is the return of the sun from some point either on the

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^60 Literally '45 minutes of the first degree of Pisces'.
^62 The reading of D,Ar at H257,18, εποχής (for ἐποχής) is possible. The meaning would be the same, but one would have to understand '[the elongation from apogee] at epoch', which is rather obscure.
^63 See HAMA 61-8, Pedersen 154-8.
^64 νυχθήμερον, literally 'a night plus a day'. See Introduction p. 23.
horizon or on the meridian to the same point. On this definition, a mean solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus approximately 0.59 time-degrees, which is the amount of the mean motion of the sun during that period; and an anomalistic solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus that stretch of the equator which rises with, or crosses the meridian with, the anomalistic motion of the sun [in that period].

This additional stretch of the equator, beyond the 360 time-degrees, which crosses [the horizon or meridian] cannot be a constant, for two reasons: firstly, because of the sun's apparent anomaly; and secondly, because equal sections of the ecliptic do not cross either the horizon or the meridian in equal times. Neither of these effects causes a perceptible difference between the mean and the anomalistic return for a single solar day, but the accumulated difference over a number of solar days is quite noticeable.

As far as the effect of the solar anomaly is concerned, the greatest [accumulated] difference occurs between the two positions of the sun where its [true] speed equals its mean speed. The sum of the [anomalistic] solar days [over either of the two such intervals] will differ from the sum of the mean solar days [over the same interval] by about 4.5 time-degrees, and from the sum of [anomalistic] solar days over the other [such] interval by twice that amount, about 9 time-degrees. For the apparent motion of the sun over the semi-circle containing the apogee is 4° less than the mean, and its apparent motion over the semi-circle containing the perigee is the same amount [4°] greater than the mean.55

As far as the effect of the variation in the time taken to cross the horizon at rising or setting is concerned, the greatest [accumulated] difference occurs between the ends of the semi-circles bounded by the solstitial points. For here too the rising-times of either of those semi-circles will differ from the 180° of the mean interval by the amount by which the longest or shortest day differs from the equinoctial day (measured in time-degrees); and they will differ from each other by the amount by which the longest day (or night) differs from the shortest. As far as the effect of the variation in the time taken to cross the meridian is concerned, the greatest [accumulated] difference will occur between two points enclosing two signs which are on either side of either a solstitial or an equinoctial point. For the sum of [the rising-times at sphaera recta] of the two such signs on either side of a solstice will differ from the mean interval by about 4° time-degrees, and from [the sum of the rising-times of] the two signs on either side of an equinox by 9 time-degrees, since the latter fall short of, and the former exceed the amount for the mean by about the same quantity.56 Hence we establish the beginning of the solar day at [astronomical] epochs from the meridian-crossing of the sun, and not from its rising or setting, since the [time-] difference with respect to the horizon can reach several hours, and is not the same everywhere but varies according to the difference in longest or shortest

55 The sun's maximum equation of anomaly is 2.23° (II 6). Thus from mean speed (90° or 270° from apogee) to mean speed the mean motion is (2 x 2.23 = 4°) greater or less than the true.

56 From the table of rising-times at sphaera recta, II 8, the sum of the rising-times of e.g. Π and Σ is 64.32 (≈ 60° + 4°), while that of e.g. Σ and Π is 55.40 (≈ 60° - 4°).
day at the different latitudes, whereas the [time-]difference with respect to the
meridian is the same at every place on earth, and is no greater than the time-
variation due to the sun's anomaly.

The greatest\(^{67}\) [accumulated] difference [between mean and anomalistic
solar days] resulting from the combination of both these effects, namely that due
to the sun's anomaly and that due to the [variation in the time of] meridian-
crossing, occurs over intervals where the above effects are either both additive
or both subtractive. Now the [maximum] subtractive result from both effects
occurs over the interval from the middle of Aquarius to [the end of] Libra, and
the [maximum] additive one over the interval from [the beginning of]
Scorpio to the middle of Aquarius. Both of these intervals produce a maximum
additive or subtractive result which is composed of about \(3^0\) due to the effect
of the solar anomaly, and about \(4^0\) due to the [variation in the time of] meridian-
crossing.\(^{68}\) Thus the maximum difference arising from the combination of both
the above effects is \(8^0\) time-degrees, or \(\frac{8}{5}\)ths of an hour, between the [true] solar
days over either of these intervals and the [corresponding] mean solar days, and
twice as much, \(16^0\) time-degrees, or \(1\frac{2}{3}\) hours, between the [true] solar days of
one such interval and those of the other. Neglect of a difference of this order
would, perhaps, produce no perceptible error in the computation of the
phenomena associated with the sun or the other [planets]; but in the case of the
moon, since its speed is so great, the resulting error could no longer be
overlooked, since it could amount to \(\frac{1}{4}\) of a degree.\(^{69}\)

Therefore, to state once for all the rule for converting any interval whatever,
given in [true] solar days (by which I mean days counted from noon to noon or
midnight to midnight), into mean solar days: we determine the ecliptic position
of the sun in both mean and anomalistic motion at the beginning and end of the
given interval of solar days; then we take the increment, in degrees, from [the
first] anomalistic (i.e. apparent) position to [the second] apparent position,
enter with it into the table of rising-times \(at\) sphaera recta, and [thus] determine
the time taken by this apparent distance [of the sun between the first and second
positions] to cross the meridian, measured in degrees of the equator. We then
take the difference between this number of time-degrees and the mean distance
[of the sun from first to second positions], measured in degrees, and convert this
difference, which is in time-degrees, to a fraction of an equinoctial hour. We
add the result to the number of [true] solar days given if the amount of the time-
degrees [corresponding to the rising-time of the apparent motion] was greater
than the mean motion, or subtract it if less. The interval we arrive at will be
corrected for expression in mean solar days. We shall use this type of interval
particularly in computing the mean motions of the moon from its tables. One
can immediately comprehend that, given mean solar days, one can find the
[corresponding] civil solar days, i.e. days defined by simple observation, by

\(^{67}\) Reading to \(\pi\lambda\epsilon\tau\sigma\tau\upsilon\delta\iota\alpha\phi\iota\rho\omicron\omicron\nu\) (with DB\(^1\)Ar) at H261.14 for \(\tau\delta\iota\alpha\phi\iota\rho\omicron\omicron\nu\) ('the difference').

\(^{68}\) For a graphical verification of the amounts and positions given here by Ptolemy see H.A.M.4 III
Fig. 57 on p. 1222.

\(^{69}\) The hourly mean motion of the moon (IV 3 p. 179) is about \(0.32,56\). So in \(1\frac{1}{4}\) hours it moves
\(0.36,36 \approx 1^0\).
performing the above computation of addition or subtraction of time-degrees in reverse.\textsuperscript{70}

At our epoch, that is, Year 1 of Nabonassar, Thoth I in the Egyptian calendar, noon, the position of the sun was in mean motion, as we showed just above, $\approx 0;45^\circ$, and in anomalistic motion about $\approx 3;8^\circ$.\textsuperscript{71}

\textsuperscript{70} If we call the interval in true solar days between times $t_1$ and $t_2$ $\Delta t$, and the interval in mean solar days $\Delta T$, then Ptolemy's rule, expressed algebraically, is $\Delta T = \Delta t + E$ ($E$ corresponds, in a certain sense, to the modern 'equation of time'), and $E = (a(t_2) - a(t_1)) - (\lambda(t_2) - \lambda(t_1))$. For proofs of the validity of this rule see \textit{H.A.M.A} 65-6, Pedersen 156-7. Pedersen shows that the rule is in fact an approximation, since one should take the motion in mean longitude, not over the interval $(t_2 - t_1) = \Delta t$, but over the interval in mean solar days $\Delta T$ (which is in practice impossible). Since, however, the difference between $\Delta t$ and $\Delta T$ never exceeds about 33 minutes, during which the sun moves less than $2'$, the error is utterly negligible. For examples of computation see \textit{H.A.M.A} 63-5 and Appendix A, Example 8.

\textsuperscript{71} Ptolemy gives the data for era Nabonassar because they will be required every time one needs to compute the lunar position accurately (i.e. in mean solar days) from his tables (e.g. for the series of observations of fixed stars with respect to the moon in VII 3). Neugebauer notes (\textit{H.A.M.A} 63) that the epoch value for the mean longitude, $\approx 0;45^\circ$, seems itself to be corrected for the equation of time, since reckoning backwards 'simply' from Ptolemy's observation would give $\approx 0;44^\circ$ to the nearest minute.
1. *The kind of observations which one must use to examine lunar phenomena*

In the preceding book we treated all the phenomena associated with the sun’s motion. We now begin our discussion of the moon, as is appropriate to the logical order. In doing so we think it our first duty not to take a naive or arbitrary approach in our use of the relevant observations. Rather, to establish our general notions [on this topic], we should rely especially on those demonstrations which depend on observations which not only cover a long period, but are actually made at lunar eclipses. For these are the only observations which allow one to determine the lunar position precisely: all others, whether they are taken from passages of the moon near fixed stars, or from [sightings with] instruments, or from solar eclipses, can contain a considerable error due to lunar parallax. It is only for particular further developments [of the theory] that we should use these other kinds of observations for our investigations. For the distance between the sphere of the moon and the centre of the earth, unlike the distance to the ecliptic, is not so great that the earth’s bulk has the ratio of a point to it. Hence it necessarily follows that the straight line drawn from the centre of the earth (which is the centre of the ecliptic) through the centre of the moon to a point on the ecliptic, which determines the true position (as it does for all bodies), does not in this case always coincide, even sensibly, with the line drawn from some point on the earth’s surface, that is, the observer’s point of view, to the moon’s centre, which determines its apparent position. Only when the moon is in the observer’s zenith do the lines from the earth’s centre and the observer’s eye through the moon’s centre to the ecliptic coincide. But when the moon is displaced from the zenith position in any way whatever, the directions of the above lines become different, and hence the apparent position cannot be the same as the true, but [differs from it], as the [line through] the observer’s eye assumes various positions with respect to the line drawn through the centre of the earth, [by an amount] proportional to the varying angle of inclination [between the two lines].

This is the reason why in the case of solar eclipses, which are caused by the

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2 Reading ὀπὸ τοῦ κέντρου τῆς γῆς τούτῃ τοῦ ζωδιακοῦ διὰ τοῦ κέντρου τῆς σελήνης (with D, Ar) for ὀπὸ τοῦ κέντρου τῆς σελήνης ("the straight line drawn from the moon’s centre", which is nonsense) at H266.5. The error in most Greek mss. is due to haplography, and is an important indication that all except D and its descendants come from a single (?Byzantine) ms. Corrected by Manitius.
moon passing below and blocking [the sun] (for when the moon falls into the cone from the observer's eye to the sun it produces the obscuration which lasts until it has passed out [of the cone] again), the same eclipse does not appear identical, either in size or in duration, in all places. For the moon does not produce obscuration for all observers, for the reasons stated above, and [even for those for whom it does produce obscuration] does not appear to obscure the same parts of the sun [for all alike]. Whereas in the case of lunar eclipses there is no such variation due to parallax, since the observer's position is not a contributory cause to what happens at a lunar eclipse. For the moon's light is at all times caused by the illumination from the sun. Thus when it is diametrically opposite to the sun, it normally appears to us as lighted over its whole surface, since the whole of its illuminated hemisphere is turned towards us as well [as towards the sun] at that time. However, when its position at opposition is such that it is immersed in the earth's shadow-cone (which revolves with the same speed as the sun, but opposite it), then the moon loses the light over a part of its surface corresponding to the amount of its immersion, as the earth obstructs the illumination by the sun. Hence it appears to be eclipsed for all parts of the earth alike, both in the size [of the eclipse] and the length of the intervals [of the various phases].

Now to establish our general theory we need to use true, and not apparent, positions of the moon; for the ordered and regular must necessarily precede and serve as a foundation for the disordered and irregular. So, for the above reasons, we declare that we must not use, for this purpose, observations of the moon into which the observer's position enters, but only lunar eclipse observations, since [only] in these does the observer's position have no effect on the determination of the moon's position. For it is obvious that, if we find the point on the ecliptic which the sun occupies at the time of mid-eclipse (which is, as accurately as we can determine, the moment at which the moon's centre is diametrically opposite the sun's in longitude), then at the same time of mid-eclipse the precise position of the moon's centre will be the point diametrically opposite.

2. *On the periods of the moon*

The above may serve as an outline of the kind of observations which must be examined to determine the general theory of the moon. We shall now endeavour to describe the method which was used by the ancients in their attempts at establishing a [lunar] theory, and which we will find a most convenient tool in deciding which hypotheses accord with the phenomena.

The moon's motion appears anomalistic both in longitude and in latitude: the time it takes to traverse the ecliptic is not constant, and neither is the time it

\footnote{Reading τὰς αὔτὰς (with D, Ar) for ταῦτας ('these eclipses') at H267,4. Corrected by Manitius. 
\footnote{duration': the Greek has the vague 'times' (τοῖς χρόνοις). This is elucidated by H268,1 τοῖς τῶν διαστάσεων χρόνοις,'the duration of the intervals [of partial and total phases].' Ptolemy may also be alluding, in both places, to the fact that the actual moments of e.g. the beginning or middle of a solar eclipse are different at different places, and by an amount which does not correspond directly to the difference in longitude.}
takes to return to the same latitude.\(^5\) Now unless one finds the period of its return in anomaly it is, necessarily, impossible to determine the period of the other motions [in longitude and latitude]. However, from individual observations it is apparent that the moon's mean speed can occur in any part of the ecliptic, as can its greatest speed and its least speed, and that it can reach its greatest northern or southern latitude, or appear exactly in the ecliptic, anywhere, too. Hence the ancient astronomers, with good reason, tried to find some period in which the moon's motion in longitude would always be the same, on the grounds that only such a period could produce a return in anomaly. So they compared observations of lunar eclipses (for the reasons mentioned above), and tried to see whether there was an interval, consisting of an integer number of months, such that, between whatever points one took that interval of months,\(^6\) the length in time was always the same, and so was the motion [of the moon] in longitude, [i.e.] either the same number of integer revolutions, or the same number of revolutions plus the same arc.

The even more ancient [astronomers] used the somewhat crude estimate that such a period could be found in 6585\(\frac{1}{2}\) days. For they saw that in that interval occurred approximately 223 lunations, 239 returns in anomaly, 242 returns in latitude, and 241 revolutions in longitude plus 10°, which is the amount the sun travels beyond the 18 revolutions which it performs in the above time (that is when the motion of sun and moon is measured with respect to the fixed stars). They called this interval the 'Periodic', since it is the smallest single period which contains (approximately) an integer number of returns of the various motions.\(^7\) In order to obtain a period with an integer number of days, they tripled the 6585\(\frac{1}{2}\) days, obtaining 19756 days, which they called 'Exeligmos'. Similarly, by tripling the other numbers, they obtained 669 lunations, 717 returns in anomaly, 726 returns in latitude, and 723 revolutions in longitude plus 32°, which is the amount the sun travels beyond its 54 revolutions.\(^8\)

However, Hipparchus already proved, by calculations from observations made by the Chaldaeans and in his time, that the above relationships were not accurate. For from the observations he set out he shows that the smallest constant interval defining an ecliptic period in which the number of months and the amount of [lunar] motion is always the same, is 126007 days plus 1 equinoctial hour. In this interval he finds comprised 4267 months, 4573 complete returns in anomaly, and 4612 revolutions on the ecliptic less about \(\frac{7}{4}\) which is the amount by which the sun's motion falls short of 345 revolutions (here too the revolution of sun and moon is taken with respect to the fixed stars). (Hence, dividing the above number of days by the 4267 months, he finds the

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\(^5\) Reading κατά πλάτος (with D) for κατά τὸ πλάτος at H269.9.

\(^6\) 'months' here means 'true synodic months'. This is generally true throughout the Almagest (except where the context makes it obvious that the reference is strictly calendaric). In the translation I usually make the meaning explicit.

\(^7\) This period, generally, but wrongly, called 'Saros' in modern times (see Neugebauer[1]), was well-known in Babylonian astronomy. See \textit{H.AM.4} 497 ff. We do not know to whom Ptolemy refers by 'the even more ancient people', except that they are earlier than Hipparchus.

\(^8\) The ἕξις ἐξηλιγμὸς (meaning 'turn of the wheel') is also mentioned by Geminus (Cap. XVIII. ed. Manitius pp. 200–2), who gives exactly the same numbers as Ptolemy, including the excess in sidereal longitude of 32°.
mean length of the [synodic] month as approximately 29;31,50,8,20 days). He shows, then, that the corresponding interval between two lunar eclipses is always precisely the same when they are taken over the above period [126007d1h]. So it is obvious that it is a period of return in anomaly, since [from whatever eclipse it begins], it always contains the same number [4267] of months, and 4611 revolutions in longitude plus 352°, as determined by its syzygies with the sun.

But if one were to look for the number of months [which always cover the same time-interval], not between two lunar eclipses, but merely between one conjunction or opposition and another syzygy of the same type, he would find an even smaller integer number of months containing a return in anomaly, by dividing the above numbers by 17 (which is their only common factor). This produces 251 months and 269 returns in anomaly.

However, it was found that the above period [of 126007d1h] did not contain an integer number of returns in latitude too. For it was apparent that the [pairs of] corresponding eclipses exhibited equality only with respect to the interval [between the pair] in time and revolution in longitude, but not with respect to the size and type of the obscuration,® which is the criterion for [a return in] latitude. Nevertheless, having already determined the period of return in anomaly, Hipparchus again adduces intervals containing [an integer number of] months which have at each end eclipses which were identical in every respect, both in size and in duration [of the various phases], and in which there was no difference due to the anomaly. Thus it is apparent that there is a return in latitude too. He shows that such a period is contained in 5458 months and 5923 returns in latitude.¹⁰

That, then, is the method which our predecessors used for the determination of such [periods]. It is not simple or easy to carry out, but demands a great deal of extraordinary care, as we can see from the following considerations.¹¹ Let us grant that [two] intervals [between pairs of eclipses] are found to be precisely equal in time. In the first place, this is no use to us unless the sun too exhibits no effect due to anomaly, or exhibits the same over both intervals: for if this is not the case, but instead, as I said, the equation of anomaly has some effect, the sun will not have travelled equal distances over [the two] equal time-intervals, nor, obviously, will the moon. For example, let us suppose that each of the two intervals being compared comprises half a year beyond the same number of complete years, and that in this time the motion of the sun in the first interval

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³By 'type' Ptolemy means whether the obscuration begins from the north or south of the lunar disk.

¹⁰Ptolemy's account here is not historically accurate. In fact Hipparchus took from Babylonian sources the parameters [1] 1 synodic month = 29;31,50,8,20°, [2] 251 synodic months = 269 anomalistic months, and [3] 5458 synodic months = 5923 returns in latitude (Kugler, *Babylonische Mondrechnung* 4-46). Multiplying [2] by 17, he constructed an eclipse-period (Aaboe[1955]), whence *H.I.M.* 1.310-2. An input of some value for the length of the year produced the solar motion over this period, rounded by Hipparchus to the nearest 1°sign (on which see Neugebauer[2], 251). Then Hipparchus confirmed (not derived, as Ptolemy says) the above by comparison of eclipses from his own time with Babylonian ones 345 years earlier (see Toomer[11] for the method and identification of the eclipses he used).

¹¹The following (to p. 178) is well explained and illustrated by Neugebauer. *H.I.M.* 71-2.
starts from the position of mean speed in Pisces, and in the second interval from the position of mean speed in Virgo. Then over the first interval the sun will have traversed about 42° less than a semi-circle [beyond complete revolutions], but over the second about 42° more than a semi-circle. Thus the moon too will have traversed over the first interval 175° beyond complete revolutions and over the second 184°, although both intervals cover an equal time. Therefore we define as the first necessary condition [for a return in lunar anomaly] that the intervals must exhibit one of the following characteristics with respect to the sun:

1. It must complete an integer number of revolutions [in both intervals]; or
2. traverse the semi-circle beginning at the apogee over one interval and the semi-circle beginning at the perigee over the other; or
3. begin from the same point [of the ecliptic] in each interval; or
4. be the same distance from apogee (or perigee) at the first eclipse of one interval as it is at the second eclipse of the other interval, [but] on the other side.

For only under one of these conditions will there be no effect due to the anomaly, or the same effect over both intervals, so that the arc traversed beyond complete revolutions over one interval is equal to that traversed over the other, or even equal to the mean motion of the sun [over the intervals] as well.

Secondly, it is our opinion that we must pay no less attention to the moon’s [varying] speed. For if this is not taken into account, it will be possible for the moon, in many situations, to cover equal arcs in longitude in equal times which do not at all represent a return in lunar anomaly as well. This will come to pass

1. if in both intervals the moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed; or
2. if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed; or
3. if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the end of the other interval, [but] on the other side.

In each of these situations there will again be either no effect or the same effect [in both intervals] of the lunar anomaly, and hence equal increments in longitude will be produced [over both intervals], but there will be no return in anomaly at all. So the intervals adduced must avoid all the above situations if

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12 That is, from the positions where the equation of anomaly reaches its positive maximum (Pisces) and negative maximum (Virgo). Illustrated by *H.A.M.1* Fig. 59 p. 1223.

13 That is, if the sun has an anomaly of α° at the beginning of the first interval, it must have an anomaly of (360−α)° at the end of the second interval. This situation (and the others listed here) is illustrated by *H.A.M.1* Fig. 60 p. 1223.

14 δρόμος is often used in early Greek astronomy for the (varying) amount which the moon travels in one day. The earliest example seems to be the ‘Eudoxus’ papyrus (ed. Blass p. 14). Where Ptolemy uses δρόμος for the moon (e.g. V 2, H355.14; V 3, H361.16) ‘speed’ seems the best translation. For a special use of the term by Hipparchus see V 3 p. 224 with n.14.

15 Illustrated (in the order [1], [3], [2]) by *H.A.M.1* Fig. 61 p. 1224, which utilizes the lunar epicycle model. One must presume that Ptolemy avoids talking in geometrical terms (which is the most convenient way to visualize the situation) because he has not yet established a lunar model. However, it is hard to give any sense to εκατέρωθεν (literally ‘on opposite sides’, translated here as ‘on the other side’) which does not involve an epicycle model.
they are to provide us directly with a period of return in anomaly. On the contrary, we should select intervals [the ends of which are situated] so as to best indicate [whether the interval is or is not a period of anomaly], by displaying the discrepancy [between two intervals] when they do not contain an integer number of returns in anomaly. Such is the case when the intervals begin from speeds which are not merely different, but greatly different either in size or in effect. By ‘in size’ I mean when in one interval [the moon] starts from its least speed and does not end at the greatest speed, while in the other it starts from its greatest speed and does not end at its least speed. For in this case, unless the intervals contain an integer number of revolutions in anomaly, the difference in the increments in longitude over the two intervals will be very great; when the increment in anomaly is about one or three quadrants of a revolution, the intervals will differ by twice the [maximum] equation of anomaly. By ‘in effect’ I mean when [the moon] starts from mean speed in both positions, not, however, from the same mean speed, but from the mean speed during the period of increasing speed at one interval, and from that during the period of decreasing speed at the other. Here too, if there is not a return in anomaly, there will be a great difference in the increment in longitude [over the two intervals]; again, when the increment in anomaly is one or three quadrants of a revolution, the difference will again amount to twice the [maximum] equation of anomaly. And when the increment in anomaly is a semi-circle, the difference will be four times that amount.  

That is why, as we can see, Hipparchus too used his customary extreme care in the selection of the intervals adduced for his investigation of this question: he used [two intervals], in one of which the moon started from its greatest speed and did not end at its least speed, and in the other of which it started from its least speed and did not end at its greatest speed. Furthermore he also made a correction, albeit a small one, for the sun’s equation of anomaly, since the sun fell short of an integer number of revolutions by about \( \frac{1}{4} \) of a sign, and this sign was different, and produced a different equation of anomaly, in each of the two intervals.

We have made the above remarks, not to disparage the preceding method of determining the periodic returns, but to show that, while it can achieve its goal if applied with due care and the appropriate kind of calculations, if any of the conditions we set out above are omitted from consideration, even the least of them, it can fail utterly in its intended effect; and that, if one does use the proper criteria in making one’s selection of observational material, it is difficult to find corresponding [pairs of eclipse] observations which precisely fulfil all the required conditions.

In any case, when we take the above periodic returns, as determined by Hipparchus’ calculations, we find that the period [containing an integer number] of months has, as we said, been calculated as correctly as possible, and has no perceptible difference from the true value. But there is an error in the

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16 These two situations (of maximum effect due to the anomaly when there is not a return in anomaly) are illustrated by *H.4.M.1* Fig. 62 p. 1225.

17 On the eclipses used by Hipparchus see Toomer[11].
periods of anomaly and latitude, so considerable as to become quite apparent to us from the procedures we devised to check these values in simpler and more practical ways; we shall soon explain these, in connection with our demonstration of the size of the lunar anomaly. But first, for convenience [of calculation] in what follows, we set out the individual mean motions [of the moon] in longitude, anomaly and latitude, in accordance with the above periods of their returns, and [also the mean motions] calculated on the basis of the corrections which we shall derive later.  

3. [On the individual mean motions of the moon]

If, then, we multiply the mean daily motion of the sun which we derived, ca. 0;59,8,17,13,12,31\textsuperscript{a}, by the number of days in one [mean synodic] month, 29;31,50,8,20\textsuperscript{b}, and add to the result the 360° of one revolution, we will get the mean motion of the moon in longitude during one synodic month as ca. 389;6,23,1,24,2,30,57°. Dividing this by the above number of days in a month, we get the mean daily motion of the moon in longitude as ca. 13;10,34,58,33,30,30°.

Next, multiplying the 269 revolutions in anomaly by the 360° of one revolution, we get 96840°. Dividing this by the number of days in 251 months, 7412;10,44,51,40\textsuperscript{d}, we get the mean daily motion in anomaly as 13;3,53,56,29,38,38°.

Similarly, multiplying the 5923 returns in latitude by the 360° of one revolution, we get 2132280°. Dividing this by the number of days in 5458 months, 161177;58,58,3,20\textsuperscript{e}, we get the mean daily motion in latitude as 13;13,45,39,40,17,19°.

Next, subtracting the mean daily motion of the sun from the mean daily motion of the moon in longitude, we get the mean daily motion in elongation as 12;11,26,41,20,17,59°.

However, from the methods which, as we said, we shall employ in what follows for investigation of this topic, we find that the mean daily motion in longitude (and hence, obviously, that in elongation), is practically identical to the above, but the mean daily motion in anomaly is 0;0,0,0,11,46,39° less: thus it is 13;3,53,56,17,51,59°; and the mean daily motion in latitude is 0,0,0,8,39,18° more; thus it is 13;13,45,39,48,56,37°.\textsuperscript{19}

Using the latter daily motions, and taking \(\frac{1}{5}\)th of each, we get the following mean hourly motions:

- in longitude: 0;32,56,27,26,23,46,15°
- in anomaly: 0;32,39,44,50,44,39,57,30°
- in latitude: 0;33,4,24,9,32,21,32,30°
- in elongation: 0;30,28,36,43,20,44,57,30°.

\textsuperscript{18}Ptolemy's corrections to the mean motions in anomaly and latitude, given below, are justified at IV 7 (p. 204) and IV 9 (p. 207).

\textsuperscript{19}All the above computations have been carried out very precisely, and are correct to the nearest sixth (60° degree). In the following computations of the mean motions for the greater units, however, Ptolemy operates as if the last place in the mean daily motions were precisely correct, i.e. no account is taken of the accumulated error for months, years, etc.
Multiplying the daily motions by 30 and subtracting complete revolutions, we get the following monthly mean increments:

- **in longitude**: 35;17,29,16,45,15°
- **in anomaly**: 31;56,58,55,59,30°
- **in latitude**: 36;52,49,54,28,18,30°
- **in elongation**: 5;43,20,40,8,59,30°.

Next, multiplying the daily motions by the 365 days of the Egyptian year, and subtracting complete revolutions, we get the following yearly mean increments:

- **in longitude**: 129;22,46,13,50,32,30°
- **in anomaly**: 88;43,7,28,41,13,55°
- **in latitude**: 148;42,47,12,44,25,5°
- **in elongation**: 129;37,21,28,29,23,55°.

Next, multiplying the yearly motions by 18 (this number is chosen, as we said, for convenience in tabulation), after subtracting complete revolutions we get the following mean increments over an eighteen-year period:

- **in longitude**: 168;49,52,9,9,45°
- **in anomaly**: 156;56,14,36,22,10,30°
- **in latitude**: 156;50,9,49,19,31,30°
- **in elongation**: 173;12,26,32,49,10,30°.

As in the case of the sun, we will again set out three tables arranged in 45 lines, with 5 columns in each. The first column will contain the time-divisions appropriate to each table, in the first table the 18-year periods, in the second the years, again followed by the hours, in the third the months, again followed by the days. The remaining four columns will contain the degrees [and their subdivisions] corresponding to the appropriate argument: the second column, longitude, the third, anomaly, the fourth, latitude, and the fifth, elongation. The layout of the tables is as follows.

4. *Tables of the mean motions of the moon*

[See pp. 182-7.]

Our next task is to demonstrate the type and size of the moon's anomaly. For the time being we shall treat this as if it were single and invariant. It is apparent that this anomaly, namely the one with a period corresponding to the above period of return, is the only one which our predecessors (just about all of them)

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20 Reading Σ for Δ ("31") in the last place at H280,5, with D, Ar (cf. also the tables IV 4). Corrected by Manitius.
21 See Pedersen 166-7.
22 Reading καὶ τὴν οὖτη (with BD) for ταύτης ("as if this were single") at H294,6. Ar read ταύτης.
have hit upon. Later, however, we shall show that the moon also has a second
anomaly, linked to its distance from the sun; this [second anomaly] reaches a
maximum round about both [waxing and waning] half-moons, and goes
through its period of return twice a month, [being zero] precisely at conjunction
and opposition.\footnote{Reference to V 2-4.} We adopt this order of procedure in our demonstration
because it is impossible to determine the second [anomaly] apart from the first,
which is always combined with it, whereas the first can be found apart from the
second, since it is determined from lunar eclipses, at which there is no
perceptible effect of the anomaly connected with [the distance from] the sun.

In this first part of our demonstrations we shall use the methods of establishing
the theorem which Hipparchus, as we see, used before us.\footnote{On Hipparchus' determination of the lunar parameters see further IV 11, Toomer[8] and
Toomer[2].} We too, using three
lunar eclipses, shall derive the maximum difference from mean motion and the
epoch of the [moon's position] at the apogee, on the assumption that only this
[first] anomaly is taken into account, and that it is produced by the epicyclic
hypothesis. It is true that the same phenomena would result from the eccentric
hypothesis, but we shall find the latter more suitable to represent the second
anomaly, which is connected with the sun, when we come to combine both
anomalies. However, the same phenomena will in all cases result from both the
hypotheses we have described, whether, as in the situation described for the sun,
the period of return in anomaly and the period of return in the ecliptic [i.e. in
longitude] are both equal, or whether, as in the case of the moon, they are
unequal, provided only that the ratios [of epicycle to deferent and eccentricity
to eccentre] are taken as identical. We can see this from the following, in which
we use the above-mentioned simple anomaly of the moon for our examination.

Since the moon completes its return with respect to the ecliptic sooner than its
return with respect to this anomaly, it is clear that, in the epicyclic hypothesis,
over a given period of time, the epicycle will always traverse a greater arc\footnote{\textquoteleft a greater arc	extquoteright: literally \textquoteleft an arc greater than the one similar to [the arc].'} of
the circle concentric to the ecliptic than the arc of the epicycle traversed by the
moon in the same time; in the eccentric hypothesis, the arc traversed by the
moon on the eccentre will be similar to the arc traversed by it on the epicycle [in
the epicyclic hypothesis], while the eccentre will move about the centre of the
ecliptic in the same direction as the moon by an amount equal to the increment
of the motion in longitude over the motion in anomaly [in the same time](this
corresponds to the increment of the arc of the deferent over the arc of the
epicycle [in the epicyclic hypothesis]). In this way we can preserve the equality
of the periods of both motions [i.e. in longitude and anomaly], as well as
equality of the ratios, in both hypotheses.

With the above as a necessary basis (as is obvious from logic), let [Fig. 4.1] the
circle concentric with the ecliptic be ABG on centre D and diameter AD, and
let the epicycle be EZ on centre G. Let us suppose that when the epicycle was at
A, the moon was at E, the apogee of the epicycle, and that in the same time as
the epicycle has traversed arc AG, the moon has traversed arc EZ. Join ED, GZ.
### TABLES OF THE MOON'S MEAN MOTIONS

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### Lunar Mean Motion Tables

**18-Year Periods**

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## IV 4. Lunar mean motion tables

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<td>36 52 49 54 28 18 30</td>
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Then, since arc AG > arc EZ, cut off arc BG || arc EZ, and join BD. Then it is clear that, in the same time, the eccentre will have moved through $\angle ADB$, which represents the difference between the two motions, and its centre and apogee will lie along line BD.

This being so, let DH = GZ. Join ZH, and with centre H and radius HZ draw the eccentre ZΘ.

I say, that

$$ZH:HD = DG:GZ,$$

and that in this hypothesis too the moon will be at point Z, i.e. arc ZΘ || arc EZ.

[Proof:] Since $\angle BDG = \angle EGZ$, GZ is parallel to DH.

But GZ = DH [by construction]. Therefore ZH too is equal and parallel to GD.\(^{26}\)

\[\therefore ZH:HD = DG:GZ.\]

Furthermore, since DG is parallel to HZ, $\angle GDB = \angle ZHΘ$; and, by hypothesis, $\angle GDB = \angle EGZ$.

\[\therefore \text{arc } ZΘ \parallel \text{arc EZ}.\]

Therefore the moon has reached point Z in the same time according to either hypothesis, since the moon itself has traversed arc EZ on the epicycle and arc $\Theta Z$ on the eccentre, which we have shown to be similar, while the epicycle

---

\(^{26}\) Euclid I 33: straight lines joining equal and parallel lines are themselves equal and parallel.
centre has moved through arc AG, and the centre of the eccentre through arc AB, which is the increment of arc AG over arc EZ. Q.E.D.

Moreover, even if [the members of] the ratios are unequal, and the eccentre is not the same size as the deferent, the same phenomena will result, provided the ratios are similar, as will be clear from the following.

Draw each of the hypotheses in a separate figure. Let [Fig. 4.2] the circle concentric to the ecliptic be ABG on centre D and diameter AD, and the epicycle EZ on centre G. Let the moon be at Z. Let [Fig. 4.3] the eccentre be HΘK on centre L and diameter ΘLM, with the centre of the ecliptic at M. Let the moon be at K. In the first figure join DGE,GZ,DZ, and in the second figure join HM, KM, KL.

Let DG:GE = ΘL:LM.

Let us suppose that in the same time as the epicycle has moved through ∠ADG, the moon has again moved through ∠EGZ, the eccentre through ∠HMΘ, and the moon, again, through ∠ΘLK.

Therefore, because of the assumed relationship between the motions,

∠EGZ = ∠ΘLK,

and ∠ADG = ∠HMΘ + ∠ΘLK.

This being so, I say that the moon will again appear to have traversed an equal arc in the same time according to either hypothesis, i.e.

∠ADZ = ∠HMK

(for at the beginning of the time-interval the moon was at the apogee and appeared along lines DA and MH, while at the end it was at points Z and K and appeared along lines ZD and MK).

[Proof:] Let arc BG again be similar to arc ΘK (or arc EZ). Join BD.

Then, since DG:GZ = KL:LM,

and the angles at G and L are equal,
IV 6. Derivation of lunar anomaly from 3 eclipses

triangle GDZ || triangle KLM (sides about equal angles proportional), and the angles opposite the corresponding sides are equal.

\[ \angle GZD = \angle LMK. \]

But \( \angle BDZ = \angle GZD. \)

for GZ is parallel to BD, since, by hypothesis, \( \angle ZGE = \angle BDG. \)

\[ \angle ZDB = \angle LMK. \]

But, by hypothesis, \( \angle ADB, \) the difference between the motions [in longitude and anomaly] equals \( \angle HM\Theta, \) the motion of [the centre of] the eccentric. Therefore, by addition,

\[ \angle ADZ = \angle KMH. \]

Q.E.D.

6. [Demonstration of the first, simple anomaly of the moon]\(^{27}\)

Let the preceding suffice us as preliminary theory. We shall now demonstrate the lunar anomaly in question, by means of the epicyclic hypothesis, for the reason mentioned. [For this purpose] we shall use, first, among the most ancient eclipses available to us, three [which we have selected] as being recorded in an unambiguous fashion, and, secondly, [we shall repeat the procedure] using, among contemporary eclipses, three which we ourselves have observed very accurately. In this way our results will be valid over as long a period as possible, and in particular it will be apparent that approximately the same [maximum] equation of anomaly results from both demonstrations, and that the increment in the mean motions [between the two sets of eclipses] agrees\(^ {28}\) with that computed from the above periods (as corrected by us).

\(^{27}\) See H.I.M.1 73–8, Pedersen 169–79.

\(^{28}\) Reading σύμφωνος (with D, Ar) for σύμφωνος αἱ ἀ ("always agrees") at H301,10.
For the purposes of demonstrating the first anomaly, considered separately, the epicyclic hypothesis which we mentioned can be described as follows. Imagine a circle in the sphere of the moon which is concentric to and lies in the same plane as the ecliptic. Inclined to this, at an angle corresponding to the amount of its [maximum] deviation in latitude, is another circle, which moves uniformly in advance (with respect to the centre of the ecliptic) with a speed equal to the difference between the motions in latitude and longitude. On this inclined circle we suppose the so-called 'epicycle' to be carried, with a uniform motion, towards the rear with respect to the heavens, corresponding to the motion in latitude. (This motion, obviously, will represent the [mean] motion in longitude with respect to the ecliptic). On the epicycle itself [we suppose] the moon to move, in such a way that on the arc near the apogee its motion is in advance with respect to the heavens, at a speed corresponding to the period of return in anomaly. However, for the purposes of the present demonstration we shall suffer no ill consequences if we neglect the advance motion in latitude and the inclination of the moon's orbit, since such a small inclination has no noticeable effect on the position in longitude.29

First, the three ancient eclipses which are selected from those observed in Babylon.

The first is recorded as occurring in the first year of Mardokempad, Thoth [I] 29/30 in the Egyptian calendar [-720 Mar. 19/20]. The eclipse began, it says, well over an hour after moonrise, and was total.

Now since the sun was near the end of Pisces, and [therefore] the night was about 12 equinoctial hours long, the beginning of the eclipse occurred, clearly, 4½ equinoctial hours before midnight, and mid-eclipse (since it was total) 2½ hours before midnight.30 Now we take as the standard meridian for all time determinations the meridian through Alexandria, which is about ¾ of an equinoctial hour in advance [i.e. to the west] of the meridian through Babylon.31 So at Alexandria the middle of the eclipse in question was 3½ equinoctial hours before midnight, at which time the true position of the sun, according to the [tables] calculated above, was approximately $\Xi 24^\circ$.

The second eclipse is recorded as occurring in the second year of the same Mardokempad, Thoth [I] 18/19 in the Egyptian calendar [-719 Mar. 8/9]. The [maximum] obscuration, it says, was 3 digits 32 from the south exactly at midnight. So, since mid-eclipse was exactly at midnight at Babylon, it must

29 I.e. for the purposes of computing the longitude the moon's orbit is treated as if it lay in the plane of the ecliptic. The maximum resulting error (for $i \approx 5^\circ$) is about 6' (cf. HAMA 83). Ptolemy himself (VI 7 p. 297) estimates it as 5'.

30 A total eclipse of the moon is assumed to last 4 hours from start to finish. This agrees fairly well with the duration one derives from Ptolemy's own eclipse tables (VI 8) and with the actual maximum possible duration. The duration of the eclipse in question (Oppolzer no. 741) was in fact about 3½.

31 This time difference corresponds to a longitudinal difference of 12½°. The actual time difference is about 58½ minutes. In the Geography Ptolemy amended the difference, in the right direction but by far too much, to 11½ hours (8:20:27), corresponding to the difference between the longitudes there assigned to Alexandria (60°, 4:5:9) and Babylon (79°, 5:20:6).

32 Modern calculations give a considerably smaller eclipse: Oppolzer (no. 743) 1.6 digits, P.V. Neugebauer 1.5 digits. However Ptolemy's own tables give about 2½ digits: see Appendix A, Example 11.
have been seen before midnight at Alexandria, at which time the true position of the sun was \( \geq 13^\circ \).

The third eclipse is recorded as occurring in the (same) second year of Mardokempad, Phamenoth [VII] 15/16 in the Egyptian calendar [-719 Sept. 1/2]. The eclipse began, it says, after moonrise, and the [maximum] obscuration was more than half [the disk] from the north. So, since the sun was near the beginning of Virgo, the length of night at Babylon was about 11 equinoctial hours, and half the night was \( 5\frac{1}{2} \) [equinoctial] hours. Therefore the beginning of the eclipse was about 5 equinoctial hours before midnight (since it began after moonrise), and mid-eclipse about 3\frac{1}{2} hours before midnight (for the total time for an eclipse of that size must have been about 3 hours).\(^3\) So in Alexandria mid-eclipse occurred \( 4\frac{1}{2} \) equinoctial hours before midnight, at which time the true position of the sun was about \( 3\frac{1}{2}^\circ \).

Then it is clear that the motion of the sun (which is the same as that of the moon apart from complete revolutions) is

- from the middle of the first eclipse to the middle of the second: \( 349;15^\circ \)
- from the middle of the second eclipse to the middle of the third: \( 169;30^\circ \).

The time intervals are:

\[
\begin{align*}
\text{from first to second} & : 345^d 21^h \text{ reckoned simply} \\
& : 345^d 21\frac{1}{2}^h \text{ reckoned in mean solar days} \\
\text{from second to third} & : 176^d 20^h \text{ reckoned simply} \\
& : 176^d 20\frac{1}{2}^h \text{ reckoned in mean solar days}.
\end{align*}
\]

Over such short intervals it will make no appreciable difference if one uses approximate periods [to determine the moon’s mean motions].\(^4\) The moon’s mean motions are, then, (beyond complete revolutions), approximately

\[
\begin{align*}
in 354^d 2\frac{1}{2}^h & : 306;25^\circ \text{ in anomaly} \\
& : 345;51^\circ \text{ in longitude} \\
in 176^d 20^h & : 150;26^\circ \text{ in anomaly} \\
& : 170;7^\circ \text{ in longitude}.
\end{align*}
\]

Thus it is clear that the motion on the epicycle of \( 306;25^\circ \) over the first interval has produced an increment of \( [349;15^\circ - 345;51^\circ] = 3;24^\circ \) over the mean motion, and the motion [on the epicycle] of \( 150;26^\circ \) over the second interval has produced a decrement from the mean motion of \( [169;30^\circ - 170;7^\circ] = 0;37^\circ \).

With the above as data, let [Fig. 4.4] the moon’s epicycle be [circle] ABG, on

\(^3\) At a lunar eclipse the moon is diametrically opposite the sun. Therefore moonrise coincided with sunset, which was \( 5\frac{1}{2} \) equinoctial hours before midnight. Ptolemy allows 1-hour to account for ‘after moonrise’. He estimates a duration of 3 hours for an eclipse of more than 6 digits (according to Oppolzer, no. 744, this eclipse had a magnitude of 6.4 digits and a duration of about 2:36\(^h\); P.V. Neugebauer calculates 6.1 digits and 2.4\(^h\)). Obviously this eclipse is hardly ‘recorded in an unambiguous fashion’ (p. 190).

\(^4\) This is a point of methodology. Ptolemy’s mean motion tables are based, not on the exact periods he took from Hipparchus, but (for the anomaly) on a correction applied to the number derived from those periods (IV 7). However, the correction is itself based in part on the parameters derived here. It is therefore important to note that the correction makes no difference over the short intervals considered here (between the first and second eclipses it is only about 1 second of arc). From IV 11 it is clear that Hipparchus had already established the principle that it was necessary to use an eclipse triple close in time, so that any long-term error in the mean motions would have a minimal effect.
which point A is the location of the moon at the middle of the first eclipse, B its position at the middle of the second eclipse, and G its position at the middle of the third eclipse. We must imagine the moon to move on the epicycle from B to A and from A to G in such a way that arc AGB, which is its increment in motion between the first and second eclipses, is 306°25' and produces an increment of 3°24' over the mean motion, while arc BAG, which is its increment in motion between the second and third eclipses, is 150°26', and produces a decrement of 0°37' from the mean motion. Hence the motion from B to A is 53°35' and produces a decrement of 3°24' from the mean motion, and the motion from A to G is 96°51' and produces an increment of 2°47' over the mean motion.

Now the perigee of the epicycle cannot lie on arc BAG. This is clear because this arc has a subtractive effect, and is less than a semi-circle, while the greatest speed occurs at the perigee. Since, then, [the perigee] necessarily lies on arc GEB, let us take the centre of the ecliptic, which is also the centre of the deferent, as point D, and draw lines DA, DEB and DG to the points representing [the positions of the moon at] the three eclipses. In order to make the sequence of the proof readily transferable for computations of this kind, whether we use the epicyclic hypothesis (as now) for our demonstration, or the

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35 For a detailed argument about the location of the observer with respect to the points on the epicycle representing the three eclipses see HAMA 74.
iv 6. Construction of figures for both hypotheses

eccentric hypothesis, in which case [see Fig. 4.5] centre D is taken inside the circle, we give the following generally applicable description.

Produce one of the three straight lines drawn [DA, DB, DG] to the opposite circumference (in this case we already have DEB drawn to E from point B of the second eclipse), and draw a line joining the points of the other two eclipses (here AG). From the point where the first line produced cuts the circumference again (here E) draw lines to the other two points (here EA, EG), and [from the same point] drop perpendiculars on to the lines between the other two points and the centre of the ecliptic (here EZ on to AD and EH on to GD). From one of these two points (here G) drop a perpendicular on to the line drawn from the other (here A) to the extra intersection [with the circumference] (here E) resulting from [the first straight line, DB,] being produced (in this case, we drop Gθ on to AE). Whichever point we start drawing the figure from, we shall find that the same ratios result from the numbers used in the demonstration. Our choice [of starting-point] is guided merely by convenience.

So, since we found that arc BA subtends 3;24° of the ecliptic, the angle at its centre, \( \angle BDA = \begin{cases} 3;24° & \text{where 4 right angles} = 360° \\ 6;48° & \text{where 2 right angles} = 360°. \end{cases} \)

Therefore in the circle about right-angled triangle DEZ.

\[ \text{arc } EZ = 6;48° \]

and \( EZ = 7;7,0° \) where hypotenuse DE = 120°.
Similarly, since arc BA = 53;35°, the angle [it subtends] at the circumference, 
\[ \angle BEA = 53;35^\circ \] where 2 right angles = 360°.

But, in the same units, \[ \angle BDA = 6;48^\circ \].

Therefore, by subtraction, \[ \angle EAZ = 46;47^\circ \] in the same units.

Therefore in the circle about right-angled triangle AEZ,
\[ \text{arc } EZ = 46;47^\circ \]
and \[ EZ = 47;38,30^\prime \] where hypotenuse EA = 120°.

Therefore where \[ EZ = 7;7,0^\circ \text{ and } ED = 120^\circ, \]
\[ AE = 17;55,32^\circ. \]

Again, since arc BAG subtends 0;37° of the ecliptic,
the angle at its centre, \[ \angle BDG = \begin{cases} 0;37^\circ \text{ where 4 right angles = 360°} \\ 1;14^\circ \text{ where 2 right angles = 360°}. \end{cases} \]

Therefore in the circle about right-angled triangle DEH,
\[ \text{arc EH} = 1;14^\circ \]
and \[ EH = 1;17,30^\prime \] where hypotenuse DE = 120°.

Similarly, since arc BAG = 150;26°,
the angle [it subtends] at the circumference, 
\[ \angle BEG = 150;26^\circ \] where 2 right angles = 360°.

But \[ \angle BDG = 1;14^\circ \] in the same units.

Therefore, by subtraction, \[ \angle EGD = 149;12^\circ \] in the same units.

Therefore in the circle about right-angled triangle GEH,
\[ \text{arc EH} = 149;12^\circ \]
and \[ EH = 115;41.21^\prime.16 \] where hypotenuse GE = 120°.

Therefore where \[ EH = 1;17,30^\circ \text{ and } DE = 120^\circ, \]
\[ GE = 1;20,23^\circ, \]

and, as we showed, \[ EA = 17;55,32^\circ \] in the same units.

Again since, as we showed, arc AG = 96;51°,
the angle [subtended by it] at the circumference, 
\[ \angle AEG = 96;51^\circ \] where 2 right angles = 360°.

Therefore in the circle about right-angled triangle GEΘ,
\[ \text{arc } GΘ = 96;51^\circ \]
and \[ arc \Theta G = 83;9^\circ \] (complement).

So the corresponding chords
\[ GΘ = 89;46,14^\prime \]
and \[ EΘ = 79;37,55^\prime \] where hypotenuse GE = 120°.

Therefore where \[ GE = 1;20,23^\circ \]
\[ GΘ = 1;0,8^\circ \]
and \[ EΘ = 0;53,21^\circ. \]

And, in the same units, the whole line EA was found to be 17;55,32°.

Therefore, by subtraction, \[ ΘA = 17;2,11^\circ \] where \[ GΘ = 1;0,8^\circ. \]

And the square on \[ AΘ \] is 290;14,19
while the square on \[ GΘ \] is 1;0,17.

But \[ AG^2 = AΘ^2 + GΘ^2 = 291;14,36. \]
Therefore AG = 17°3.57" where DE = 120° and GE = 1°20.23".
But, where the diameter of the epicycle is 120°, AG = 89°46.14°
(for it subtends arc AG, which is 96°51').
Therefore where AG = 89°46.14° and the epicycle diameter is 120°,
DE = 631°13.48°
and GE = 7°2.50".
Therefore arc GE of the epicycle = 6°44.1°.
And, by hypothesis, arc BAG = 150°26°.

Therefore, by addition, arc BGE = 157°10.1°,
so its chord, BE = 117°37.32° where the epicycle diameter is 120° and ED = 631°13.48°.

Now if we had found BE equal to the diameter of the epicycle, the epicycle
centre would, obviously, lie on it, and we would immediately get the ratio
between the diameters [of epicycle and deferent]. Since, however, it is less than
the diameter, and also arc BGE is less than a semi-circle, it is clear that the
centre of the epicycle will fall outside segment BAGE.

Let it be [Fig. 4.6] in point K, and draw the line DMKL from D, the centre of
the ecliptic, through K. Thus point L represents the apogee of the epicycle and
M its perigee. Then

$$BD \cdot DE = LD \cdot DM;$$  

$$37$$

Fig. 4.6

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37 Euclid III 36: the rectangle contained by any line drawn from a point outside the circle and the
segment of that line outside the circle equals the square on the tangent to the circle from that point.
and we have shown that where the epicycle diameter $LKM = 120'$,
$BE = 117;37,32''$ and $ED = 631;13,48''$.
Therefore, by addition, $BD = 748;51,20''$.
Therefore $LD.DM = BD.DE = 472700;5,32''$.  
Furthermore, since $LD.DM + KM^2 = DK^2$, and the radius of the epicycle, $KM = 60''$,
$$KM^2 = 3600'',$$
and $DK^2 = 472700;5,32'' + 3600'' = 476300;5,32''$.  
Therefore $DK$, the radius of the deferent circle concentric to the ecliptic, is $690;8,42''$ where $KM$, the radius of the epicycle, is $60''$.  
So, where the radius of the deferent, the centre of which coincides with the observer, is $60''$, the radius of the epicycle is about $5;13''$.  

Repeating the same figure [Fig. 4.7], drop perpendicular $KNX$ from centre $K$ on to $BE$, and join $BK$.

Now, where $DK = 690;8,42''$,
we found that $DE = 631;13,48''$
and $NE = \frac{1}{2}BE = 58;48,46''$.
Therefore, by addition, $DEN = 690;2,34''$.

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Fig. 4.7

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38 Euclid II 6: if a straight line (LM) be bisected and a straight line (DM) added to it, the rectangle contained by the whole plus the added line (LD) and the added line (DM), together with the square on the half (KM) is equal to the square on the line (DK) made up of the half (KM) and the added line (DM).
Therefore in the circle about right-angled triangle DNK,
\[ \text{DN} = 119;58,57^\circ \] where hypotenuse \( \text{DK} = 120^\circ \),
and arc \( \text{DN} \approx 178;2^\circ \).
\[ \therefore \angle \text{DKN} = \begin{cases} 178;2^\circ & \text{where 2 right angles = } 360^\circ \\ 89;1^\circ & \text{where 4 right angles = } 360^\circ. \end{cases} \]
Therefore arc XM of the epicycle = 89;1°,
and arc LBX = 90;59° (complement),
\[ \text{arc XB} = \frac{1}{3} \text{arc BXE} = 78;35^\circ \] (for arc BE was determined [p. 196] as about 157;10°).

Therefore, by subtraction, arc LB of the epicycle, which is the distance of the moon from the apogee of the epicycle at the middle of the second eclipse in question, is 12,24°.

Similarly, since, as we showed,
\[ \angle \text{DKN} = 89;1^\circ \] where 4 right angles = 360°,
by subtraction, \( \angle \text{KDN} \), which represents the equation of anomaly (which is subtractive with respect to the mean motion) corresponding to the epicycle arc LB, is 0;59° (complement of \( \angle \text{DKN} \)). Therefore the mean position of the moon at the middle of the second eclipse was \( 14;44^\circ \), since its true position was \( 13;45^\circ \), corresponding to the position of the sun in Pisces.

Let us now turn to the three eclipses which we have selected from those very carefully observed by us in Alexandria.

The first occurred in the seventeenth year of Hadrian, Pauni \([X]\) 20:21 in the Egyptian calendar [133 May 6:7]. We computed the exact time of mid-eclipse as \( \frac{1}{3} \) of an equinoctial hour before midnight. It was total.\(^{39}\) At that time the true position of the sun was about \( 8^\circ \) 13\( \frac{1}{2} \).\(^{39}\)

The second occurred in the nineteenth year of Hadrian, Choiak \([IV]\) 2:3 in the Egyptian calendar [134 Oct. 20/21]. We computed that mid-eclipse occurred \( 1 \) equinoctial hour before midnight. [The moon] was eclipsed \( \frac{1}{3} \) of its diameter from the north.\(^{40}\) At that time the true position of the sun was about \( \approx 25^\circ \).\(^{40}\)

The third eclipse occurred in the twentieth year of Hadrian, Pharmouthi \([VIII]\) 19/20 in the Egyptian calendar [136 Mar. 5/6]. We computed that mid-eclipse occurred \( 4 \) equinoctial hours after midnight. [The moon] was eclipsed \( \frac{1}{2} \) of its diameter from the north.\(^{41}\) At that time the position of the sun was about \( \approx 14r\).\(^{41}\)

It is clear that here too the mean motion [in longitude] of the moon, beyond complete revolutions, is equal to that of the sun, and is:

\[ \text{from middle of the first eclipse to middle of the second: } 161;55^\circ \]
\[ \text{from middle of the second eclipse to middle of the third: } 138;55^\circ. \]

The length of the first interval is:

1 Egyptian year 166 days 23\( \frac{1}{2} \) equinoctial hours reckoned simply
1 Egyptian year 166 days 23\( \frac{1}{2} \) equinoctial hours reckoned accurately.

\(^{39}\)Oppolzer no. 2071, the circumstances of which agree well with Ptolemy's report.

\(^{40}\)Oppolzer no. 2074, the circumstances of which agree extremely well with Ptolemy's report.

\(^{41}\)Oppolzer no. 2075; circumstances: mid-eclipse 1:43 a.m. \( \approx 3\frac{1}{2} \) hours after midnight Alexandria, magnitude 5.5 digits.
The length of the second interval is:
1 Egyptian year 137 days 5 equinoctial hours reckoned simply
1 Egyptian year 137 days $5\frac{1}{4}$ equinoctial hours reckoned accurately.

The approximate mean motion of the moon (beyond complete revolutions) is:

\[ \begin{align*}
&\text{in } 1^\circ 166^d 23^h \\
&\text{110;21° in anomaly} \\
&169;37° \text{ in longitude} \\
&\text{and in } 1^\circ 137^d 5^h \\
&81;36° \text{ in anomaly} \\
&137;34° \text{ in longitude.}
\end{align*} \]

Therefore, clearly, the 110;21° of motion on the epicycle over the first interval have produced a decrement from the mean motion of [161;55° - 169;37° =] 7;42°, while the 81;36° of motion on the epicycle over the second interval have produced an increment to the mean motion of [138;55° - 137;34° =] 1;21°.

With the above data, let the moon's epicycle [Fig. 4.8] be ABG. Let A be the point in which the moon was at the middle of the first eclipse, B its location at the middle of the second eclipse, and G its position at the middle of the third.

![Fig. 4.8](image-url)
motion; thus the remaining arc GA is 168;3° and produces an increment to the mean motion of 6;21°, which is the difference [between 7;42° and 1;21°].

It is clear that the apogee must lie on arc AB, since it can lie neither on arc BG nor on arc GA, both of which produce an additive effect and are less than a semi-circle. In the same way [as before],12 take the centre of the ecliptic and the circle carrying the epicycle as D, and draw from it, to the points representing the 3 eclipses, lines DEA, DB, DG. Join BG and draw from point E to B and G lines EB and EG, and drop on to lines BD and DG perpendiculars EZ and EH. Also drop perpendicular GΘ from G on to BE.

Then, since arc AB subtends 7;42° on the ecliptic, the angle at the centre of the ecliptic,

\[ \angle ADB = \begin{cases} 7;42° & \text{where 4 right angles = 360°} \\ 15;24° & \text{where 2 right angles = 360°} \end{cases} \]

Therefore in the circle about right-angled triangle13 DEZ,

arc EZ = 15;24°

and EZ = 16;4,42° where hypotenuse DE = 120°.

Similarly, since arc AB = 110;21°,

the angle [subtended by it] at the circumference,

\[ \angle AEB = 110;21°° \text{ where 2 right angles = 360°°} \]

But \[ \angle ADB = 15;24°° \text{ in the same units.} \]

Therefore, by subtraction, \[ \angle EBD = 94;57°° \text{.} \]

Therefore in the circle about right-angled triangle14 BEZ,

arc EZ = 94;57°

and EZ = 88;26,17° where hypotenuse BE = 120°.

Therefore where EZ = 16;4,42° and DE = 120°,

BE = 21;48,59°.

Furthermore, since, as we showed, arc GEA subtends 6;21° of the ecliptic, the angle at the centre of the ecliptic also,

\[ \angle ADG = \begin{cases} 6;21° & \text{where 4 right angles = 360°} \\ 12;42° & \text{where 2 right angles = 360°} \end{cases} \]

Therefore in the circle about right-angled triangle DEH,

arc EH = 12;42°

and EH = 13;16,19° where hypotenuse DE = 120°.

Similarly, since arc ABG = 191;57°,

the angle [subtended by it] at the circumference,

\[ \angle AEG = 191;57°° \text{ where 2 right angles = 360°°} \]

But \[ \angle ADG \text{ was found to be 12;42°° in the same units.} \]

Therefore, by subtraction, \[ \angle EGD = 179;15°° \text{ in the same units.} \]

Therefore in the circle about right-angled triangle GEH,

arc EH = 179;15°

and EH = 119;59,50° where hypotenuse GE = 120°.

12 Reading ὁμοιώς for ὁμας ως μη ὑποκειμένου τούτου at H317.4-5. This would mean 'Nevertheless, without this as an assumption'; but the location of the apogee on arc AB is (and must be) assumed in Fig. 4.8. I suppose that ὁμοιώς ('similarly') was corrupted to ὁμας ('however') and the rest then added as an ancient gloss.

13 Reading ὀθογόνον (with D, Ar) for τρίγωνον at H317.25. So too at H319.4 and 319.14.

14 Reading BEZ ὀθογόνον (with D, Ar) for BEZ at H318.8.
Therefore where \( \text{EH} = 13;16,19^o \) and \( \text{DE} = 120^o \),
\( \text{GE} = 13;16,20^o \).
And, as we showed, \( \text{BE} = 21;48,59^o \) in the same units.
Furthermore, since \( \text{arc BG} = 81;36^o \),
the angle [subtended by it] at the circumference,
\[ \angle \text{BEG} = 81;36^o \] where 2 right angles = \( 360^o \).
Therefore in the circle about right-angled triangle \( \text{GE}0 \),
\( \text{arc G}0 = 81;36^o \)
and \( \text{arc E}0 = 98;24^o \) (supplement).
Therefore the corresponding chords
\[ \text{G}0 = 78;24,37^o \text{ and E}0 = 90;50,22^o \] where hypotenuse \( \text{EG} = 120^o \).

Therefore where \( \text{GE} = 13;16,20^o \),
\( \text{G}0 = 8;40,20^o \) and \( \text{E}0 = 10;2,49^o \).
And the whole line \( \text{EB} \) was found to be \( 21;48,59^o \) in the same units.
Therefore, by subtraction [of \( \text{E}0 \) from \( \text{EB} \)],
\( \text{O}B = 11;46,10^o \) where \( \text{G}0 = 8;40,20^o \).

But where the diameter of the epicycle is \( 120^o \),
\( \text{BG} = 78;24,37^o \) (chord of arc \( \text{BG} \), which is \( 81;36^o \)).
Therefore where \( \text{BG} = 78;24,37^o \) and the epicycle diameter is \( 120^o \),
\( \text{DE} = 643;36,39^o \) and \( \text{GE} = 71;11,4^o \).
Therefore arc \( \text{GE} \) of the epicycle = \( 72;46,10^o \).
And, by hypothesis, arc \( \text{GEA} = 168;3^o \).
Therefore, by subtraction, arc \( \text{EA} = 95;16,50^o \)
and therefore its chord \( \text{AE} = 88;40,17^o \)
where the epicycle diameter is \( 120^o \) and where \( \text{ED} = 643;36,39^o \).
Furthermore, since arc \( \text{EA} \) was shown to be less than a semi-circle, the centre of the epicycle will, obviously, fall outside segment \( \text{EA} \). Take the centre as point \( K \) [Fig. 4.9], and draw line \( \text{DMKL} \), so that, again, point \( L \) represents the apogee and point \( M \) the perigee. Then
\[ \text{AD} \cdot \text{DE} = \text{LD} \cdot \text{DM} \],
and we have shown that, where the epicycle diameter \( \text{LKM} = 120^o \),
\( \text{AE} = 88;40,17^o \) and \( \text{ED} = 643;36,39^o \).
\( \therefore \text{LD} \cdot \text{DM} = \text{AD} \cdot \text{DE} = 471304;46,17^o \).

Again, since
\[ \text{LD} \cdot \text{DM} + \text{KM}^2 = \text{DK}^2 \],
and \( \text{KM} \), the radius of the epicycle, is \( 60^o \),
if we add the \( 3600^o \) (of \( \text{KM}^2 \)) to the above \( 471304;46,17^o \),
we find \( \text{DK}^2 = 474904;46,17^o \).

45 Reading ἡ δὲ \( \Delta \) ἔστει χθη for ἡ δὲ \( \Delta \) ἔστει χθη βρόκα (all mss.) at H319,7. The latter would mean 'where \( \text{DE} \), as was shown, equals 120°', which is nonsense, since this is assumed, not proven. D,Ar have the same nonsensical ἔστει χθη at H318,11.
46 Reading τοῦ ἐπικύκλου τῶν αὐτῶν ἐστὶν \( \frac{1}{3} \), ἐὰν τὰ \( \frac{1}{3} \) τοῦ τετραγώνου (with D,Ar) for τοῦ
Therefore the radius of the deferent, concentric with the ecliptic,

\[ DK = 689;8^\circ \]

where the radius of the epicycle, \( KM = 60^\circ \).

Therefore where the line joining the centres of ecliptic and epicycle is \( 60^\circ \),

the radius of the epicycle is \( 5;14^\circ \).

This ratio is very nearly the same as that derived just above from the more ancient eclipses.

So, in the same figure [Fig. 4.10] drop perpendicular \( KNX \) from centre \( K \) on to \( DEA \), and join \( AK \).

Then, as we showed, where \( DK = 689;8^\circ \), \( DE = 643;36,39^\circ \);
and \( NE = \frac{1}{2}AE = 44;20,8^\circ \) in the same units.

Therefore, by addition, \( DEN = 687;56,47^\circ \).

Therefore, where hypotenuse \( DK = 120^\circ \), \( DN = 119;47,36^\circ \),
and in the circle about right-angled triangle \( DKN \),

\[ \text{arc } DN \approx 173;17^\circ. \]

\( \therefore \angle DKN = \begin{cases} 173;17^\circ \text{ where } 2 \text{ right angles } = 360^\circ \\ 86;38,30^\circ \text{ where } 4 \text{ right angles } = 360^\circ. \end{cases} \)
\[ \therefore \text{arc MEX of the epicycle} = 86;38.30^\circ, \]
\[ \quad \text{and arc LAX} = 93;21,30^\circ \text{ (supplement)}, \]
\[ \quad \text{and arc } AX = \frac{1}{2} \text{ arc } AE \approx 47;38,30^\circ. \]

Therefore, by subtraction, arc AL = 45;43°.

But, by hypothesis, the whole arc AB = 110;21°.

Therefore, by subtraction, arc LB = 64;38°.

This is the distance of the moon from the apogee at the middle of the second eclipse determined above.

Similarly, as we showed,
\[ \angle DKN \approx 86;38^\circ, \]
\[ \text{so } \angle KDN = 3;22^\circ \text{ (complement)}, \]

and, by hypothesis, \[ \angle ADB = 7;42^\circ. \]

Therefore, by subtraction, \[ \angle LDB = 4;20^\circ. \]

This angle subtends the arc of the ecliptic representing the equation of anomaly (which is subtractive with respect to the mean motion) resulting from arc LB of the epicycle.

Therefore the mean position of the moon at the middle of the second eclipse was \( \Upsilon 29;30^\circ \), since its true position was \( \Upsilon 25;10^\circ \), corresponding to the position of the sun in Libra.
IV 7. Correction of mean motion in anomaly of moon.

7. {On the correction of the mean positions of the moon in longitude and anomaly}^47

Now we have shown that the mean position of the moon at the middle of the second of the [three] ancient eclipses was:

- in longitude: \(14;44^o\)
- in anomaly: \(12;24^o\) from the apogee of the epicycle;

and at the second of the three eclipses in our time:

- in longitude: \(29;30^o\)
- in anomaly: \(64;38^o\) from the apogee.

So it is clear that in the interval between the above two eclipses the mean motion of the moon, beyond complete revolutions, was:

- in longitude: \(224;46^o\)
- in anomaly: \(52;14^o\).

Now the time between Mardokempad 2, Thoth 18/19, \(\frac{1}{2}\) hour before midnight, and Hadrian 19, Choiak 2/3, 1 hour before midnight is

- 854 Egyptian years \(73^d 23^h\) equinoctial hours reckoned simply
- 854 Egyptian years \(73^d 23^h\) equinoctial hours reckoned accurately (in mean solar days).

In days this is 311783 days \(23^h\) equinoctial hours.

In this interval we find that the increment over complete revolutions, according to the daily motions derived above from the uncorrected hypotheses, is:

- in longitude: \(224;46^o\)
- in anomaly: \(52;31^o\)^48

Thus, as we said [p. 179], we find that the increment in longitude is identical with what we derived from the above observations, but the increment in anomaly is 17 minutes too great. Hence, before constructing the [mean motion] tables, we corrected the daily motion in anomaly by dividing these 17 minutes by the above total in days, and subtracting the resulting correction for 1 day (of \(0;0,0,0,11.46.39^o\)) from the uncorrected mean daily motion in anomaly. The corrected motion is \(13;3,53,56,17,51,59^o\), which is the basis of the other entries, derived by accumulation, in the tables.

8. {On the epoch of the mean motions of the moon in longitude and anomaly}^49

In order to establish the epochs of these [mean motions] for the same first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, we took the time-interval from that moment to the middle of the second eclipse of the first trio (which is the nearer [to the epoch]). This, as we said, took place in the second year of Mardokempad, Thoth 18/19 in the Egyptian calendar, \(\frac{1}{2}\)th of an equinoctial hour before midnight. This interval is computed as 27 Egyptian years, 17 days

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^47 On chs 7 and 8 see *HAMA* 78-9, Pedersen 180-2.

^48 If one computes accurately with Ptolemy's mean daily motions (p. 179) one finds \(224,47,15^o\) (cf. *HAMA* 79) and \(52,32,18^o\) respectively, i.e. in each case one minute more (not utterly negligible in this context). I suspect that Ptolemy computed, not for \(23;20^h\), but for \(23;18^h\), i.e. his correction for the equation of time was not precisely \(-1^h\), but \(-32\) mins. (Accurate computation gives \(-28\frac{1}{2}\) mins.)
and 11 \frac{1}{2} hours both by the simple and (approximately) by the accurate reckoning. To this interval corresponds (beyond complete revolutions) 123;22° in longitude, and 103;35° in anomaly. Subtracting each of these values from the corresponding one at the middle of the second eclipse [m 14;44° and 12;24°, p. 198], we find for the mean positions of the moon in the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon: in longitude: 8 11;22° in anomaly: 268;49° from the apogee of the epicycle in elongation: 70;37° (for, as we showed, the [mean] position of the sun at the same moment was X 0;45°).

9. \{On the correction of the mean positions in latitude of the moon, and their epochs\}  

By the above methods we have established the periodic motions and epochs [of the moon] in longitude and anomaly. Concerning the corresponding amounts for its latitude, we were formerly in error, because we too adopted Hipparchus’ assumptions that [the diameter of] the moon goes approximately 650 times into its own orbit, and 2\frac{1}{2} times into [the diameter of] the earth’s shadow, when it is at mean distance in the syzygies. For once these quantities and the size of the inclination of the moon’s orbit are given, the limits of individual lunar eclipses are given. So we took [pairs of] eclipses separated by a known interval, computed (from the magnitude of the obscuration at mid-eclipse) the true distance [of the moon] from whichever of the two nodes [the eclipse was near] along its inclined circle in [argument of] latitude, determined the mean position [in latitude] from the true by applying the equation of anomaly as already determined, and thus found the mean position in latitude at the middle of each eclipse, and hence the motion in latitude (as increment over complete revolutions) during that interval.  

But now, using more elegant methods which do not require any of the previous assumptions for the solution of the problem, we have found that the motion in latitude computed by the above method is faulty. Furthermore, from

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49 The equation of time between era Nabonassar (-746 Feb. 26) and the eclipse in question (-719 Mar. 18) is in fact about -3 mins. This would make the mean motions 1 minute less in each case than Ptolemy’s figures.  
50 See HAM 80-2. Pedersen 181 is inadequate.  
51 Hipparchus’ method was first explained by Schmidt. ‘Maanens Middelbevaegelse’. Cf. HAM 313. Norman T. Hamilton has discovered the relevance of this passage to the value of the moon’s mean motion and position in latitude given in the Canobic Inscription, (Op. Min. 151-2, cf. HAM 914), and shown that these were derived by application of the method outlined here to the two eclipses Nabonassar 281 18 19 (IV 6, H303) and Nabonassar 882 IV 2/3 (IV 6, H315). The first of these had already been used by Hipparchus (cf. VI 9, H526), who had found (by this method) that the moon was 9° past the node. Applying Hipparchus’ mean motion in latitude to the interval between the eclipses, Ptolemy found that the moon should have been 5° past the node at the second eclipse. However, from the observed magnitude he computed that it must rather be 6° past the node, and thus ‘corrected’ Hipparchus’ mean motion by adding 1°, to be distributed over the intervening 311.784 days. Cf. IV 7. This produces exactly the value found in the Canobic Inscription.
the motion in latitude computed from our new method without those assumptions, we have proven that those very assumptions concerning sizes and distance are false, and have corrected them. We have done something similar with the hypotheses for Saturn and Mercury, changing some of our earlier, somewhat incorrect, assumptions because we later got more accurate observations. For those who approach this science in a true spirit of enquiry and love of truth ought to use any new methods they discover, which give more accurate results, to correct not merely the ancient theories, but their own too, if they need it. They should not think it disgraceful, when the goal they profess to pursue is so great and divine, even if their theories are corrected and made more accurate by others beside themselves. As for those topics [corrections to the theories of Saturn and Mercury], we will explain how we deal with them at the proper places in the later part of our treatise. For the time being, to preserve the proper order of procedure, we will turn to the demonstration of the position in latitude, which is by the following method.

First, then, to correct the actual mean motion in latitude, we looked for [pairs of] lunar eclipses (among those securely recorded) separated by as great an interval as possible, at both of which
[1] the size of obscuration was equal,
[2] the eclipses took place near the same node,
[3] the eclipse was from the same side (either both from the north or both from the south) and
[4] the moon was at about the same distance [from the earth].

If these conditions are fulfilled the moon’s centre must be the same distance from the same node, and on the same side, at both eclipses, and hence its true motion in latitude during the interval between the observations contains an integer number of revolutions in latitude.

The first eclipse we used is the one observed in Babylon in the thirty-first year of Darius I, Tybi [V] 3–4 in the Egyptian calendar, [−490 Apr. 25–26] at the middle of the sixth hour [of night]. It is reported that at this eclipse the moon was obscured 2 digits from the south.

The second eclipse we used is the one observed in Alexandria in the ninth year of Hadrian, Pachon [IX] 17–18 in the Egyptian calendar [125 Apr. 5–6], 3½ equinoctial hours before midnight. At this eclipse too the moon was obscured 2 digits from the south.

The position of the moon in latitude was near the descending node at each
eclipse (such conclusions can be drawn even from quite crude hypotheses). The distance [of the moon] was about the same [at both eclipses], and a little closer to the perigee than the mean distance. This too can be shown from our previous determination of the anomaly. Now, when the moon is eclipsed from the south, its centre is north of the ecliptic. So it is clear that at both eclipses the moon’s centre was an equal amount in advance of the descending node. In the first eclipse the distance of the moon from the apogee of the epicycle was 100;19°. (For the time of mid-eclipse was 1-hour before midnight at Babylon, and [hence] 1 3/4 equinoctial hours before midnight at Alexandria, from the Nabonassar epoch the time comes to

\[
256 \text{ years } 122 \text{ days } \begin{cases} 10^1 \text{ hours reckoned simply} \\ 10^4 \text{ hours reckoned in true solar days.} \end{cases}
\]

Therefore the true position was 5° less than the mean. In the second eclipse the moon was 251;53° from the apogee of the epicycle. (For in this case the time from epoch to the middle of the eclipse comes to

\[
871 \text{ years } 256 \text{ days } \begin{cases} 8^1 \text{ equinoctial hours reckoned simply} \\ 8^4 \text{ equinoctial hours reckoned accurately.} \end{cases}
\]

Therefore the true position was 4;53° more than the mean. Therefore, in the interval between the two eclipses, which comprises 615 Egyptian years, 133 days and 21 3/4 equinoctial hours, the true motion of the moon in latitude comprises an integer number of revolutions, while its mean motion fell short of a complete revolution by 9:53°, which is the sum of both [equations of] anomaly. But according to the mean motions derived from Hipparchus’ hypotheses, as set out above, in that interval it falls short of a complete revolution by about 10:2°. Thus the mean motion in latitude is greater than one would expect from his hypotheses by 9 minutes.

We divided these 9 minutes by the total of days in the above interval (approximately 224609), and added the resulting 0:0:0:8.39.18° to the mean daily motion [in latitude] derived above from those hypotheses; thus we found the corrected mean motion of 13:13.45.39.48.56.37°, which we again used as the basis for the other accumulated totals in the tables.

Having once, in this way, determined the mean motion in latitude, we next proceeded to establish its epoch position. For this purpose we looked for another pair of accurately observed eclipses at a known interval, in which all the same conditions were fulfilled as in the previous pair (namely, for both eclipses the distance of the moon was approximately equal, and [the magnitude of] the obscuration was equal and from the same side (either from the north or from the south for both), except that here the eclipses were near opposite nodes instead of near the same node.

55 For an example of how this can be done see H.A.M.A 81 n.4.
56 It is not clear whether Ptolemy takes the time of the observation to be given in seasonal or equinoctial hours. However, the sun is close enough to the equinox that (for 4-hour) the difference is minimal.
57 The simplest way to check this (and the corresponding amount at the second eclipse) is to use the equation table (IV 10) with arguments 100:19° and 251:53°.
58 The corrections for equation of time are computed rather inaccurately, being about 4 minutes too great at both eclipses. However, these inaccuracies cancel out in the computation of the interval.
The first of these eclipses is the one which we also used for our demonstration of the anomaly [p. 191]. It occurred in the second year of Mardokempad, Thoth[1] 18/19 in the Egyptian calendar [-719 Mar. 8/9], at midnight in Babylon, and \( \frac{1}{2} \) of an equinoctial hour before midnight at Alexandria; at this eclipse it is recorded that the moon was obscured 3 digits from the south.

The second, which Hipparchus too used, occurred in the twentieth year of that Darius who succeeded Kambyses, Epiphi [XI] 28/29 in the Egyptian calendar [-501 Nov. 19/20], when 6\( \frac{1}{2} \) equinoctial hours of the night had passed; at this eclipse the moon was, again, obscured from the south \( \frac{1}{2} \) of its diameter. The middle of the eclipse was \( \frac{1}{2} \) of an equinoctial hour before midnight in Babylon (for the length of half the night was about 6\( \frac{1}{2} \) equinoctial hours on that date), and [hence] 1\( \frac{1}{2} \) equinoctial hours before midnight in Alexandria.®

Both of these eclipses occurred when the moon was near its greatest distance, but the first was near the ascending node, while the second was near the descending node. So here too the centre of the moon was an equal distance north of the ecliptic at both eclipses.

Then let [Fig. 4.11] the moon's inclined orbit be ABG on diameter AG. Let us take point A as the ascending node, G as the descending node, and B as the northern limit. Cut off equal arcs, AD and GE, from nodes A and G towards the northern limit B. Then in the first eclipse the centre of the moon was at D and in the second at E.

Now the time from epoch to [the middle of] the first eclipse is 27 Egyptian years, 17 days 11\( \frac{1}{2} \) equinoctial hours (reckoned both simply and accurately). Hence the moon's distance from the apogee of the epicycle was 12;24°, and the

\[58\] Reading \( \gamma \varepsilon \nu \mu \varepsilon \nu \varepsilon \nu \) with CD for \( \gamma \varepsilon \nu \mu \varepsilon \nu \varepsilon \nu \) at H332.14.

\[60\] Oppolzer no. 1090: time 21:24\( \approx 11:15 \) p.m. Alexandria), magnitude 2.1 digits.
mean position was greater than the true by 59 minutes. Likewise, the time [from
epoch] to [the middle of] the second eclipse was

245 Egyptian years, 327 days \[10\frac{1}{4}\] equinoctial hours reckoned simply
\[10\frac{3}{4}\] equinoctial hours reckoned accurately.

Hence the moon's distance from the apogee of the epicycle was 2;44°, and the
mean position was greater than the true by 13 minutes. The interval between
the observations contains 218 Egyptian years, 309 days 23½ equinoctial hours,
which produces, for the mean motion in latitude deduced above, an increment
[over complete revolutions] of 160;4°.

So, because of the above, let the mean position of the centre of the moon be at
Z [in Fig. 4.11] at the first eclipse and at H in the second. Then since

\[\text{arc } ZBH = 160;4°\]

and \[\text{arc } DZ = 0;59°\] and \[\text{arc } EH = 0;13°\],

\[\text{arc } DE = \text{arc } DZ + \text{arc } ZBH - \text{arc } EH = 160;50°.\]

\[: (\text{arc } AD + \text{arc } EG) = 19;10° \text{ (supplement).}\]

And, since they are equal, \[\text{arc } AD = \text{arc } EG = 9;35°.\]
That is the amount by which the true position of the moon at the first eclipse was
to the rear of the ascending node, and by which the true position of the moon at
the second eclipse was in advance of the descending node. Therefore, by
addition,

\[\text{arc } AZ = \text{arc } AD + \text{arc } DZ = 10;34°\]

and, by subtraction,

\[\text{arc } HG = \text{arc } EG - \text{arc } EH = 9;22°.\]

Hence the mean position of the moon at the first eclipse was 10;34° to the rear of
the ascending node, and [therefore] was 280;34° from the northern limit B, and
at the second eclipse it was 9;22° in advance of the descending node, and
[therefore] its distance from the northern limit was 80;38°.

Next, since the time from epoch to the middle of the first eclipse produces an
increment [over complete revolutions] of [mean motion in] latitude of 286;19°,
we subtract this amount from the 280;34° for the position at the first eclipse and
(after adding 360°) find, for the first year of Nabonassar, Thoth 1 in the
Egyptian calendar, noon: the mean position in latitude (counted from the
northern limit): 354;15°.

In order to be able to check calculations concerning conjunctions and
oppositions (since for those positions [of the moon] we have no need of the
second anomaly which we shall demonstrate later), we shall set out a table for
the individual [equations of anomaly]. We have calculated it geometrically, in
the same way as we already did for the sun. In this case we used the ratio 60:51
[as a basis], but, as [previously], we tabulate it at intervals of 6° for the apogee
quadrants, and of 3° for the perigee [quadrants]. Thus the layout of the table is
identical to that for the sun: it consists of 45 lines and 3 columns; the first two
columns contain the argument, in degrees of anomaly, while the third contains
the equation corresponding to each argument. In calculating the longitude and
the latitude, this equation has to be subtracted when the anomaly, counted
from the apogee of the epicycle, is up to 180°, and added when the anomaly is
more than 180°. The table is as follows.
### IV 10. Table of lunar equation (first anomaly)

#### 10. {Table of the first, simple anomaly of the moon}

<table>
<thead>
<tr>
<th>Common Numbers</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Equation</td>
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11. \{That the difference in the size of the lunar anomaly, according to Hipparchus, is due not to the different hypotheses employed, but to his calculations\}^61

Now that we have demonstrated the above, it would be quite reasonable for someone to ask why it is that the ratio \([\text{of the eccentricity}]\) found by Hipparchus from the lunar eclipses which he adduced for the determination of this anomaly is neither identical with the one determined by us, nor \([\text{consistent with itself, since}]\) the first ratio he found, using the eccentric hypothesis, differs from the second, which was calculated from the epicyclic hypothesis. For in his first demonstration he derives the ratio between the radius of the eccentric and the distance between the centres of the eccentric and the ecliptic as about 3144:327\(\frac{1}{2}\) (which is the same as 60:6;15), while in the second he finds the ratio between the line joining the centre of the ecliptic to the centre of the epicycle, and the radius of the epicycle, as 3122\(\frac{1}{2}\):247\(\frac{1}{2}\) (which is the same as 60:4;46). Now the maximum equation of anomaly for a ratio of 60:6\(\frac{1}{2}\) is 5;49°; for a ratio of 60:4;46 it is 4;34°, while our ratio of 60:5\(\frac{1}{2}\) produces a maximum equation of about 5°.^62

Such a discrepancy cannot, as some think, be due to some inconsistency between the \([\text{epicyclic and eccentric}]\) hypotheses. Not only have we shown this by logical argument just above \([\text{IV 5}]\), from the perfect agreement between the phenomena resulting from both hypotheses, but numerically too, if we wanted to carry out the calculations, we would find that the same ratio results from both hypotheses, provided we use the same set of data for both, and not, like Hipparchus, different sets. For in that case \([\text{if different sets of \text{éclipses} are used as basis}]\), the discrepancy can occur \([\text{through errors}]\) in the actual observations or in the computations of the intervals. At any rate, we will find that in the case of those eclipses \([\text{used by Hipparchus}]\) the syzygies were observed correctly, and are in agreement with our proven theories for the mean and anomalistic motions, but the computations of the intervals \([\text{on which the demonstration of the size of the ratio depends}]\) were not carried out as carefully as possible. We shall demonstrate both of these assertions, beginning with the first three eclipses.

He says that these three eclipses which he adduces are from the series brought over from Babylon, and were observed there; that the first occurred in the archonship of Phanostratos at Athens, in the month Poseideon;^63 a small section of the moon's disk was eclipsed from the summer rising-point \([\text{i.e. the northeast}]\) when half an hour of night was remaining. He adds that it was still eclipsed

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61 See HAMA 317-19.
62 There are some inaccuracies here: 3122\(\frac{1}{2}\) : 247\(\frac{1}{2}\) \(\approx\) 60 : 4;45,21. The maximum equation resulting from an eccentricity of 4;46 in 60 is not 4;34°, but 4;33° to the nearest minute. These inaccuracies could be eliminated by changing 3122\(\frac{1}{2}\) to 3112\(\frac{1}{2}\) \([\text{cf. p. 215 n.75}]\), but ms. authority is unanimous at all places. Even more inaccurate is the 5;49° of the maximum equation resulting from 60 : 6\(\frac{1}{2}\). Correct (to the nearest minute) is 5;59°, and perhaps we should so emend it \([\text{vθ for μθ at H338.23}]\).
63 It is practically certain that this and the corresponding dates for the other two eclipses are in the astronomical Metonic calendar \([\text{see Introduction p. 12}]\) rather than the Athenian civil calendar, for at the time when the Babylonian observations were \"brought over\", the equation with the old Athenian civil calendar could hardly have been determined, and certainly was of no interest to the users of the observations.
when it set. Now this moment is in the 366th year from Nabonassar, in the Egyptian calendar (as Hipparchus himself says) Thoth 26/27 [-382 Dec. 22/23], 5½ seasonal hours after midnight (since half an hour of night was remaining). When the sun is near the end of Sagittarius, 1 hour of night in Babylon is 18 time-degrees (for the night is 14½ equinoctial hours long). So 5½ seasonal hours produce 6½ equinoctial hours. Therefore the beginning of the eclipse was 18½ equinoctial hours after noon on the 26th. And since a small section [of the disk] was obscured, the duration of the whole eclipse must have been about 1½ hours, so the middle of the eclipse, obviously, must have been 19½ equinoctial hours after [noon]. Therefore mid-eclipse at Alexandria was 18½ equinoctial hours after noon on the 26th. The time from epoch in the first year of Nabonassar to the moment in question is

365 Egyptian years 25 days \[ 18\frac{1}{2} \text{ equinoctial hours reckoned simply} \] \[ 18\frac{1}{2} \text{ equinoctial hours reckoned accurately} \]

At this moment, using our hypotheses as set out above, we find

the true position of the sun as \( \odot 28;18^\circ \),
the mean position of the moon as \( \Pi 24;20^\circ \),
and its true position as \( \Pi 28;17^\circ \)

(for its distance in anomaly from the apogee of the epicycle is 227;43°).

He says that the next eclipse occurred in the archonship of Phanostratos at Athens, in the month Skirophorion, Phamenoth 24/25 in the Egyptian calendar, and that [the moon] was eclipsed from the summer rising-point [i.e. the north-east] when the first hour [of night] was well advanced. This moment is in the 366th year from Nabonassar, Phamenoth [VII] 24/25 [-381 June 18/19], about 5½ seasonal hours before midnight. When the sun is near the end of Gemini, one hour of the night at Babylon is 12 time-degrees. Therefore the 5½ seasonal hours produce 4½ equinoctial hours. So the beginning of the eclipse was 7½ equinoctial hours after noon on the 24th. And since the duration of the whole eclipse is recorded as three hours, mid-eclipse, obviously, occurred 9½ equinoctial hours after [noon]. So in Alexandria it must have occurred about 8½ equinoctial hours after noon on the 24th. The time from epoch is

365 Egyptian years 203 days \[ 8\frac{1}{2} \text{ equinoctial hours reckoned simply} \] \[ 7\frac{1}{2} \text{ equinoctial hours reckoned accurately} \]

For this moment we find:

true longitude of the sun: \( \Pi 21;46^\circ \)

These figures agree well enough with those derivable from the rising-time table (II8) for Clima IV (Rhodes, \( M = 14^\circ, \phi = 36^\circ \)), for \( \lambda_0 = \odot 28;18^\circ \). In the Geography (5.20.6) Ptolemy assigns Babylon a latitude of 35°.

Oppolzer no. 1275: time 5:55 (≈ 7 a.m. Alexandria), magnitude 2.6 digits, half-duration 52 mins. P.V. Neugebauer calculates c. 8 a.m. Babylon (≈ 7 a.m. Alexandria), magnitude 3.0 digits, duration 1.8°.

I.e. here (and in the other five eclipses) the true moon and true sun, as calculated from Ptolemy’s hypotheses, are almost exactly 180° apart, thus giving further confirmation of those hypotheses. In fact more accurate calculation gives rather worse agreement (e.g. here the discrepancy is about 4½ minutes of arc rather than 1°), but in no case is the difference greater than could be explained by the vagueness of the time given in the eclipse report.

Oppolzer no. 1276: time 18:31 (≈ 8:30 p.m. Alexandria), half-duration 1:15°. P.V. Neugebauer calculates the beginning of the eclipse at Babylon as 19.8°, mid-eclipse as ca. 21.1° (≈ 8 p.m. Alexandria), duration 2.7°.
mean longitude of the moon: $23;58^\circ$
true longitude of the moon: $21;48^\circ$
(for its distance from the apogee of the epicycle in anomaly was $27;37^\circ$).
The intervals between the first and second eclipses are:

- [time:] 177\textsuperscript{d} 13\textfrac{1}{2} equinoctial hours
- motion of the sun in longitude: 173;28°,

whereas Hipparchus carried out his demonstration on the basis of the intervals:

- [time:] 177\textsuperscript{d} 13\textfrac{1}{2} equinoctial hours
- [longitude:] 173° – 1°.

He says that the third eclipse occurred in the archonship of Euandros at Athens, in the month Poseideon I, Thoth 16/17 in the Egyptian calendar, and that [the moon] was totally eclipsed, beginning from the summer rising-point [i.e. the north-east], after 4 hours [of night] had passed.\footnote{This moment is in the 367th year from Nabonassar, Thoth [I] 16/17 [–381 Dec. 12/13], about 2\textfrac{1}{2} hours before midnight. Now when the sun is about two-thirds through Sagittarius, one hour of night at Babylon is about 18 time-degrees. So 2\textfrac{1}{2} seasonal hours produce 3 equinoctial hours. Therefore the beginning of the eclipse was 9 equinoctial hours after noon on the 16th. And since the eclipse was total, its duration was about 4 equinoctial hours. So mid-eclipse, clearly, was about 11 hours after noon. Therefore in Alexandria mid-eclipse must have occurred 10\textfrac{1}{2} equinoctial hours after noon on the 16th.\footnote{The time from epoch [to this moment] is 366 Egyptian years 15 days
\begin{align*}
&\text{10\textfrac{1}{2} equinoctial hours reckoned simply} \quad 366 \text{ Egyptian years 15 days} \\
&\text{9\textfrac{1}{2} equinoctial hours reckoned accurately.}
\end{align*}
}
}

For this moment we find:

- true longitude of the sun: $17;30^\circ$
- mean longitude of the moon: $17;21^\circ$
- true longitude of the moon: $17;28^\circ$

(for its distance from the apogee of the epicycle in anomaly was $181;12^\circ$).

The intervals from the second to the third eclipse are:

- [in time:] 177\textsuperscript{d} 2 equinoctial hours
- [in longitude:] 175:44°,

whereas Hipparchus assumed the following intervals:

- [in time:] 177\textsuperscript{d} 1\textfrac{1}{2} hours
- [in longitude:] 175\textfrac{1}{2}°.

Thus it is apparent that he committed errors in his computations of the intervals of $\frac{3}{4}$th and $\frac{3}{4}$rd of an equinoctial hour in time, and about $\frac{1}{4}$ of a degree [in...
longitude] in each interval. Errors of this amount can produce a considerable discrepancy in the size of the ratio [derived].

We will pass to the second set of three eclipses he set out, which he says were observed in Alexandria. He says that the first of these occurred in the 54th year of the Second Kallippic Cycle, Mesore [XII] 16 in the Egyptian calendar [-200 Sept. 22]. In this eclipse the moon began to be obscured half an hour before it rose, and its full light was restored in the middle of the third hour [of night]. Therefore mid-eclipse occurred at the beginning of the second hour, 5 seasonal hours before midnight, and also 5 equinoctial hours, since the sun was near the end of Virgo. So mid-eclipse at Alexandria occurred 7 equinoctial hours after noon on the 16th. And the time from epoch in the first year of Nabonassar is

$$546 \text{ Egyptian years } 345 \text{ days} \left\{ \begin{array}{l}
7 \text{ equinoctial hours reckoned simply} \\
6\frac{1}{2} \text{ equinoctial hours reckoned accurately.}
\end{array} \right.$$ 

For this moment we find:

- true longitude of the sun: $\equiv 26;6^\circ$
- mean longitude of the moon: $\equiv 22^\circ$
- true longitude of the moon: $\equiv 26;7^\circ$

(for its distance in anomaly from the apogee of the epicycle was 300;13').

He says that the next eclipse occurred in the 55th year of the same cycle, Mechir [V] 9 in the Egyptian calendar [-199 Mar. 19], that it began when 3 hours of night had passed, and was total. So the beginning of the eclipse was 11\frac{1}{2} equinoctial hours after noon on the 9th (since the sun was near the end of Pisces), and mid-eclipse was 13\frac{1}{2} equinoctial hours after noon, (since the whole moon was eclipsed). The time from epoch to this moment is

$$547 \text{ Egyptian years } 158 \text{ days} \left\{ \begin{array}{l}
13\frac{1}{2} \text{ equinoctial hours, whether reckoned simply or accurately.}
\end{array} \right.$$ 

For this moment we find:

- true longitude of the sun: $\equiv 26;17^\circ$
- mean longitude of the moon: $\equiv 1;7^\circ$
- true longitude of the moon: $\equiv 26;16^\circ$

(for its distance in anomaly from the apogee was 109;28').

The intervals from first to second eclipse are:

- [in time:] 178\textsuperscript{d} 6\textsuperscript{t} equinoctial hours
- [in longitude:] 180;11°.

\(^{51}\) Oppolzer no. 1545: time 17;2\textsuperscript{h} (= 7 p.m. Alexandria), half-duration 1;29\textsuperscript{h}.

\(^{52}\) Ideler, Untersuchungen 216-17, emended '55th' to '54th' here (H345.12) and was consequently forced to excise av6 ("the same") in the year designation of the third eclipse at H346.13. His argument was that the year begins at the summer solstice in the Kallippic calendar (see Introduction p. 12). Since year 1 of Cycle I begins at the summer solstice of -329, year 54 of Cycle II goes from June -200 to June -199, and thus includes this eclipse of March -199. However, the two passages H345.12 and 346.13 confirm one another, and we must allow the possibility that Hipparchus, who was using the Egyptian calendar within the framework of the Kallippic cycle, began the year, not at the summer solstice, but at Thoth 1. Thus in his reckoning year 55 of Cycle II would run from Oct. of -200 to Oct. of -199, and would include both the second and third eclipses. It is true that this kind of reckoning cannot be applied to the Kallippic years of the equinoxes listed in III 1, but that was in another work of Hipparchus, and there is no mention of the Egyptian calendar there. See also V 3 p. 224 with n.13.

\(^{53}\) Oppolzer no. 1546: time 23;7\textsuperscript{h} (= 1 a.m. Alexandria), half-duration 1;48\textsuperscript{h}. 
whereas Hipparchus carried out his demonstration on the basis of the following intervals:

[in time:] 178° 6 equinoctial hours
[in longitude:] 180;20°.

He says that the third eclipse occurred in the same (55th) year of the Second Cycle, on Mesore [XII] 5 in the Egyptian calendar [-199 Sept. 11] and that it began when 6½ hours of the night had passed, and was total. He also says that mid-eclipse occurred at about 8½ hours of night, that is 2½ seasonal hours after midnight. Now when the sun is near the middle of Virgo, one hour of the night in Alexandria is 14³₂ time-degrees. So 2½ seasonal hours produce about 2⅔ equinoctial hours. So mid-eclipse was 14 ½ equinoctial hours after noon on the 5th. The time from epoch to this moment is

547 Egyptian years 334 days \{ 14 ½ equinoctial hours reckoned simply
13 ½ equinoctial hours reckoned accurately.\}

For this moment we find:
true position of the sun: \( \mu \); 15;12°
true position of the moon: \( \nu \); 10;24°
true position of the moon: \( \nu \); 15;13°

(For its distance in anomaly from the apogee of the epicycle was 249;9°).

The interval from second to third eclipse is:
[in time:] 176° 3 equinoctial hour
[in longitude:] 168;55°,
whereas Hipparchus assumed the following intervals:
[in time:] 176° 1½ equinoctial hours
[in longitude:] 168;33°.

Here too, then, it is apparent that he committed errors of about ³2° and 3° [in longitude], and about 6 and 75 ( 6 + ½ ) equinoctial hours [in time]. These errors too can result in a considerable discrepancy in the ratio calculated for the [particular] hypothesis.

²⁴ Oppolzer no. 1547: time Sept. 12 0:28° (= 2:30 a.m. Alexandria), half-duration 1:50°. Note that for Hipparchus the whole eclipse took place on Mesore 5, although it did not begin until after midnight (what Ptolemy would call 'the midnight which lies towards the sixth'). See Introduction p. 12.

²⁵ Reading ημισει και τριτω και ημισει και τριτω και δεκατω for ημισει και τριτω και δεκατω ( ½ and ½ and 1) at H347, 16-17. The difference between Ptolemy's and Hipparchus' time intervals are: I-II: 6²⁸ - 6²⁸ = 6°; II-III: 1²⁸ - 1²⁸ = 1²⁸ = (6 + ½)°. The emendation is certain and simple, but appears never to have been made. (In the Arabic tradition, T, Q, occurs the almost correct variant ' ½ + ½ + ½'.) Manitus noticed the discrepancy, but was led astray by his misunderstanding at H347, 13-14 of μιας τριτου ώρας, which he took to mean 'a third of one hour'. Thus he supposed the difference between Ptolemy's and Hipparchus' intervals (II-III) to be (½ - ½) = 4 minutes = ½ hour, and emended Heiberg's δεκατω to δωδεκατω (the reading of D). I carelessly followed his interpretation and emendation in Toomer[2], in which I used Hipparchus' intervals to recompute the ratios for the eccentric and epicyclic models. The result was that, while I found fairly good agreement with the ratio 3144:3271 for the eccentric model, using the first triple of eclipses, I could derive a value close to the ratio 3122 3:2475 for the epicyclic model and the second eclipse triple only by attributing a computational error to Hipparchus. Now, however, using the correct time interval of 1²⁸ for II-III, I find much better agreement with the above ratio, as I shall show in detail elsewhere. (If the ratio were 3112 2:475, agreement would be almost perfect, and this also provides a better fit with the equivalences given by Ptolemy.) These calculations not only vindicate Hipparchus' computational abilities, but cast doubt on my claim that he was operating with a chord table with base \( R = 3438°\).
Thus we have plainly displayed the reason for the above discrepancy, and it is clear that we can have even more confidence than before in the correctness of the ratio we deduced for the anomaly at lunar syzygies, since we have found these very same eclipses agreeing closely with our hypotheses.
1. (On the construction of an 'astrolabe' instrument)¹

As far as concerns the [moon's] syzygies with the sun at conjunction and opposition, and the eclipses which occur at such syzygies, we find that the hypothesis set out above for the first, simple anomaly is sufficient, even if we employ it just as it is, without any change. But for particular positions [of the moon] at other sun-moon configurations one will find that it is no longer adequate, since as we said [p. 181], we have discovered that there is a second lunar anomaly, related to its distance from the sun. This anomaly is reduced to the first [i.e. becomes zero] at both syzygies, and reaches a maximum at both quadratures. We were led to awareness of and belief in this [second anomaly] by the observations of lunar positions recorded by Hipparchus,² and also by our own observations, which we made by means of an instrument which we constructed for this purpose. The makeup of the instrument is as follows.

We took two rings of an appropriate size, with their surfaces precisely turned on the lathe so as to be squared off [i.e. with rectangular cross-sections], equal and similar to each other in all dimensions. We joined them together at diametrically opposite points, so that they were fixed at right angles to each other, and their corresponding surfaces coincided: thus one of them [Fig. F,3] represented the ecliptic, and the other [Fig. F,4] the meridian through the poles of the ecliptic and the equator [i.e. a colure]. On the latter, using the side of the [inscribed] square [as measure], we marked the points representing the poles of the ecliptic, and pierced each point with a cylindrical peg [Fig. F,5] projecting beyond both outer and inner surfaces. On the outer [projections] we pivoted another ring [Fig. F,5] the concave [inner] surface of which fitted closely on the convex [outer] surface of the two joined rings, in such a way that it could move freely about the above-mentioned poles of the ecliptic in the

¹On the instrument described in this chapter the only good discussion is that of Rome[4], to which the reader is referred for all details of its construction and use. My Fig. F is based on the drawing there. The numbers and letters designating the rings and other parts of the instrument also follow Rome’s notation. In modern terms, it is an ‘armillary sphere’. The adjective ‘astrolabe’ applied to it and to its parts simply means ‘for taking the [the position of] the stars’, and has nothing to do with the instrument to which the name ‘astrolabe’ is now usually applied (on which see HAMA II 868–79). The latter was called the ‘small astrolabe’ by Theon of Alexandria: see Rome[1] I 4 n.0; by Ptolemy it was apparently called ‘horoscopic instrument’ (see HAMA II 866).

²Examples of these are preserved at V 3 p. 224 and V 3 pp. 227 and 230. It is notable that these are the latest three known observations of Hipparchus. The obvious conclusion is that towards the end of his career he suspected that the ‘simple’ lunar hypothesis was inadequate for positions outside the syzygies, and was making observations to check this.
longitudinal direction. Similarly we pivoted another ring [Fig. F.2] on the inner [projections]; this too fitted the two [joined] rings closely, its convex surface to their concave, and, like the outer ring, moved freely in longitude about the same poles. We marked on this inner ring, and also on the ring representing the ecliptic, the divisions indicating the standard 360 degrees of the circumference, and as small subdivisions of a degree as was practical. Then we fitted snugly inside the inner of the two [movable] rings another thin ring [Fig. F.1] with sighting-holes [Fig. F,b,b] projecting from it at diametrically opposite points. [This ring was constructed] so that it could move laterally in the plane of the ring it was fitted into, towards either of the above-mentioned poles, in order to allow observation of the variation in latitude.

Having completed the above construction, we marked off from both poles of the ecliptic, on the ring representing the circle through both poles [Fig. F.4], an arc equal to the distance between the poles of ecliptic and equator (as determined above). At the ends of these arcs (which were, again, diametrically opposite) we again inserted pivots [Fig. F,d,d], attaching them to a meridian ring [Fig. F,6] similar to that described at the beginning of this treatise [pp. 61-2] for making observations of the arc of the meridian between the solstitial points. This meridian ring was set up in the same position as the earlier one, perpendicular to the plane of the horizon and at an elevation of the pole appropriate for the place in question, and also parallel to the plane of the actual meridian [at that place]. Thus the inner rings [Fig. F,4 etc.] were set up so as to

3 Reading τοῦ ἐν ἀρχή τῆς συντάξεως ἀποδειγμένη (with D,Ar) for τοῦ ἐν ἀρχή τῆς συντάξεως ἰποδειγμένην (which is untranslatable) at H353.1-2.
revolve about the poles of the equator, from east to west, following the first motion of the universe.

Once we had set up the instrument in the way described, whenever we had a situation in which both sun and moon could be observed above the earth at the same time, we set the outer astrolabe ring [Fig. F,5] to the graduation [on the ecliptic ring, fig. F,3] marking, as nearly as possible, the position of the sun at that moment. Then we rotated the ring through the poles [Fig. F,4] until the intersection [of outer astrolabe ring and ecliptic ring] marking the sun’s position was exactly facing the sun, and thus both the ecliptic ring [Fig. F,3] and the [ring] which goes through the poles of the ecliptic [Fig. F,5] cast its shadow exactly on itself. 1 Or, if we were using a star as sighting [i.e. orienting] object, we set the outer [astrolabe] ring to the position assumed for that star on the ecliptic-ring, [and then rotated the ring Fig. F,4 to such a position] that when we applied one eye to one face of the outer ring [Fig. F,5] the star appeared fastened, so to speak, to both [nearer and farther] surfaces of that face, 2 and thus was sighted in the plane through them. Then we rotated the other, inner astrolabe ring [Fig. F.2] towards the moon (or any other object we desired) so that the moon (or any other desired object) was sighted through both sighting-holes on the inmost ring at the same time as the sun (or the other sighting-star) was being sighted [as described above].

In this way we read off the position [of the moon or any other desired object] in longitude on the ecliptic, from the graduation occupied by the inner [astrolabe] ring [Fig. F.2] on the ring representing the ecliptic[Fig. F.3], and its deviation to north or south [of the ecliptic] along the circle through the poles of the ecliptic, from the graduation of the inner astrolabe ring [Fig. F.2]: the latter is given by the distance between the mid-point of the upper 3 sighting-hole on the inmost rotating ring [Fig. F.1] and the line drawn through the centre of the ecliptic ring.

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1 According to Ptolemy’s instructions, one has to compute the solar longitude, set the outer astrolabe ring [Fig. F.5] to that position on the ecliptic ring [Fig. F.3], and then, keeping the two in that position relative to each other, swing both until one can sight the sun along the outer astrolabe ring. Both rings should then shade themselves. Theoretically, even without knowing the sun’s position, one could set up the instrument by sighting the sun along the outer astrolabe ring and then moving the ecliptic ring relative to the latter until it shaded itself. Cf. p. 224 n.11.

2 Reading ὀπερ κεκολλημένος ἀμφότερας αὐτῆς ταῖς ἐπιφανείαις for καὶ διὰ τῆς ἀπεναντίων καὶ παραλλήλων τοῦ κύκλου πλευρᾶς ὀπερ κεκολλημένος ἀμφότερας αὐτῶν ταῖς ἐπιφανείαις at H33.24-334.1. The latter would mean ‘when we applied one eye to the [nearer] face of the outer ring and [looked] along the opposite, parallel face of the ring, the star appeared fastened, so to speak, to the surfaces of both those faces’. The words καὶ διὰ . . . πλευρᾶς are a foolish explanatory interpolation by someone who misinterpreted ἀμφότερας ταῖς ἐπιφανείαις to mean ‘the opposite faces’ of the ring instead of ‘the two parts of the same face nearer to and farther from the eye’; then αὐτῆς (referring to τῇ ἐτέρᾳ τῶν πλευρῶν) was changed to αὐτῶν (referring to both πλευραί), or possibly αὐτῶν was simply interpolated. Quite apart from the technical problem, the text as printed by Heiberg is extraordinarily clumsy. The interpolation is quite early, since it is also in the Arabic tradition. Pappus’ commentary to the passage betrays no hint that he read the interpolation, but is not sufficiently close to the Almagest to allow us to say that he did not.

3 ‘upper’: literally ‘above the earth’. Since the centre of all the rings represents the centre of the earth, the sight nearer the observer’s eye is notionally ‘below the earth’, the other ‘above the earth’. 
When this type of observation was made without further analysis, it was found, both from the observations recorded by Hipparchus and from our own, that the distance of the moon from the sun was sometimes in agreement with that calculated from the above [simple] hypothesis, and sometimes in disagreement, the discrepancy being at some times small and at other times great. But when we paid more attention to the circumstances of the anomaly in question, and examined it more carefully over a continuous period, we discovered that at conjunction and opposition the discrepancy [between observation and calculation] is either imperceptible or small, the difference being of a size explicable by lunar parallax; at both quadratures, however, while the discrepancy is very small or nothing when the moon is at apogee or perigee of the epicycle, it reaches a maximum when the moon is near its mean speed and [thus] the equation of the first anomaly is also a maximum; furthermore, at either quadrature, when the first anomaly is subtractive the moon's observed position is at an even smaller longitude than that calculated by subtracting the equation of the first anomaly, but when the first anomaly is additive its true position is even greater [than that calculated by adding the equation of the first anomaly], and the size of this discrepancy is closely related to the size of the equation of the first anomaly. From these circumstances alone we could see that we must suppose the moon's epicycle to be carried on an eccentric circle, being farthest from the earth at conjunction and opposition, and nearest to the earth at both quadratures. This will come about if we modify the first hypothesis along somewhat the following lines.

Imagine the circle (in the inclined plane of the moon) concentric with the ecliptic moving in advance, as before [p. 191], (to represent the [motion in] latitude) about the poles of the ecliptic with a speed equal to the increment of the motion in latitude over the motion in longitude. Imagine, again, the moon traversing the so-called epicycle (moving in advance on its apogee arc) with a speed corresponding to the return of the first anomaly. Now, in this inclined plane, we suppose two motions to take place, in opposite directions, both uniform with respect to the centre of the ecliptic: one of these carries the centre of the epicycle towards the rear through the signs with the speed of the motion in latitude, while the other carries the centre and apogee of the eccentric, which we assume located in the same [inclined] plane, (the centre of the epicycle will at all times be located on this eccentric), in advance through [i.e. in the reverse order of] the signs) by an amount corresponding to the difference between the motion in latitude and the double elongation (the elongation being the amount by which the moon's mean motion in longitude exceeds the sun's mean motion).

Thus, to give an example, in one day the centre of the epicycle traverses about 13;14° in motion of latitude towards the rear through the signs, but appears to have traversed 13;11° in longitude on the ecliptic, since the whole inclined circle [of the moon] traverses the difference of 0;3° in the opposite direction, [i.e.] in advance; [meanwhile] the apogee of the eccentric, in turn, travels 11;9°.

\footnote{On chs. 2-4 see H:4.M.1 84-8, Pedersen 184-9.}
in the opposite direction, (again in advance): this is the amount by which the double elongation, $24;23^\circ$, exceeds the motion in latitude, $13;14^\circ$. The combination of both of these motions, which take place in opposite directions, as we said, about the centre of the ecliptic, will produce the result that the radius carrying the centre of the epicycle and the radius carrying the centre of the eccentre will be separated by an arc which is the sum of $13;14^\circ$ and $11;9^\circ$, and is twice the amount of the elongation (which is approximately $12;11\frac{1}{2}^\circ$). Hence the epicycle will traverse the eccentre twice during a mean [synodic] month. We assume that it returns to the apogee of the eccentre at mean conjunction and opposition.

In order to illustrate the details of the hypothesis, imagine [Fig. 5.1] the circle in the moon's inclined plane concentric with the ecliptic as $ABGD$ on centre $E$ and diameter $AEG$. Let the apogee of the eccentre, the centre of the epicycle, the northern limit, the beginning of Aries and the mean sun [all] be located at point $A$ at the same moment. Then I say that in the course of one day the whole [inclined] plane moves in advance from $A$ towards $D$ about centre $E$, by about $3'$; thus the northern limit (which is [still represented by] $A$) reaches $\xi 29;57^\circ$. The two opposite motions are carried out by the radius corresponding to $EA$ [moving] uniformly about $E$, the centre of the ecliptic. Thus I say that in the course of one day the radius through the centre of the eccentre corresponding to $EA$ rotates uniformly in advance [i.e. in the reverse order] of the signs to the position $ED$, carrying the apogee of the eccentre to $D$, and making arc $AD$

\[\text{H358}\]

\[\text{H358.20-21. This would mean 'and describing eccentre DH about centre Z'. This is nonsense: EA does not 'describe the eccentre' (since it is not a radius of the eccentre), but merely marks the position of the apogee of the eccentre. If Ptolemy wanted to refer to the eccentre here, he would presumably have written (as}\]
In the same time the radius through the centre of the epicycle [corresponding to EA] rotates uniformly, again about E, towards the rear through the signs to the position EB, carrying the centre of the epicycle to H, and making arc AB 13;14°. Thus the apparent distance of H, the centre of the epicycle, is 13;14° (in motion of latitude) from the northern limit A, 13;11° (in longitude) from the beginning of Aries (for the northern limit A has moved to X 29;57° in the same time), and 24;23° (the sum of arc AD and arc AB, and twice the mean daily elongation) from the apogee of the eccentre D. Since, in this way, the motion through B and the motion through D meet each other once in half a mean [synodic] month, it is obvious that these motions will always be diametrically opposite at intervals of a quarter and three-quarters of that period, i.e. at the mean quadratures. At those times the centre of the epicycle, located on EB, will be diametrically opposite the apogee of the eccentre, located on ED, and [thus] will be at the perigee of the eccentre.

It is also clear that under these circumstances the eccentre itself (that is, the fact that the arc DB is not similar to arc DH) will not produce any correction to the mean motion. For the uniform motion of the line EB is counted, not along arc DH of the eccentre, but along arc DB of the ecliptic, since it rotates, not about the centre of the eccentre Z, but about E. The only [correction] which will result is that due to the difference in the effect of the epicycle: as the epicycle moves towards the perigee it produces a continuous increase in the equation of anomaly (subtractive and additive alike), since the angle formed by the epicycle at the observer’s eye is greater at positions [of the epicycle] nearer the perigee. On the other hand, there will, in general, be no difference from the first hypothesis when the centre of the epicycle is at the apogee A, which is the situation at the mean quadratures and oppositions.

For if [Fig. 5.2] we draw epicycle MN about point A, AE:AM is the same ratio as that which we demonstrated from the eclipses. The greatest difference will be when the epicycle reaches H, the perigee of the eccentre (as XO here). This occurs at the mean quadratures. For the ratio XH:HE is greater than that at any other position, since XH, the radius of the epicycle, is always a constant length, while EH is the shortest of all lines drawn from the centre of the earth to the eccentre.

3. {On the size of the anomaly of the moon which is related to the sun}

In order to see what the maximum equation of anomaly is when the epicycle is at the perigee of the eccentre, we sought observations of the distance of the moon from the sun under the following conditions:

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*The figure given by Heiberg (p. 360), which is taken from the ms. tradition represented by A, is wrong in making E the centre of the circle and adding a point K above it. My figure agrees with the text and with part of the Arabic tradition (e.g. P), except that all Arabic mss. have the equivalent of Θ for O. Manitius already made the same correction, except that he unnecessarily added the point Z (unattested in the mss.) as the centre of the circle.
V 3. Determination of size of second lunar anomaly

The moon's speed was about at the mean (for that is when the equation of anomaly is maximum).

The mean elongation of the moon from the sun was about a quadrant (for then the epicycle was near the perigee of the eccentric).

In addition to the above, the moon had no longitudinal parallax.

If these conditions are fulfilled, the apparent observed longitudinal distance is the same as the true, and thus we can safely infer the size of the second anomaly which we are seeking. When we investigate on the basis of the above kind of observations, we find that, when the epicycle is closest to the earth, the greatest equation of anomaly is about 7° with respect to the mean position (or 2° different from [the corresponding equation of] the first anomaly).

We will illustrate the way in which this kind of determination is made from one or two observations by way of example. We sighted sun and moon in the 2nd year of Antoninus, Phamenoth [VII] 25 in the Egyptian calendar [139, Feb. 9], after sunrise, and 2½ equinoctial hours before noon. The sun was sighted in = 18½°, and ♀ 4 was culminating. The apparent position of the moon was n. 9½°, and that was its true position too, since when it is near the beginning of Scorpius, about 1½ hours to the west of the meridian at Alexandria, it has no noticeable parallax in longitude. ¹⁰ Now the time from epoch in the first year of

¹⁰ i.e. at that situation the angle between ecliptic and altitude circle (derived from Table II 13) is about 90°, hence the parallax affects only the latitude, not the longitude. Interpolation in the tables
Nabonassar to the observation is

H363 885 Egyptian years 203 days 181/3 equinoctial hours (whether reckoned simply or accurately).

For this moment we find:

- mean position of the sun: $\approx 16;27^\circ$
- true position of the sun: $\approx 18;50^\circ$ (in accordance with its sighted position according to the astrolabe).\(^1\)

From the first hypothesis we find the mean position of the moon at that moment as $m = 17;20^\circ$ (thus its mean elongation from the sun was about a quadrant), and the moon's distance in anomaly from the apogee of the epicycle as $87;19^\circ$ (which is near the position of maximum equation). Thus the true position of the moon was less than the mean by 7° (instead of the 5° of the first anomaly).\(^2\)

Again, to display the amount of the equation under similar conditions which is derived from Hipparchus' observations of such positions, we will adduce one of these. He says that he made the observation in the fifty-first year\(^3\) of the Third Kallippic Cycle, Epiphi [XI] 16 in the Egyptian calendar [-127 Aug. 5], when of the first hour had passed. 'The speed was [that of day] 241',\(^4\) he says, 'and while the sun was sighted in Leo $\theta_2^\circ$ the apparent position of the moon was Taurus $12^\circ$, and its true position was approximately the same'. So the true observed distance between moon and sun was 86;15°. But when the sun is near the beginning of Leo, at Rhodes (where the observation was made), 1 hour of the day is $17\frac{1}{2}$ time-degrees. So the $5\frac{1}{2}$ seasonal hours (which make up the interval to [the following] noon) produce 61/3 equinoctial hours. Therefore the

\(^{1}\) Is this meant as a confirmation of the accuracy of the observation? This would imply that Ptolemy set up the instrument by using the shadow (cf. p. 219 n.4). It may, however, merely mean that this computation is the basis of the position to which Ptolemy set the instrument.

\(^{2}\) Precise computation: mean elongation $= 16;27^\circ - m = 17;20^\circ = 89;7^\circ$; equation $= m = 9;40^\circ - m = 17;20^\circ = -7;40^\circ$; equation from first hypothesis (from Table V 10/1. a: 87;19°) $= -4;57^\circ$. However, Ptolemy is operating with rounded numbers, quite properly here.

\(^{3}\) I have, doubtfully, accepted the emendation $\nu\psi$ for $\nu\nu\psi$ (fiftieth) at H363.16. The Julian date of the observation, -127 Aug. 5, is guaranteed both by the astronomical data and by Ptolemy's reckoning in the era Nabonassar. Ideler 'Historische Untersuchungen 217-18' made the emendation because he calculated, correctly, from the known epoch of the Kallippic cycles that this must fall in the fifty-first year. In this case (cf. p. 214 n.72) using the Egyptian calendar makes no difference. However, I suspect that the error, if it is one, lies not with the scribes but with Ptolemy or even Hipparchus, and that possibly there is no error, but another method of counting which eludes us.

\(^{4}\) Literally 'The true daily motion $\delta\rho\omicron\omicron\omicron\omicron\omicron$ was the 241st'. Hipparchus is referring to a table of the true motion of the moon over 248 days (9 anomalistic months), in which the moon was supposed to return to the same velocity. Such a table is extant on a cuneiform tablet. ACT no. 190 (III p. 131). If Hipparchus was using that table the motion on day 241 would be 13;30° or 13;31,10° according to whether one starts at the beginning or goes in reverse from the end, i.e. close to the mean, as our passage requires. The historical interest of this passage has been missed because '241' has hitherto been interpreted as 'degrees of anomaly' (and hence 'emended', to '259' by Manitius and to $\mu\omicron\omicron\omicron\omicron\omicron\omicron$ 'mean', by Halma). I think it likely that Hipparchus was the channel through which use of the 248-day lunar anomaly period was transmitted from Mesopotamia to the Greek world (e.g. Vettius Valens I 4-5, ed. Kroll 20-1, and P. Ryl. 27, on which see H.11.1.1808 II), and ultimately to India (the Vākya system, see H.11.1.1817 II). See provisionally Toomer (11) p. 108 n.12.
observation occurred $6\frac{2}{3}$ equinoctial hours before noon on the sixteenth, while $9^\circ$ was culminating. Thus in this case the time from epoch to the observation is

$$17\frac{2}{3} \text{ equinoctial hours reckoned simply}$$

and

$$17\frac{2}{3} \text{ equinoctial hours reckoned accurately.}$$

For this moment we find from our hypotheses (since the meridian through Rhodes is the same as that through Alexandria):$^{16}$

- mean position of the sun: $\Omega 10;27^\circ$
- true position of the sun: $\Omega 8;20^\circ$
- mean position of the moon in longitude: $8 4;25^\circ$

(Thus the mean elongation was again nearly a quadrant)

mean distance of the moon from the apogee of the epicycle in anomaly: $257;47^\circ$

(which is again near the position of the maximum equation of the anomaly due to the epicycle).

So the distance from the mean moon to the true sun is calculated as $93;55^\circ$. And the observed distance from the true moon to the true sun was $86;15^\circ$.$^{17}$ Therefore the true position of the moon was greater than the mean, again by $7\frac{2}{3}^\circ$, instead of the $5^\circ$ of the first hypothesis. And it is [further] evident, that of these two observations taken near the second quadrature, ours was found to be less than the position computed from the first anomaly by $2^\circ$, while Hipparchus' was greater by the same amount, since the total equation of anomaly was subtractive at our observation and additive at Hipparchus'.

From numerous other similar observations also we find that the greatest equation of anomaly is about $7\frac{2}{3}^\circ$ when the epicycle is at the perigee of the eccentre.

4. \textit{(On the ratio of the eccentricity of the moon's circle)}

With this as a datum, let [Fig. 5.3] the moon's eccentric circle be $ABG$ on centre $D$ and diameter $ADG$, on which $E$ is taken as the centre of the ecliptic. Thus $A$ is the apogee of the eccentre and $G$ the perigee. On centre $G$ draw the moon's epicycle $ZH$, draw $E\Theta B$ tangent to it, and join $G\Theta$.

Then since the greatest equation of anomaly occurs when the moon is at the epicycle tangent, and we have shown that this amounts to $7\frac{2}{3}^\circ$, the angle at the centre of the ecliptic.

$$\angle GE\Theta = \begin{cases} 7;40^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\ 15;20^\circ \text{ where } 2 \text{ right angles } = 360^\circ. \end{cases}$$

$^{15}$ As Neugebauer remarks, the equation of time for a solar longitude of $\Omega 8^\circ$ should be $-16$ mins. rather than $-3$ mins. For this and other inaccuracies in Ptolemy's computations see $H.A.M.A$ 92-3.

$^{16}$ In fact Rhodes is about $1.7^\circ$ west of Alexandria. The notion that they lay on the same meridian was traditional: see Strabo 2.5.7, where the same meridian is supposed to pass through Meroe, Syene, Alexandria, Rhodes, the Troad, Byzantium and the Borysthenes. This is probably derived from Eratosthenes via Hipparchus.

$^{17}$ Note that Ptolemy takes only the distance observed by Hipparchus ($86;15^\circ$) as accurate, and substitutes his own calculations of the positions of sun and moon for those observed (or calculated) by Hipparchus.
V 4. Ratio of eccentricity of moon’s eccentre

Therefore in the circle about right-angled triangle GE\(\Theta\)

\[
\text{arc } G\Theta = 15;20^\circ
\]

and the corresponding chord

\[
G\Theta \approx 16^\circ \text{ where the hypotenuse } GE = 120^\circ.
\]

So, where \(G\Theta\), the radius of the epicycle, is, as was shown, 5;15°

and EA, the distance from the centre of the ecliptic to the apogee of the eccentre, is 60°,

EG, the distance from the centre of the ecliptic to the perigee of the eccentre, is 39;22°.

Therefore, by addition, diameter AG = 99;22°,

and DA, the radius of the eccentre = 49;41°

and ED, the distance between the centres of the ecliptic and the eccentre = 10;19°.

Thus we have demonstrated the ratio of the eccentricity.

As far as concerns the phenomena at syzygies and at quadrature positions of the moon, the preceding discussion would provide a full explanation of the hypotheses elucidating the circles of the moon described above. But from individual observations taken at distances of the moon [from the sun] when it is sickle-shaped or gibbous (which occur when the epicycle is between the apogee

\[18\text{See } H\text{AMA} 88-91, \text{Pedersen 189-95.}\]
and the perigee of the eccentre), we find that the moon has a peculiar characteristic associated with the direction\(^{19}\) in which the epicycle points. Every epicycle must, in general, possess a single, unchanging point defining the position of return of revolution on that epicycle. We call this point the 'mean apogee', and establish it as the beginning from which we count motion on the epicycle. Thus point \(Z\) on the previous figure [5.3] is such a point. It is defined, for the position of the epicycle at apogee or perigee of its eccentre, by the straight line drawn through all the centres [of ecliptic, eccentre and epicycle] (DEG here). Now in all other hypotheses [involving epicycle on eccentre], we see absolutely nothing in the phenomena which would count against the following [model]: in other positions of the epicycle [outside apogee and perigee of the eccentre], the diameter of the epicycle through the above apogee, i.e. \(ZGH\), always keeps the same position relative to the straight line which carries the epicycle centre round with uniform motion (here \(EG\)), and [thus] (as one would think appropriate) always points towards the centre of revolution, at which, furthermore, equal angles of uniform motion are traversed in equal times. In the case of the moon, however, the phenomena do not allow one to suppose that, for positions of the epicycle between \(A\) and \(G\), diameter \(ZH\) points towards \(E\), the centre of revolution, and keeps the same position relative to \(EG\). We do indeed find that the direction in which [diameter \(ZH\)] points is a single, unchanging point on diameter \(AG\), but that point is neither \(E\), the centre of the ecliptic, nor \(D\), the centre of the eccentre, but a point removed from \(E\) towards the perigee of the eccentre by an amount equal to \(DE\). We shall show that this is so, again, by setting out from among the numerous [relevant] observations, two which are particularly suitable for illustrating our point, since the epicycle at these observations was at distances halfway [between apogee and perigee of the eccentric], and the moon was near apogee or perigee of the epicycle; for in these situations occur the greatest effects of the above direction [of the epicycle diameter].

Now Hipparchus records that he observed the sun and the moon with his instruments\(^{20}\) in Rhodes in the 197th year from the death of Alexander, Phormouthi [VIII] 11 in the Egyptian calendar [-126 May 2], at the beginning of the second hour. He says that while the sun was sighted in \(\psi\) 72°, the apparent position of the centre of the moon was \(\Psi 21\frac{1}{2}^\circ\), and its true position was \(\Psi 21\frac{1}{2}^\circ + \frac{1}{2}^\circ [21;27\frac{1}{2}^\circ]\).\(^{21}\) Therefore at the moment in question the distance of the true moon from the true sun was about 313°, [counting] towards the rear. Now the observation was made at the beginning of the second hour, about 5 seasonal hours (which correspond to about 5½ equinoctial hours in Rhodes on

\(^{19}\)\(\pi\rho\sigma\sigma\nu\epsilon\upsilon\circ\varsigma\), used by Neugebauer and Pedersen as a technical term ('prosneusis') for this element of Ptolemy's lunar theory. However, it is hardly that for Ptolemy, as he applies the word in many other contexts (see p. 43 n.38).

\(^{20}\) It is usually assumed that by this is meant an armillary sphere similar to that described by Ptolemy in \(V\) 1 (and often, that Hipparchus was the inventor of that instrument). That may be true, but the vague expression here certainly does not require it, and whether the data described below do is doubtful. I consider it possible that Hipparchus used a dioptra of the type described by Heron ('Dioptra', ed. Schöne, 187 ff.).

\(^{21}\) On the correction for parallaxis made by Hipparchus here (which is fairly accurate) see \textit{HAMA} 92.
that date) before noon on the 11th. So the time from our epoch to the observation is

$$620 \text{ Egyptian years } 219 \text{ days} \begin{cases} 18 \frac{1}{2} \text{ equinoctial hours reckoned simply} \\ 18 \text{ equinoctial hours reckoned accurately.} \end{cases}$$

For this moment we find:

- mean sun in $\gamma$ 6;41°
- true sun in $\delta$ 7;45°
- mean moon in $\kappa$ 22;13° in longitude
- at 185;30° from mean apogee of epicycle in anomaly.

Therefore the distance of the mean moon from the true sun was 314;28°.

With these data, let [Fig. 5.4] the moon's eccentric circle be ABG on centre D and diameter ADG, on which E represents the centre of the ecliptic. On centre B draw the moon's epicycle, ZH0. Let the sense of motion of the epicycle be towards the rear from B towards A, and the sense of motion of the moon on the epicycle be from Z towards H and [then] $\Theta$. Join DB and E$\Theta$BZ.

Now in a mean [synodic] month occur two revolutions of the epicycle on the eccentric, and in the situation in question the elongation of mean moon from mean sun was 315;32°. So if we double the latter and subtract [the 360° of] a circle, we will get the elongation at that moment of the epicycle from the apogee of the eccentric, [counting] towards the rear: this is 271;4°.

$$\therefore \angle AEB = 88;56° \text{ (remainder [when 271;4° is subtracted] from 360°).}$$

So drop perpendicular DK from D on to EB.

$$\therefore \angle DEB = \begin{cases} 88;56° \text{ where 4 right angles } = 360° \\ 177;52° \text{ where 2 right angles } = 360°. \end{cases}$$
Therefore in the circle about right-angled triangle DEK,
\[ \text{arc } DK = 177;52\text{°} \]
and \[ \text{arc } EK = 2;8\text{°} \text{ (supplement)}. \]

Therefore the corresponding chords
\[ \begin{align*}
DK &= 119;59\text{°} \\
EK &= 2;14\text{°} 
\end{align*} \]
where hypotenuse \( DE = 120\text{°} \).

Therefore where \( DE \), the distance between the centres, is \( 10;19\text{°} \)
and \( DB \), the radius of the eccentric, is \( 49;41\text{°} \),
\[ \begin{align*}
DK &= 10;19\text{°} \text{ also}, \\
and \ EK &= 0;12\text{°} \\
\text{But } BK^2 &= DB^2 - DK^2. \\
\therefore BK &= 48;36\text{°} \text{ in the same units,}
\end{align*} \]
and, by addition, \( BE = BK + EK \) = \( 48;48\text{°} \).

Again, since the distance of the mean moon from the true sun was found to be \( 314;28\text{°} \), and the distance of the true moon [from the true sun] was observed
to be \( 313;42\text{°} \), the equation of anomaly is \( -0;46\text{°} \). Now the mean position
of the moon is seen along the line \( EB \). So let the moon be located at \( H \) (since
it is near the perigee), join \( EH \) and \( BH \), and drop perpendicular \( BL \) from \( B 
\) on to \( EH \) produced. Then, since \( \angle BEL \) contains the moon's equation of
anomaly,
\[ \begin{align*}
\angle BEL &= \begin{cases} 
0;46\text{° where 4 right angles} = 360\text{°} \\
1;32\text{°° where 2 right angles} = 360\text{°°}
\end{cases}
\end{align*} \]

Therefore in the circle about right-angled triangle \( EBL \),
\[ \text{arc } BL = 1;32\text{°} \]
and the corresponding chord
\[ \begin{align*}
BL &= 1;36\text{° where the hypotenuse } EB = 120\text{°}. \\
\end{align*} \]

Therefore where \( BE = 48;48\text{°} \) and \( BH \), the radius of the epicycle, is \( 5;15\text{°} \),
\[ \begin{align*}
BL &= 0;39\text{°}. \\
\end{align*} \]

Therefore where \( BH \), the radius of the epicycle, is \( 120\text{°} \),
\[ \begin{align*}
BL &= 14;52\text{°} \\
\end{align*} \]
and, in the circle about right-angled triangle \( BHL \),
\[ \text{arc } BL = 14;14\text{°} \]
\[ \therefore \angle BHL = 14;14\text{°° where 2 right angles} = 360\text{°°}, \]
\[ \text{H373} \]
and, by subtraction \[ \angle EBH = \begin{cases} 
12;42\text{°° where 2 right angles} = 360\text{°°} \\
6;21\text{° where 4 right angles} = 360\text{°}. \\
\end{cases} \]

That \( [6;21\text{°}] \), then, is the size of arc \( H\Theta \) of the epicycle, which comprises the
distance from the moon to the true perigee [of the epicycle].

But since the distance of the moon from the mean apogee at the time of the
observation was \( 185;30\text{°} \) [p. 228], it is clear that the mean perigee is in advance
of the moon, i.e. of point \( H \). Let [the mean perigee] be point \( M \), draw line \( BMN, \)
and drop perpendicular \( EX \) on to it from point \( E \).

Then since, as was shown,
\[ \text{arc } \Theta H = 6;21\text{°}, \]
and arc \( HM \), the distance from the perigee, is given as \( 5;30\text{°} \),
by addition, arc \( \Theta M = 11;51\text{°}. \)

So \( \angle EBX = \begin{cases} 
11;51\text{° where 4 right angles} = 360\text{°} \\
23;42\text{°° where 2 right angles} = 360\text{°°}. \\
\end{cases} \)
Therefore in the circle about right-angled triangle BEX, 
\[ \text{arc EX} = 23;42^\circ \]
and \[ \text{EX} = 24;39^\circ \] where hypotenuse BE = 120°.
Therefore where BE = 48;48° 
\[ \text{EX} = 10;2^\circ. \]

Again, since \{p. 228\}
\[ \angle AEB = 177;52^\circ \]
and \[ \angle EBN = 23;42^\circ \]
where 2 right angles = 360°,
by subtraction, \[ \angle ENB = 154;10^\circ. \]
Therefore in the circle about right-angled triangle ENX, 
\[ \text{arc EX} = 154;10^\circ \]
and \[ \text{EX} = 116;58^\circ \] where hypotenuse EN = 120°.
Therefore where \[ \text{EX} = 10;2^\circ \] and DE, the distance between the centres, is 10;19°,
\[ \text{EN} = 10;18^\circ. \]
Therefore the [radius of the epicycle] through the mean perigee, BM, points in a direction such that, when produced to N, it cuts off a line EN which is very nearly equal to DE.

Similarly, in order to show that we get the same result at the opposite sides of the eccentrate and epicycle, we have again selected from the distances [between sun and moon] observed by Hipparchus, as already mentioned, in Rhodes, the observation he made in the same year [as the preceding one], being the 197th year from the death of Alexander, Payni [X] 17 in the Egyptian calendar [-126 July 7], at 9½ hours. He says that while the sun was sighted at \[ \odot 10\frac{9}{10}^\circ \] the apparent position of the moon was \[ \odot 29^\circ. \] And this was its true position too; for at Rhodes, near the end of Leo, about one hour past the meridian, the moon has no longitudinal parallax.\footnote{22} Therefore the distance of the true moon from the true sun at the time in question was 48;6° towards the rear. Now since the observation was 3½ seasonal hours after noon on the 17th of Payni, which correspond to about 4 equinoctial hours in Rhodes on that date, the time from our epoch to the observation is

620 Egyptian years 286 days \[ \begin{align*}
4 \text{ equinoctial hours reckoned simply} \\
3\frac{1}{2} \text{ equinoctial hours reckoned accurately}.
\end{align*} \]

For this moment we find:
\begin{align*}
\text{mean sun at} & \odot 12;5^\circ \\
\text{true sun at} & \odot 10;40^\circ \\
\text{mean moon at} & \odot 27;20^\circ \text{ in longitude}
\end{align*}
(thus the distance of the mean moon from the true sun was 46;40°)
\begin{align*}
\text{mean moon at} & 333;12^\circ \text{ in anomaly from the apogee of the epicycle.} \footnote{23}
\end{align*}

With these data, let [Fig. 5.5] the moon's eccentric circle be ABG on centre D

\footnote{22} For verification of this see \textit{HAMA} 92.
\footnote{23} For 620°286°31° I find: \[ \lambda = 147;7^\circ, \alpha = 333;1^\circ. \] Since the differences from Ptolemy's positions represent the lunar motion over about 20 mins., it is obvious that he has carelessly calculated the positions for 4 hours after noon, i.e. without making the correction for the equation of time, which he had given, correctly, as about 20 mins. This error has a not inconsiderable effect on the final result, which would not agree nearly so neatly if the computation were carried out with the above figures.
and diameter ADG, on which the centre of the ecliptic is represented by point E. About point B draw the moon’s epicycle, ZHΘ, and join DB, EΘBZ.

Then since twice the mean elongation of sun and moon is 90;30°, from the theory already established

\[ \angle AEB = \begin{cases} 
90;30° \text{ where 4 right angles = } 360° \\
181° \text{ where 2 right angles = } 360°.
\end{cases} \]

So if we produce BE and drop perpendicular DK on to it from D,

\[ \angle DEK = 179° \text{ (supplement).} \]

Therefore in the circle about right-angled triangle DEK

\[ \text{arc } DK = 179° \]
and \[ \text{arc } EK = 1° \text{ (supplement).} \]

Therefore the corresponding chords

\[ \text{DK} = 119;59° \] \[ \text{and } EK = 1;39° \]

where hypotenuse DE = 120°.

Therefore where DE, the distance between the centres, is 10;19°
and BD, the radius of the eccenter, is 49;41°,

\[ \text{DK} \approx 10;19° \]
and \[ EK = 0;59°. \]

Now since \[ BK^2 = BD^2 - DK^2, \]
\[ BK = 48;36°, \]

and, by subtraction [of EK], \[ EB = 48;31°. \]

Furthermore, since the distance of mean moon from true sun was found to be 46;40°, and the distance of true moon [from true sun was observed as] 48;6°, the equation of anomaly is +1;26°. So let the position of the moon be at H (since it is near the apogee of the epicycle). Join EH, BH, and drop perpendicular BL from B on to EH.
Then since

\[ \angle BEL = \begin{cases} 1;26^\circ & \text{where 4 right angles} = 360^\circ \\ 2;52^\circ & \text{where 2 right angles} = 360^\circ \end{cases} \]

in the circle about right-angled triangle BEL,

\[ \text{arc BL} = 2;52^\circ \]

and \( BL = 2;59^\circ \) where hypotenuse \( EB = 120^\circ \).

Therefore where \( EB = 48;31^\circ \) and \( BH \), the radius of the epicycle, is \( 5;15^\circ \),

\( BL = 1;12^\circ \).

So in the circle about right-angled triangle BHL,

\[ \begin{align*}
BL &= 27;34^\circ & \text{where hypotenuse BH} &= 120^\circ, \\
\text{arc BL} &= 26;34^\circ.
\end{align*} \]

\[ \therefore \angle BHL = 26;34^\circ \]

and, by addition [of \( \angle BEL = 2;52^\circ \)],

\[ \angle ZBH = \begin{cases} 29;26^\circ & \text{where 4 right angles} = 360^\circ \\ 14;43^\circ & \text{where 4 right angles} = 360^\circ \end{cases} \]

That \( 14;43^\circ \) is the size of the arc \( HZ \) of the epicycle, which comprises the distance from the moon to the true apogee.

But since [the moon's] distance from the mean apogee at the time of the observation was \( 333;12^\circ \), if we put the mean apogee at \( M \), draw line \( MBN \), and drop perpendicular \( EX \) on to it from \( E \), then

\[ \text{arc HZM} = 26;48^\circ \] (by subtraction [of \( 333;12^\circ \) from the circle],

and, by subtraction [of arc \( HZ = 14;43^\circ \)], \( arc ZM = 12;5^\circ \).

\[ \therefore \angle MBZ = \angle EBX = \begin{cases} 12;5^\circ & \text{where 4 right angles} = 360^\circ \\ 24;10^\circ & \text{where 2 right angles} = 360^\circ \end{cases} \]

Therefore in the circle about right-angled triangle \( BEX \)

\[ \text{arc EX} = 24;10^\circ \]

and \( EX = 25;7^\circ \) where hypotenuse \( BE = 120^\circ \).

Therefore where \( BE = 48;31^\circ \) and \( DE \), the line between the centres, is \( 10;19^\circ \).

\( EX = 10;8^\circ \).

Again, since \( \angle AEB \) is given as \( 181^\circ \) where 2 right angles = \( 360^\circ \),

and we have shown that \( \angle EBN = 24;10^\circ \),

by subtraction, \( \angle ENB = 156;50^\circ \) in the same units,

and, in the circle about right-angled triangle \( ENX \),

\[ \text{arc EX} = 156;50^\circ \]

and \( EX = 117;33^\circ \) where hypotenuse \( EN = 120^\circ \).

Therefore where \( EX = 10;8^\circ \) and \( DE \), the line between the centres, is \( 10;19^\circ \).

\( EN = 10;20^\circ \).

So from this calculation too it turns out that \( MB \), [the radius of the epicycle] through \( M \), the mean apogee, points in a direction such that, when produced to \( N \), it cuts off a line \( EN \) approximately equal to \( DE \), the distance between the centres.

We also find that approximately the same ratio results by calculation from a number of other observations. Thus these observations confirm the peculiar characteristic of the direction of the epicycle in the hypothesis of the moon: the

\[ 24 \times 120/5:15 = 27:25.43 \] Ptolemy was obviously operating, not with the value 1:12, but with 1:12.22 (which leads to 27:34:5), which is in fact what one finds from the immediately preceding calculation. 2.59 \times 48:31:120.
6. \textit{How the true position of the moon can be calculated geometrically from the periodic motions}\textsuperscript{25}

Now that we have demonstrated the above, the appropriate sequel is to show how, for a particular position of the moon, given the amounts of the various mean motions, we can find from the amount of the elongation and of the moon's motion in anomaly on the epicycle the amount due to the equation of anomaly which should be added to or subtracted from the mean motion in longitude. If one uses strictly geometrical methods, the way to solve such a problem is via theorems similar to those already set out.

Let us use the last of the above figures [5.5] as an example, and take as a basis of calculation the same periodic motions in elongation and anomaly, namely double elongation: 90\textdegree30\textprimeminutes

anomaly counted from the mean epicyclic apogee: 333\textdegree12\textprimeminutes.

[See Fig. 5.6.] We drop perpendicular NX (instead of EX) and perpendicular HL (instead of BL). Then, by the same computation as before [p. 231], since we are given

[1] The angles at centre E;
[2] hypotenuse DE and hypotenuse EN (which are equal),
\[DK = NX \approx 10\textdegree19\textprimeminutes\]

\textsuperscript{25} See \textit{H.A.M.A} 93, Pedersen 194-5.
where DB, the radius of the eccentre = 49;41°
and BH, the radius of the epicycle = 5;15°
and EK = EX = 0;5°.
Hence, as shown before [p. 231] BK = 48;36°
and similarly, [by subtraction of EK] BE = 48;31°
So, since $BX^2 + BX^2 = BN^2$,
$BN = 49;31°$ where $NX = 10;19°$.
Therefore, in the circle about right-angled triangle BNX,
where hypotenuse $BN = 120°$
$NX = 25°$,
and arc $NX = 24;3°$
$\therefore \angle NBX = \angle ZBM = \begin{cases} 24;3° & \text{where 2 right angles = 360°} \\ 12;1° & \text{(approximately) where 4 right angles = 360°} \end{cases}$
That $[12;1°]$ is the size of the arc $ZM$ of the epicycle.
But since the distance of point H, representing the moon, from M, the mean
apogee, is one revolution minus [the mean anomaly of 333;12°], i.e. 26;48°,
by subtraction [of arc $ZM$ from arc MH], arc $HZ = 14;47°$.
$\therefore \angle HBZ = \begin{cases} 14;47° & \text{where 4 right angles = 360°} \\ 29;34° & \text{where 2 right angles = 360°} \end{cases}$
and, in the circle about right-angled triangle HBL,
arc $HL = 29;34°$
and arc $LB = 150;26°$ (supplement).
Therefore where hypotenuse $BH = 120°$, the corresponding chords
arc $HL = 30;37°$ and $LB = 116;2°$.
Therefore where $BH$, the radius of the epicycle, is 5;15°
and (as was shown) $BE = 48;31°$,
arc $HL = 1;20°$ and $LB = 5;5°$.
$H383$
Therefore, by addition, $EBL = 53;36°$ where $LH = 1;20°$.
And since $EL^2 + LH^2 = EH^2$
$EH \approx 53;37°$ in the same units.
Therefore in the circle about right-angled triangle EHL,
where hypotenuse $EH = 120°$,
arc $HL = 2;59°$
and arc $HL = 2;52°$.
Therefore the equation of anomaly,
$\angle HEL = \begin{cases} 2;52° & \text{where 2 right angles = 360°} \\ 1;26° & \text{where 4 right angles = 360°} \end{cases}$
Q.E.D.

7. \textit{(Construction of a table for the complete lunar anomaly)}^{26}

In order again to provide a ready means of computing the individual additive
or subtractive equations by setting out a table, we have supplemented the table

\[^{26}\text{See } HAMA \text{ 93–5, Pedersen 195–202.}\]
for the simple hypothesis set out above [IV 10] with columns which enable one
to correct easily for the second lunar anomaly. For this purpose we again used
the same geometrical methods [as explained above]. After the first two columns
containing the argument, we inserted a third column containing the equation
to be added to or subtracted from the anomaly in order to reduce the mean
motion counted from M [in Fig. 5.6], the mean apogee, to Z, the true apogee.
[E.g.] above [p. 234], for the elongation of 90°30', we showed that arc ZM is
12;1°, and thus, since the distance of the moon from M, the mean apogee, was
333;12°, we find that its distance from Z, the true apogee, was, obviously,
345;13°, which we must use as argument for the epicyclic equation correcting
the mean motion in longitude. In the same way, for other elongations, taken at
intervals appropriate [for the table], we calculated the corresponding amount
of the equation in question. We did this by the same method [as above], (to cut a
long story short), and entered the amount corresponding to each [tabulated]
argument in the third column. Of the succeeding columns, the fourth will
contain the equations of the epicyclic anomaly (already set out in the previous
table [IV 10]), where the maximum equation reaches approximately 5;1°,
corresponding to the ratio 60 : 5;15. The fifth column will contain the
increments in the equations due to the second anomaly as compared with the
first, in the situation where the maximum equation is 71°, corresponding to the
ratio 60 : 8.27 Thus the fourth column is for the situation of the epicycle at
the apogee of the eccentre (which occurs at the syzygies), and the fifth column is
for the increments [to the equations] accruing from [the position of the epicycle]28
at the perigee of the eccentre (which occurs at the quadratures).

In order to enable one to find the proportion of these tabulated increments [in
the fifth column] corresponding to a position of the epicycle in between those
two locations [at apogee and perigee of the eccentre], we have added a sixth
column. This contains, for each tabulated argument of elongation, the
corresponding fraction (given in sixtieths) of the tabulated increment which
must be added to the equation of anomaly tabulated in the fourth column. We
have calculated these fractions in the following manner.

[See Fig. 5.7.] Let the moon's eccentre again be ABG on centre D and
diameter ADG, on which E is taken as the centre of the ecliptic. Mark off arc
AB, draw the epicycle, ZHΘK, on centre B, and draw line EBZ. Let the
elongation be given, e.g., as 60°.

Hence by the same argument as before

$$\measuredangle AEB = \text{double the given elongation} = 120°.$$  

Drop perpendicular DL from D on to BE produced, and draw HBKD. Suppose
that the line from centre E to the moon, EMN, is tangent to the epicycle,

27 The ratio is 39;22 (the distance from the earth to the perigee of the moon's eccentre, p. 226) to
5;15 (the radius of the moon's epicycle). This is approximately equal to 60 : 8.

28 Excising άνωμαλίας at H385,7. Heiberg's text would have to mean 'accruing from the anomaly
which is produced at the perigee of the eccentre, at the quadratures'. Besides being an exceedingly
clumsy expression, this ruins the parallelism of the sentence. It is obvious that Ptolemy intended to
contrast the two different positions of the epicycle, at apogee and perigee of the eccentre (cf. ον των δύο
του τεσσαρων θέσεων, H385,8-9). τής (H385,6) refers to θέσεως (understood from above; for
άποτελέσθαι used with θέσεως cf. H394,11-12). The interpolation of άνωμαλίας is the work of
someone who looked for something for τής to refer to, but misunderstood this.
producing a maximum equation of anomaly, and join BM. Then since

\[ \angle AEB = \begin{cases} 120° & \text{where 4 right angles} = 360° \\ 240° & \text{where 2 right angles} = 360° \end{cases} \]

\[ \angle DEL = 120° \] (supplement).

Therefore in the circle about right-angled triangle DEL,

- arc DL = 120°
- arc EL = 60° (supplement).

So the corresponding chords

- EL = 60°
- DL = 103°55°

Therefore where DE = 10°19° and DB = 49°41°,

- EL = 5°10°
- DL = 8°56°.

And, since BL² = BD² - DL²,

- BEL = 48°53°.

and, by subtraction of EL, EB = 43°43°.

where MB, the radius of the epicycle, is 5°15°.

Therefore in the circle about right-angled triangle BEM,

- where hypotenuse EB = 120°,
- BM = 14°25°
- and arc BM = 13°48°.

Therefore the maximum equation of anomaly,

\[ \angle BEM = \begin{cases} 13°48° & \text{where 2 right angles} = 360° \\ 6°54° & \text{where 4 right angles} = 360° \end{cases} \]

Thus, at this distance in elongation, the equation of anomaly differed from the 5°1° (of maximum equation) at the apogee (of the eccentric) by 1°53°. But the total difference (between maximum equation at apogee and) at perigee (of the
eccentre] is 2;39°. So, where the total difference is 60, \(1,53°\) will be \(42;38\). This is the amount which we will put in the sixth column corresponding to \(120°\) of [double] elongation.

In exactly the same way we computed, for the other tabulated arguments, the fractions of the difference between the two maximum equations of anomaly, obtained in the above manner, and entered them, expressed in sixtieths of that difference, opposite the corresponding argument. It is obvious that the total 60 [sixtieths] correspond to the double of \(90°\) of elongation, which is at \(180°\) of the eccentric, the location of the perigee.

We also added a seventh column containing the position of the moon in latitude, on either side of the ecliptic, as measured along a circle through the poles of the ecliptic, i.e. the arc of the latter circle cut off between the ecliptic and the inclined circle of the moon on the same centre [as the ecliptic], for each [tabulated] position of the moon on its inclined circle. For this we have used the same procedure as we did to calculate the arcs of the circle through the poles of the equator [which are cut off] between the equator and the ecliptic [I 14]. Here, however, we took the arc between the ecliptic and the northern or southern limit of the inclined circle, as measured along the great circle through both their poles, as \(5°\). For, like Hipparchus, we find by calculation from the moon's most northerly and southerly apparent positions that its greatest deviation either side of the ecliptic is approximately that amount. Furthermore, almost all circumstances of observations of the moon, whether taken with respect to the stars, or taken with instruments, fit a maximum latitudinal deviation of that amount, as will become clear from subsequent demonstrations.

The table of the complete lunar anomaly is as follows.

8. {Table of the complete lunar anomaly}{10} [See p. 238.] {H390-1}

9. {On the complete calculation of the moon's position} {H392}

So, whenever we choose to calculate the moon's anomalistic position by means of the table set out, we take, for the moment in question at Alexandria, the mean

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{10} The only details of an observation which confirm \(t \approx 5°\) for the lunar orbit are at V 12 p. 247.

In general the entries in this table are correct to within \(\pm 1\) in the second place. However, in col. 3, arguments 123-9, 147-53 and 171-7 the error reaches \(-3\) or \(-4\), possibly because of interpolation between computed values. In col. 5 the first 9 values (from arguments 6 to 54 inclusive) are all too big, and the first 7 of them fit a ratio (radius of epicycle: distance of epicycle centre) of .136 (instead of .133 \(\approx 5;15\) : 39;22 which Ptolemy's text requires and which underlies all values from argument 60 on). This could be derived from a distance of 38;36° or an epicycle radius of 5;21°, neither of which has any motivation. I cannot explain this discrepancy, but it is too consistent to be the result of mere inaccurate calculation. In col. 6 the calculation to two sexagesimal places gives a quite illusory accuracy, and Ptolemy's results (for the second place) bear little relationship to what one gets with accurate calculation. However, this has a negligible effect on the accuracy of computations carried out with the table. In the Handy Tables Ptolemy quite properly tabulated only one place in this and the corresponding column in the planetary tables.
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motions of the moon in longitude, elongation, anomaly and latitude, in the way explained. Then we always, first, double the figure computed for the elongation, and (after subtracting 360°, if necessary), enter with this into the table of anomaly and take the corresponding amount in the third column. If the double elongation is less than 180° we add the amount [in the third column] to the mean anomaly, but if the double elongation is greater than 180° we subtract the amount from the mean anomaly. We enter with the resulting true anomaly into the same table, and take the corresponding equation in the fourth column and also the corresponding increment in the fifth column, and write [both] down separately. Next we enter with the doubled mean elongation into the same table, take the sixtieths corresponding to it in the sixth column, multiply the increment which we wrote down separately by that number of sixtieths, and always add the result to the previously computed equation from the fourth column. If the true anomaly is less than 180°, we subtract this sum from the mean longitude and mean [argument of] latitude, but add it to them if the true anomaly is greater than 180°. Thus we have [two] numbers: we add the one for the longitude to the position [of the mean moon] at epoch: the result will be the true position of the moon. With the one for the [argument of] latitude, counted from the northern limit, we enter into the same table: the number corresponding to it in the seventh column will be the distance of the moon’s centre from the ecliptic, measured along the great circle through the poles of the ecliptic. If the argument falls within the first 15 lines, it will be to the north [of the ecliptic], but if it falls below the first 15 lines, it will be to the south. The first column of argument comprises the moon’s motion from north to south, and the second column its motion from south to north.

10. {That the difference at the syzygies due to the moon’s eccentricity is negligible}\(^{32}\)

Now it is likely that some people will suspect that the moon’s eccentric circle might also have a considerable effect at conjunctions and oppositions and the eclipses occurring at them, since the centre of the epicycle does not always under all circumstances stand exactly at the apogee at those times, but can be removed from the apogee by an arc [of the eccentricity] of considerable size, because location precisely at the apogee occurs at the mean syzygies, whereas the determination of true conjunction and opposition requires taking the anomalies of both luminaries into account. Therefore we shall try to show that this difference cannot produce any considerable error in [calculation of] the phenomena at syzygies, even if the correction due to the eccentricity is not taken into account.

\(^{31}\) Ptolemy has not in fact explained how to do this, but the essence of the procedure is the same as that explained for the sun at III 8. Note here, however, that the ‘mean motions’ in elongation, anomaly and latitude must include the epoch positions, whereas, according to the procedure in the text, the ‘mean motion in longitude’ does not include the epoch position, which is added only at a later stage. For the procedure in general see HAMA 193-6, Pedersen 197-9 and, for a worked example, HAMA 96 or my Appendix A Example 9.

\(^{32}\) See HAMA 98-9.
Let [Fig. 5.8] the moon's eccentric circle be $ABG$ on centre $D$ and diameter $ADG$, on which the centre of the ecliptic is taken at point $E$, and the point of 'direction', opposite to $D$, as $Z$. Cut off arc $AB$ from the apogee $A$, and draw the epicycle, $H\Theta KL$, on centre $B$. Join $BD$, $HBK\Theta$ and $BLZ$.

Now the size of the [equation of] anomaly can differ from that of the apogee situation of the epicycle (at $A$) in two ways:

1. because the epicycle is removed towards the perigee, the epicycle subtends a larger angle at $E$;
2. the direction in which the diameter through mean apogee and perigee [of the epicycle] points is no longer towards $E$ but towards $Z$.

The effect from the first factor is a maximum when the moon's equation of anomaly is a maximum, while the effect of the second factor is a maximum when the moon is near the apogee or perigee of the epicycle. Hence it is clear that when the maximum effect of the first factor occurs, the effect of the second factor will be quite negligible, since the moon's equation of anomaly hardly varies for a considerable distance either side of its situation on the tangent to the epicycle. However, [in this situation] the true syzygy can differ from the mean by the sum of the equations of the two luminaries, if one is additive and the other subtractive. On the other hand, when the maximum effect of the second factor, the difference due to the direction, occurs, then again the effect of the first factor is negligible, since the complete equation of anomaly is either zero or very small when the moon is near the apogee or perigee of the epicycle. But [in this case] the true syzygy will differ from the mean only by the sun's equation of anomaly.

Fig. 5.8

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\(^{33}\) Fig. 5.8 is wrongly drawn in Heiberg's text, where $D\Theta$ has been connected instead of the tangent $E\Theta$. This is an error of Heiberg's, unsupported by the mss., and corrected by Manitius.

\(^{34}\) πρόσευσις. See p. 227 n.19.
Let us suppose, then, that the sun has a maximum additive equation of $2;23^\circ$, and (first) that the moon too has a maximum (but subtractive) equation of $5;1^\circ$. Thus $\angle AEB$ contains twice the sum of the above, $7;24^\circ$, i.e. $14;48^\circ$. Draw $E\Theta$ from $E$ tangent to the epicycle, and drop perpendicular $B\Theta$ on to it, and also perpendicular $DM$ from $D$ on to $BE$. Then since

$$\angle AEB = \begin{cases} 14;48^\circ & \text{where 4 right angles = 360°} \\ 29;36^\circ & \text{where 2 right angles = 360°} \end{cases}$$

in the circle about right-angled triangle $DEM$

arc $DM = 29;36^\circ$

and arc $EM = 150;24^\circ$ (supplement).

Therefore the corresponding chords

$$\begin{align*}
DM &= 30;39^\circ \\
EM &= 116;1^\circ
\end{align*}$$

and $90^\circ$ where 2 right angles = 360°.

Therefore where $DE$, the distance between the centres, is $10;19^\circ$, and $BD$, the radius of the ecnode, is $49;41^\circ$,

$$\begin{align*}
DM &= 2;38^\circ \\
EM &= 9;59^\circ
\end{align*}$$

And since $BM^2 = BD^2 - DM^2$,

$BM = 49;37^\circ$.

and, by addition of $EM$, $BME = 59;36^\circ$.

$\angle BE\Theta$ where 2 right angles = 360°,

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$$\begin{align*}
\angle BE\Theta &= \begin{cases} 10;6^\circ & \text{where 2 right angles = 360°} \\ 5;3^\circ & \text{where 4 right angles = 360°} \end{cases} \\
\angle BE\Theta &= 10;34^\circ \\
\text{arc } BE\Theta &= 10;6^\circ
\end{align*}$$

Therefore the angle of the maximum equation of anomaly,

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in the circle about right-angled triangle $BE\Theta$.

where $\text{hypotenuse } EB = 120^\circ$.

$B\Theta = 10;34^\circ$.

and arc $B\Theta = 10;6^\circ$.

$\angle AEB$ = $4;46^\circ$, $9;32^\circ$ where 4 right angles = 360°.

in the circles about right-angled triangles $EDM$ and $EZX$.

arc $DM = \text{arc } ZX = 9;32^\circ$

and arc $EM = \text{arc } EX = 170;28^\circ$ (supplements).

$\because \text{Crd arc } DM = \text{Crd arc } ZX = 9;58^\circ$

and $\text{Crd arc } ME = \text{Crd arc } EX = 119;35^\circ$

$\text{H399} = 120^\circ$.

$35$ In the time of an eclipse. See p. 136 n.16.
Therefore where $DE = EZ = 10;19''$
and $DB$, the radius of the eccentre, is $49;41''$,
$DM = ZX = 0;51''$
and $ME = EX = 10;17''$.
And since $BM^2 = BD^2 - DM^2$,
$BM \approx 49;41''$.
$BE = [BM + ME =] 59;58''$,
and, by addition [of $EX$], $BX = 70;15''$ where $ZX = 0;51''$.

Therefore by the same argument, hypotenuse $BZ$ [of triangle $BZX$] will be
approximately the same size [as $BX$], $70;15''$.
And $BZ:ZX = BL:LN$ and $BZ:BX = BL:BN$.
Therefore where $BL$, the radius of the epicycle, is $5;15''$.
and $BE$, as was shown $= 59;58''$,
$LN = 0;4''$ and $BN \approx 5;15''$.
and, by subtraction [of $BN$ from $BE$], $NE = 54;43''$ where $LN = 0;4''$.
And since, from the preceding, hypotenuse $EL$ [of triangle $ELN$] is not noticeably different from this amount of $54;43''$, it follows that, where hypotenuse $EL = 120''$,

$$LN \approx 0;8''$$
and, in the circle about right-angled triangle $ELN$,
arc $LN = 0;8''$.
Therefore the difference in the moon's position due to the direction towards $Z$,
$$\angle BEL = \begin{cases} 0;8^{\circ}\circ & \text{where 2 right angles} = 360^{\circ}\circ \\ 0;4^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

Ptolemy's final result is correct (to the nearest minute), but some of the intermediate results are inaccurate. E.g. just above in the computation of $LN$, $0;4 \times 120 = 54;43$ is much closer to $0;9$ than to
Thus here too the difference in the moon’s equation of anomaly is [only] 4
minutes of arc; and even this does not produce a significant error in the
phenomena at the syzygies, since it cannot reach as much as 1/8th of an hour, an
amount one may expect to encounter frequently as a purely observational
error.

We made the above argumentation, not to show that one cannot take these
differences into account, very small though they be, for the computation of
syzygies too, but to show that we committed no noticeable error in our previous
demonstrations using lunar eclipses when we used the [simple hypothesis], and
not that supplemented by introducing the eccentric.

11. {On the moon’s parallaxes}^37

With the above we have about disposed of the [elements] necessary for finding
the true positions of the moon. However, in the case of the moon there is the
additional problem that its apparent position does not coincide with its true
position, even to the senses. For, as we said [IV 1 p. 173], the earth does not bear
the ratio of a point to the distance of the moon’s sphere. Hence it is both
necessary and appropriate to discuss the lunar parallaxes, especially in order to
deal with the theory of solar eclipses, amongst other phenomena. By means of
the lunar parallaxes it will be possible, given a true position [of the moon], [i.e.
its position] with respect to the centre of the earth and of the ecliptic, to
determine its position as seen from the standpoint of the observer, that is from
some point on the earth’s surface, and, vice versa, to determine the true position
from the apparent position. Now it is a feature of this kind of enquiry that one
cannot find the amount of the parallax for individual situations unless one is
first given the ratio of the distance [of the body to the earth’s radius], nor can
one find the ratio of the distance without the parallax for some particular
situation being given. Hence for those bodies with no perceptible parallax,
namely, those to [the distance of] which the earth bears the ratio of a point, it is,
obviously, impossible to find the ratio of the distance. But in the case of those
bodies, like the moon, which do exhibit a parallax, the only appropriate
procedure is, first given some particular parallax, to find the ratio of the
distance. For it is possible to make an observation of a [particular] parallax of
this kind by itself, but quite impossible to determine the amount of the distance
[by itself].

Now Hipparchus used the sun as the main basis of his examination of this
problem. For since it follows from certain other characteristics of the sun and
moon (which we shall discuss subsequently) that, given the distance to one of
the luminaries, the distance to the other is also given, Hipparchus tries to
demonstrate the moon’s distance by guessing at the sun’s. First he supposes that
the sun has the least perceptible parallax, in order to find its distance, and then

0:8. It looks as if he computed to two sexagesimal fractional places, and then fudged the results
somewhat in the presentation.

^37 On chs. 11 and 12 see H.A.M.I 100-1, Pedersen 203-4.
he uses the solar eclipse which he adduces; at one time he assumes that the sun has no perceptible parallax, at another that it has a parallax big enough [to be observed]. As a result the ratio of the moon's distance came out different for him for each of the hypotheses he put forward; for it is altogether uncertain in the case of the sun, not only how great its parallax is, but even whether it has any parallax at all.  

We, in contrast, to avoid taking any uncertain factors into our examination of this topic, constructed an instrument to enable us to observe as accurately as possible the amount of the moon's parallax, and its zenith distance, along the great circle through the poles of the horizon and the moon.

We made two rods [Fig. G.1.2], rectangular [in cross-section], no less than 4 cubits long, so as to admit finer graduation, and with a cross-section of sufficient size that they were not distorted because of their length, but each side conformed very strictly to a straight line. Then we drew a straight line along the middle of the broader side of each rod, and affixed to one of them [Fig. G.2], at each end, centred on the line, and perpendicular [to it], two rectangular plates, of equal size and parallel to each other [Fig. G.a,b]; each plate had an aperture exactly in the centre, the aperture at the eye being small, and that towards the moon being greater, in such a way that when one eye was placed at the plate with the smaller aperture, the whole of the moon would be visible through the aperture on the other plate, which was aligned [with the first aperture]. We made a perforation of equal size through both rods at the end of the median line near the plate with the larger hole, and fitted a peg [Fig. G.c] through both perforations in such a way that the sides of the rods inscribed with the lines were fastened together round the peg as a centre, but the rod with the plates could rotate freely in all directions without distortion. We wedged the rod with no plates on it [Fig. G,1] into a base [Fig. G,4]. On the median line of each rod, at the end by the base, we took a point as far as possible from the centre of the peg (the same distance from it [on both rods]), and, on the rod with the base, divided the line so defined into 60 sections, subdividing each section into as many subdivisions as possible. We also attached to the back of the same rod, at its end, [two] plates [Fig. G,d,d] having their corresponding faces aligned with

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18 This passage is supplemented by Pappus' commentary ad loc. (Rome[1] 167-8), which extracts some details of the two procedures of Hipparchus from Books 1 and 2 respectively of the latter's 'On sizes and distances'. For details of the important historical consequences which can be drawn see Toomer[9] (showing that the solar eclipse referred to is that of -189 Mar. 14), which builds on the work of Swerdlow. 'Hipparchus'.

19 On the instrument described in this chapter (known in the middle ages as a 'triquetrum') see Price, 'Precision Instruments' 589-90 with Fig. 344. My Fig. G is based on the text of the Almagest rather than on the figure provided by Pappus in his commentary (Rome[1] I p. 71, with a modern reconstruction; see also Rome's notes on pp. 70-5).

20 The faces of rods 1 and 2 inscribed with the lines cannot be flush with one another, as is clear from Fig. G. Ptolemy seems to mean only that one views the inscribed faces of the two rods as radii of a circle with centre peg c.
each other, and each being equidistant in all respects from that same median line, so that when a plumb-line was suspended between them, the rod could be set up exactly perpendicular to the plane of the horizon. We also had a meridian line [Fig. G,e] ready drawn in the plane parallel to that of the horizon in an unshaded place. We set the instrument upright in such a way that the sides of

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41 Excising the words \(\pi\rho\omicron\ \zeta\ \tau\omicron\ \\nu\upsilon\theta\omicron\ \gamma\rho\alpha\mu\mu\eta\) at H404,17–18. That would mean 'each having that face which was on the same side as the [graduated] line aligned with the other'. But this is impossible, since the plates are not to one side of the face with the graduations, but 'on the back', i.e. on the face opposite the graduated line. This is also clear from Pappus' detailed description (Rome
the rods which were held flush with each other by the peg lay in the meridian, being parallel to the meridian line, and the rod with the base was fixed exactly perpendicular, in a firm and immovable position, while the other rod could move in the plane of the meridian about the peg, responding to the pressure [of the user].\(^2\) We also added another thin, straight rod, [Fig. G,3] attached by a small pin [Fig. G,f] at the base end of the graduated line, so that it too could be rotated, and long enough to reach the end of the line on the other rod equidistant [from the peg] when it was rotated to its maximum distance [from the base];\(^3\) thus by rotating it at the same time as the latter, one could use it to show the straight-line distance between the ends [of the centre-lines on the two rods].

We made our observations of the moon as follows. The moon had to be located on the meridian, and near the solstices on the ecliptic, since at such situations the great circle through the poles of the horizon and the centre of the moon very nearly coincides with the great circle through the poles of the ecliptic, along which the moon's latitude is taken. Furthermore the true distance [of the moon] from the zenith can also be conveniently determined from the same situation. When the moon was precisely in the meridian, we moved the rod with the [sighting-] plates on it round to the position in which the centre of the moon, when sighted through both apertures, was in the centre of the larger aperture. We marked on the thin rod the distance between the ends of the lines on the [two] rods, then applied the distance [marked on the thin rod] to the line on the upright rod graduated into 60 sections. Thus we found the amount of that distance in those units of which the radius of the circle described by the rotation [of the rod with the sighting-plates] in the plane of the meridian contains 60. By calculating the arc corresponding to that chord, we found the angular distance of the apparent centre of the moon from the zenith, measured along the great circle through the poles of the horizon and the moon's centre, which coincided at that moment with the [great circle] through the poles of the equator and the ecliptic, [i.e.] the meridian.

In order, first, to determine the precise amount of the moon's greatest deviation in latitude, we made sightings when the moon was simultaneously

\(^{2}\) I.e. the peg held the rods together tightly enough so that rod 2 would not move under its own weight, but loosely enough so that it could be rotated by the user.

\(^{3}\) This rod has indeed to be 'thin', since it has to pass between the two rods 1 and 2, the faces of which are supposed to be flush. Pappus overcomes this difficulty by saying that rod 2 has to be hollowed out along its length to the depth of the thickness of rod 3 (Rome p. 73). There is the further difficulty that according to Ptolemy's instructions rod 3 has to be long enough to reach to the end of rod 2 at the maximum rotation, presumably 90°; hence its length should be \(\sqrt{2} \times \) length of the graduated line. But since one measures the chord of the zenith distance, not directly on rod 3, but by marking it on rod 3 and then measuring it on the scale on rod 1, no zenith distance greater than 60° (the chord of which is 60°) can be measured. Hence, presumably, Pappus (p. 73) says that rod 3 should be less than the length of the graduated line. Rome (p. 73 n.0) suggests that Ptolemy deliberately chose this limit to avoid the complications of refraction near the horizon. It seems more likely that it is simply a by-product of Ptolemy's construction, and that Pappus' shortening of the rod was done to avoid the difficulties which would result from trying to apply rod 3 to the graduated line if it were 60° or more.
near the summer solstice and near the northern limit of its inclined circle. For in the region of those points the moon's latitude remains sensibly the same over a considerable interval, and furthermore, since the moon is then very near the zenith at the parallel through Alexandria (at which we made our observations), its apparent position is approximately the same as its true position. At such situations it was found that the distance of the centre of the moon from the zenith was always about 12°. Hence by this method too the moon's greatest latitude either side of the ecliptic is shown to be 5°. For the zenith distance of the equator at Alexandria has been shown to be 30°;58°; if we subtract from this the 21° (which is the apparent distance [of the centre of the moon from the zenith]), the result [28;504°] is about 5° greater than the distance from the equator to the summer solstice, which was shown to be 23°;51°.

Then, in order to attack the problem of the parallaxes, we observed the moon in the same way, but this time when it was near the winter solstice, both for the reason already mentioned [above] and because its distance from the zenith in that situation is the greatest of all such meridian positions, and thus provides us with a greater and more easily determinable parallax. We will set out one of a number of parallax observations which we made at such situations. By this means we shall display the method of calculation and at the same time provide a demonstration of the rest of what is to follow in the appropriate order.

13. {Demonstration of the distances of the moon} 

In the twentieth year of Hadrian, Athyr [III] 13 in the Egyptian calendar [135 Oct. 1], 5½ equinoctial hours after noon, just before sunset, we observed the moon when it was on the meridian. The apparent distance of its centre from the zenith, according to the instrument, was 50½°. For the distance [measured] on the thin rod was 5½ of the 60 subdivisions into which the radius of revolution had been divided, and a chord of that size subtends an arc of 50½°. Now the time from epoch in the first year of Nabonassar to the moment of the above observation is 882 Egyptian years 72 days

882 Egyptian years 72 days

For this moment we find:

- mean longitude of the sun: ▲ 7;31°
- true longitude of the sun: ▲ 5;28°
- mean longitude of the moon: ♢ 25;44°
- elongation: 78;13°
- distance [in anomaly] from mean apogee of epicycle: 262;20°

44 Since the revolution of the node takes place once in about 18½ years, this situation occurs 9½ years earlier or later than the similar situation of the moon near the winter solstice, observed by Ptolemy (V 13) in Oct. 135. Therefore these observations were made either in the summer of 126, or in the spring of 145. This is the only useful conclusion that can be drawn from the confused discussion of Newton, 184–6.

45 See HAMA 101–3, Pedersen 204–7.
Hence the complete equation of anomaly, derived from the appropriate table, was $+7;26^\circ$, so that the true position of the moon at that moment was:

- in longitude: $\psi 3;10^\circ$
- in [argument of] latitude on the inclined circle: $2;6^\circ$ from the northern limit
- in latitude on the great circle
  - through the poles of the ecliptic (which almost coincided at that moment with the meridian): $4;59^\circ$ north of the ecliptic.

Now $\psi 3;10^\circ$ is $23;49^\circ$ south of the equator on the same [meridian] circle, and the equator is, likewise, $30;58^\circ$ south of the zenith at Alexandria. Therefore the true distance of the centre of the moon from the zenith was $[23;49 + 30;58 - 4;59 =] 49;48^\circ$. And its apparent distance was $50;55^\circ$. Therefore the moon’s parallax at the distance [of the moon from the earth] corresponding to the position in question was $1;7^\circ$ along the great circle through the moon and the poles of the horizon, when its true distance from the zenith was $49;48^\circ$.

Now that we have established that, draw [Fig. 5.10] in the plane of the great circle through the poles of the horizon and the moon the following great circles, on the same centre:

- that of the earth, AB;
- that through the centre of the moon at the [above] observation, GD;
- the great circle to which the earth bears the ratio of a point, EZH$\Theta$.

---

*For the moon was almost at the winter solstice (cf. p. 247).*
Let their common centre be K, and let the line through the points at the zenith be KAGE. Let us assume that the same distance of the moon, D, from the zenith at G is the amount already determined, 49°48'. Join KDH, ADΩ, and furthermore from point A, which represents the observer's eye, draw AL as perpendicular to KB, and AZ as parallel to KH.

Then it is obvious that for an observer at point A the moon's parallax was arc HΩ. So arc HΩ is 1°7′, according to the calculation from the observation. But since arc ZΩ is negligibly greater than arc HΩ (for the whole earth bears the ratio of a point to circle EZHΩ), arc ZHΩ is very nearly the same, 1°7′. And since, again, point A is negligibly different from the centre of circle ZΩ,

$$\angle ZAΩ = \begin{cases} 1°7′ & \text{where 4 right angles} = 360° \\ 2°14′ & \text{where 2 right angles} = 360° \end{cases}$$

And $$\angle ADL = \angle ZAΩ = 2°14′.$$ Therefore in the circle about right-angled triangle ADL,

arc AL = 2°14′

and Crd arc AL = 2°21′ where hypotenuse AD = 120°.

But LD is negligibly smaller than AD.

Therefore where LA = 2°21′, LD ≈ 120°.

Furthermore since, by hypothesis, arc GD = 49°48′, the angle at the centre of the circle.

$$\angle GKD = \begin{cases} 49°48′ & \text{where 4 right angles} = 360° \\ 99°36′ & \text{where 2 right angles} = 360° \end{cases}$$

Therefore in the circle about right-angled triangle ALK

arc AL = 99°36′

and arc LK = 80°24′ (supplement).

Therefore the corresponding chords

AL = 91°39′

and LK = 77°27′ where hypotenuse AK = 120°.

Therefore where AK, the radius of the earth, is 1°,

AL = 0°46′

and KL = 0°39′.

But where AL = 2°21′, LD, as was shown, = 120°.

Therefore where AL = 0°46′, LD = 39°6′.

And, in the same units, KL = 0°39′.

and the radius of the earth, KA = 1°.

Therefore where KA, the radius of the earth, is 1°, by addition, KLD, which represents the distance of the moon at the observation, is 39°45′.\footnote{There is an accumulated error here, due to a series of small inaccuracies and roundings. More accurate would be 39°50′.}

Now that we have demonstrated this, let [Fig. 5.11] the moon's eccentre be ABG on centre D and diameter ADG, on which E is taken as the centre of the ecliptic, and Z as the point towards which [the mean apogee diameter of] the epicycle is directed. Draw the epicycle, HΩKL, on point B, and join HBΩE, BD and BKZ. Let L represent the position of the moon at the observation in
question, and draw perpendiculars to BE, DM from D and ZN from Z.

Then since the amount of the elongation at the time of the observation was 78;13° [p. 247], it follows from the theory previously established that

\[ \angle AEB = 156;26° \text{ where 4 right angles } = 360°; \]

hence its supplement, \[ \angle ZEN = \angle DEM = \begin{cases} 23;34° \text{ where 4 right angles } = 360° \\ 47;8° \text{ where 2 right angles } = 360° \end{cases} \]

Therefore in the circles about the corresponding right-angled triangles, [ZEN, DEM], since DE = EZ,

\[
\begin{align*}
\text{arc } DM &= \text{arc } ZN = 47;8° \\
\text{and arc } EM &= \text{arc } EN = 132;52° \text{ [supplements].}
\end{align*}
\]

Therefore the corresponding chords

\[
\begin{align*}
DM &= ZN = 47;59° \\
\text{and } EM &= EN = 110;0°
\end{align*}
\]

where hypotenuse DE = hypotenuse EZ = 120°.

Therefore where DE = EZ = 10;19° and DB, the radius of the eccentre, is 49;41°,

\[
\begin{align*}
DM &= ZN = 4;8° \\
\text{and } EM &= EN = 9;27°.
\end{align*}
\]

And since \( BM^2 = BD^2 - DM^2 \),

\( BM = 49;31° \).

And \( BE = [BM - EM =] 40;4° \),

and, by subtraction [of EN from BE], \( BN = 30;37° \) where ZN = 4;8°.

And since \( BN^2 + ZN^2 = BZ^2 \),

hypotenuse \( BZ = 30;54° \).

Heiberg rightly excised \( \varepsilon \kappa \beta \lambda \pi θες \) ('extended') at H413,7 as an unnecessary gloss which disturbs the sentence structure. Transferring it after BE (as Halma and Manitius) is no improvement, since the perpendicular from Z is not on the extension of BE.
Therefore in the circle about right-angled triangle BZN, where hypotenuse BZ = 120°, 
ZN = 16;2°
and arc ZN = 15;21°.

\[ \therefore \angle ZBN = \begin{cases} 
15;21° & \text{where 2 right angles} = 360° \\
about 7;40° & \text{where 4 right angles} = 360°.
\end{cases} \]

That [7;40°], then, is the size of arc ΘK of the epicycle.

Next, the distance of the moon from the mean apogee of the epicycle at the moment of the observation was 262;20° [p. 247], and, obviously, its distance from K, the mean perigee, was 82;20° (by subtraction of a semi-circle).

Therefore arc KL = 82;20°
and arc ΘKL = (arc ΘK + arc KL) = 90;0°.

So \( \angle ΘBL \) is a right angle.

\[ \therefore EL^2 = BL^2 + EB^2, \]
and where DB, the radius of the eccentre, is 49;41°
and BL, the radius of the epicycle, is 5;15°
EB, as we showed = 40;4''.

\[ \therefore EL = 40;25''. \]

Therefore the distance of the moon at the observation is 40;25'', where BL, the radius of the epicycle, is 5:15''
and where EA, the distance from the centre of the earth to the apogee of the eccentre, is 60°,
and where EG, the distance from the centre of the earth to the perigee of the eccentre, is 39;22''.

But we showed that the moon’s distance at the observation, that is EL, was 39;45'' where the radius of the earth is one.

Therefore where EL, the distance of the moon at the observation, is 39;45°, and the earth’s radius is 1°,

EA, the mean distance at the syzygies = 59;0°,49
EG, the mean distance at the quadratures = 38;43°,
and the radius of the epicycle = 5;10°.

Q.E.D.
too can readily be performed geometrically, if we are given, in addition to the
distances of the moon at the syzygies, the sizes of the angles formed at the
(observer's) eye at the syzygies by the diameters of the sun, moon and shadow.

Of the various methods used to solve the latter problem, we have rejected
those claiming to measure the luminaries by measuring [the flow of] water or by
the time [the bodies] take to rise at the equinox,\(^5\) since such methods cannot
provide an accurate result for the matter in hand. Instead, we too constructed
the kind of dioptra which Hipparchus described, which uses a four-cubit rod,\(^6\)
and, observing with this, found that the sun's diameter always subtends
approximately the same angle, there being no noticeable difference due to [the
variation in] its distance, but that the moon subtends the same angle as the sun
only when it is at its greatest distance from the earth (i.e. the apogee of the
epicycle) at full moon, in contradiction to the hypotheses of my predecessors,
[who assumed that it subtends the same angle as the sun at full moon] when it is
at mean distance.\(^5\) Furthermore, we find that the angles themselves are
considerably smaller than those traditionally accepted.\(^\) However our com­
putation of the latter rests, not on measurement with the dioptra, but on certain
lunar eclipses. For although it was possible to determine readily from the
diopters, as constructed, when both diameters subtend the same angle (since
such a determination involves no actual measurement), the amount [of the angle
subtended] seemed utterly dubious to us, since the measurement\(^5\) involving the
positioning of the width [of the plate] which covers [the body being sighted] on

---

\(^1\) According to Pappus ad loc. (Rome[II] I 87-9) 'the more ancient astronomers' used water­
clocks to measure the time taken by the sun to cross the horizon, a procedure criticised by
Hipparchus. He refers to a lost work of Heron, περὶ ύδριων ύψοςκοπεῖον, on which see also
Proclus, Hypotyposis IV 73-6 (ed. Manitius p. 120-2). At H416.21 Heiberg rightly accents
ὑψομετρίαν (from the abstract ύψομετρια). There is no evidence for the existence of
ὑψομετρίαν, 'vessel for measuring flow of water', conjectured by LSJ s.v. In the corresponding
passage Proclus p. 120 line 14 we should read ύψομετριας. Cf. also H.I.M.I 103 n. 1.

\(^2\) There are ancient descriptions of this instrument by Pappus in his commentary ad loc.
(Rome[II] I 90-2) and by Proclus, Hypotyposis IV 87-96 (ed. Manitius pp. 126-30). See Price,
'Precision Instruments' 591, and, for modern literature, H.I.M.I 103 n.2. The essential feature is a
plate ξηραμάτων, H417.22-3) which can be moved along a graduated rod until it appears to
exactly cover the object being sighted by the eye placed at one end of the rod.

\(^3\) It was shown by Swerdlow, 'Hipparchus' 291-8, that Hipparchus was one of those who held
this. An important consequence of this hypothesis is that annular solar eclipses become possible,
whereas under Ptolemy's assumption they are impossible.

\(^4\) Hipparchus (see IV 9 p. 205) assumed that the moon at mean distance subtends a six hundred
and fiftieth of its circle, or about 0.33,14\(^\circ\); hence his figure for the sun's diameter was the same.
Ptolemy (below) finds that when moon and sun have the same apparent diameter (at maximum
distance) it is 0.31,20\(^\circ\), considerably smaller. This must be what he means here. However, his value
for the lunar diameter at mean distance, 0.33,20\(^\circ\), is negligibly different from Hipparchus'.

\(^5\) Excising πλεξίστης οὔσης at H417.23, to which I can attach no meaning (it cannot mean 'very
laborious', as Manitius translates, nor, if it could, would it be true). The variant πλεξίστης οὔσης
found in D, part of the Arabic tradition (L) and Pappus (Rome[II] I 93.21) can be translated
('involving multiple positionings'), but it is not true that sighting the moon would require more
than one positioning of the plate. Unless the corruption lies deeper (e.g. πλεξίστης has replaced a
word meaning 'delicate') one must assume that πλεξίστης οὔσης was an inept gloss intended to
explain why the process was inaccurate, and that this was corrupted to the unintelligible πλεξίστης
οὔσης by attraction to παραμετρήσεως.
the length of the rod running from the eye to the plate can be inaccurate. However, once it was determined that the moon is at its greatest distance when it subtends the same angle at the eye as the sun, we computed the size of the angle it subtends from observations of lunar eclipses in which the moon was near that [greatest] distance, and thence obtained immediately the size of the angle subtended by the sun. We shall explain the method of procedure in this by means of two of the eclipses used.

In the fifth year of Nabopolassar, which is the 127th year from Nabonassar, Athyr [III] 27/28 in the Egyptian calendar [-620 Apr. 21/22], at the end of the eleventh hour in Babylon, the moon began to be eclipsed; the maximum obscuration was \( \frac{1}{4} \) of the diameter from the south. Now, since the beginning of the eclipse occurred 5 seasonal hours after midnight, and mid-eclipse about 6 [seasonal hours after midnight], which correspond to 5\( \frac{6}{6} \) equinoctial hours at Babylon on that date (for the true position of the sun was \( \Phi 27;39^\circ \)), it is clear that mid-eclipse, which is when the greatest part of the diameter is immersed in the shadow, occurred 5\( \frac{6}{6} \) equinoctial hours after midnight in Babylon, and exactly 5 [hours after midnight] at Alexandria.\(^{56}\)

The time from epoch is

\[
\text{126 Egyptian years 86 days} \quad \begin{cases} 17 \text{ equinoctial hours reckoned simply} \\ 16\frac{1}{2} \text{ equinoctial hours in mean solar days.} \end{cases}
\]

Therefore the lunar position was as follows:

- mean position in longitude: \( \approx 25;32^\circ \)
- true position in longitude: \( \approx 27;5^\circ \)
- distance [in anomalies] from the apogee of the epicycle: 340;7°
- distance [in latitude] from the northern limit on the inclined circle: 80;40°.

Thus it is clear that when the centre of the moon near its greatest distance is 9\( \frac{6}{6} \) distant from the node, measured along its inclined circle, and the centre of the shadow lies on the great circle drawn through the moon’s centre at right angles to the inclined circle (which is the situation at which the greatest obscuration occurs), \( \frac{1}{4} \) of the moon’s diameter is immersed in the shadow.

Again, in the seventh year of Kambyses, which is the 225th year from Nabonassar, Phamenoth [VII] 17/18 in the Egyptian calendar [-522 July 16/17], 1 [equinoctial] hour before midnight at Babylon, the moon was eclipsed half its diameter from the north. Thus this eclipse occurred about 1\( \frac{5}{6} \) equinoctial hours before midnight at Alexandria.\(^{58}\) The time from epoch is

\(\text{Oppolzer no. 901: mid-eclipse 2:38 a.m., \( \approx 41^\circ \) after midnight at Alexandria}, \text{ magnitude 1.6}^4\).

P.V. Neugebauer, Spezieller Kanon, gives about 5\( \frac{1}{6} \) after midnight (Babylon) for mid-eclipse, magnitude 2.1\( ^4 \).

\(\text{Oppolzer no. 1056: mid-eclipse 21:6^d (\approx 11 \text{ p.m. Alexandria})}, \text{ magnitude 6.1}^4\). P.V. Neugebauer gives mid-eclipse as ca. 23.6^d Babylon, magnitude 6.1^d. The time used by Ptolemy is clearly in error (although the computed positions of sun and moon must have seemed to him to confirm it), but the source of his error is too complicated to discuss here. The best treatment is in Britton[1] 81-4. For this eclipse (alone of those preserved in Almagest) there is also an extant cuneiform report (published by Kugler, SSB I p. 71). According to A. J. Sachs this text should be translated as follows: ‘Year VII, month IV, night of the fourteenth, 11 double hours in the night a “total” lunar eclipse took place [with only] a little remaining [uneclipsed]. The north wind blew’. Here the time agrees with modern computations (and disagrees with Ptolemy), but the magnitude disagrees with both.
224 Egyptian years 196 days \[ \{ 10^1 \text{ equinoctial hours reckoned simply} \]
\[ 9^1 \text{ equinoctial hours reckoned accurately} \]

(for the position of the sun was \( \equiv 18;12^\circ \)).

Therefore the lunar position was as follows:

- mean position in longitude: \( \phi 20;22^\circ \)
- true position in longitude: \( \phi 18;14^\circ \)

distance [in anomaly] from the apogee of the epicycle: \( 28;5^\circ \)

distance [in latitude] from the northern limit on the inclined circle: \( 262;12^\circ \).

Hence it is clear that, when the centre of the moon, again near its greatest
distance, is \( 7\frac{1}{2}^\circ \) from the node, as measured along its inclined circle, and the centre
of the shadow has the same position relative to it as before, half of the moon’s
diameter is immersed in the shadow.

But, when the moon’s centre is \( 9\frac{1}{2}^\circ \) from the node along the inclined circle, it is
\( 48\frac{1}{2}^\circ \) from the ecliptic along the great circle drawn through it at right angles to the
inclined circle [the orbit]; and when it is \( 7\frac{1}{2}^\circ \) from the node along the inclined
circle, it is \( 40\frac{1}{2}^\circ \) from the ecliptic along the great circle drawn through it at right
angles to the inclined circle. Therefore, since the difference between [the sizes
of] the two eclipses comprises \( \frac{1}{2} \) of the moon’s diameter, and the difference
between the above distances of the moon’s centre from the ecliptic (i.e. from the
centre of the shadow) comprises \( 48\frac{1}{2} - 40\frac{1}{2} = 17^\circ \), it is obvious that the total
diameter of the moon subtends a great circle arc of \( 4 \times 7\frac{1}{2} = 31\frac{1}{2}^\circ \).

From the same data it is easy to see that the radius of the shadow at the same
greatest distance of the moon subtends \( 40\frac{1}{2}^\circ \). For when the moon’s centre was that
distance \( 40\frac{1}{2}^\circ \) from the centre of the shadow, it was touching the edge of the
shadow’s circumference, because [in that situation] half of the moon’s diameter
was eclipsed. This is negligibly less than \( 2\frac{1}{2} \) times the radius of the moon, which is
\( 15\frac{1}{2}^\circ \). The values we derive for the above quantities from a number of similar
observations are in agreement with these, hence we use them, both in other
parts of the theory, concerning eclipses, and in the following demonstration of
the solar distance, which will be along the same lines as that followed by
Hipparchus. A further presupposition [of this demonstration] is that the circles of
sun, moon and earth enclosed by the cones are not noticeably less than great
circles on their spheres, and the diameters too [not noticeably less than great
circle diameters].

59 Possibly one should read \( 18;11^\circ \) with \( \text{D}^1 \) (computed: \( 18;10^\circ \)).
60 Ptolemy has made a computing error here: correct is \( \alpha = 27;54^\circ \). Obviously, he has computed
(here only) for the uncorrected time of \( 10^1 \). However, this has no serious consequences, since it is
merely intended to show that the moon is near the apogee of the epicycle. The discrepancy in the
true position (see n. 59) cannot be explained by this error.
61 On the computation of these amounts see \textit{HAMA} 107. It seems probable that they were,
properly, computed from a spherical triangle with the right angle at the moon’s orbit (rather than
from a plane triangle or any of the other approximations suggested there). But the computations are
inaccurate: Ptolemy should have found \( 48\frac{1}{2}^\circ \) and \( 40\frac{1}{2}^\circ \) respectively. For similar computations with
the moon at the perigee of the epicycle see VI 5 pp. 284–5.
62 Although Ptolemy’s procedure for finding the apparent diameters of moon and shadow is both
elegant and theoretically correct, it suffers from serious practical disadvantages. On these, and the
inaccuracies involved in his actual computations, see \textit{HAMA} 106–8.
63 Reference to VI 5–7 and VI 11.
64 I.e. in Fig. 5.12 the cones from points \( \text{N} \) and \( \text{X} \) enclosing the spheres of sun (\( \text{ABG} \)), moon (\( \text{EZG} \))
and earth (\( \text{KLM} \)) have bases (the circles on \( \text{AG} \), \( \text{EH} \) and \( \text{KM} \)) which are not sensibly less than great
15. \{On the distance of the sun and other consequences of the demonstration of that\}^{65}

Now, given the above, and given that the greatest distance of the moon at the
syzygies is 64;10 units where the earth's radius is 1 (for we showed [p. 251] that its
mean distance is 59 of those units, and the radius of the epicycle 5;10), let us see the
size of the sun's distance which results.

[See Fig. 5.12.] Let there be the following great circles of the [various] spherical
bodies lying in the same plane: circle ABG of the sun's, on centre D, circle EZH of
the moon's at its greatest distance, on centre O, circle KLM of the earth's, on
centre N. Let AXG be the plane through the centres [in the tangent cone]
enclosing earth and sun, and ANG the plane through the centres [in the tangent
cone] enclosing sun and moon, with DNX as common axis. Let the straight
lines through the points of tangency, which are, obviously, parallel to each other,
and sensibly equal to diameters, be ADG on the sun's circle, E0H on the moon's
circle, KNM on the earth's circle, and OPR on the circle of the shadow in which
the moon is immersed at its greatest distance (thus ON equals NP, and each of
them is 64;10 units where NL, the earth's radius, is 1).

Then we have to find the ratio between ND, the distance of the sun, and NL,
the earth's radius.

Produce EH to [meet XG at] S.

Since we demonstrated [p. 254] that the moon's diameter at the distance in
question, namely the greatest distance in the syzygies, subdends 0;31,20 of the
circle drawn through the moon about the earth's centre,

\[ \angle ENH = 0;31,20^\circ \] where 4 right angles = 360°.

and \[ \angle EOH = 0;31,20^\circ \] where 2 right angles = 360°.

Therefore in the circle about right-angled triangle NH\Theta,

arc \Theta H = 0;31,20^\circ

and arc \Theta N = 179;28,40° (supplement).

Therefore the corresponding chords

\[ \Theta H = 0;32,48^\circ \] where diameter NH = 120°.

\[ \Theta N \approx 120^\circ \]

Therefore where \( \Theta N = 64;10, \Theta H = 0;17,33 \).

And NM, the radius of the earth, is 1 in the same units.

But \( PR : \Theta H \approx 2;36 : 1 \) [p. 254].

\[ \therefore PR = 0;45,38 \] in the same units.

\[ \therefore \Theta H + PR = 1;3,11 \] where \( NM = 1 \).

But \( PR + \Theta S = 2 \), since \( PR + \Theta S = 2NM \)

(for, as we said, all [three] are parallel, and NP = \( \Theta N \)).

Therefore, by subtraction [of (\( PR + \Theta H \)) from (\( PR + \Theta S \))],

\[ HS = 0;56,49 \] where \( NM = 1 \).

And \( NM : HS = NG : HG = ND : OD \).

\footnotesize{\textsuperscript{65}On chs. 15 and 16 see \textit{H.\&M.} 109-12, Pedersen 209-13.}
V 15. Geometrical determination of sun's distance
Therefore where ND = 1, DΘ = 0.56,49, and, by subtraction, ΘN = 0.3,11.

Therefore where NΘ = 64;10 and NM = 1,
the sun's distance, ND = 1210.

Similarly, as we showed, PR = 0.45,38 where NM = 1,
and NM:PR = NX:XP.

Therefore where NX = 1, XP = 0.45,38
and, by subtraction, PN = 0.14,22.

Therefore where PN = 64;10 and NM, the earth's radius, = 1,
XP = 203;50,
and, by addition, XN = 268.

Therefore we have calculated that where the earth's radius is 1
the mean distance of the moon at the syzygies is 59
the distance of the sun is 1210
and the distance from the centre of the earth to the apex of the shadow cone is 268.

16. [On the sizes of sun, moon and earth]

The ratios of the volumes of the bodies are immediately derivable from the ratios of the diameters of sun, moon and earth.

For, since we have shown that, where NM, the earth’s radius, is 1,
the moon’s radius, ΘH = 0.17.33
and NΘ = 64;10,
and since NΘ:ΘH = ND:DG,
and ND was shown to be 1210 in the same units,
the radius of the sun, DG ≈ 5½ in the same units.

So the diameters will have the same ratios.
Therefore where the moon's diameter is 1, the earth’s diameter will be about 3½, and the sun’s 18½.
Therefore the earth’s diameter is 3½ times the moon’s and the sun’s diameter is 18½ times the moon’s and 5½ times the earth’s.

And, using the same numbers,

since 1½ = 1,
and 3½ = 39½,
and 18½ = 6644½,

we conclude that, where the moon’s volume is 1,
the earth’s volume is 39½ and the sun’s 6644½.
Therefore the sun’s volume is about 170 times that of the earth.⁶⁶

⁶⁶ There is no point in estimating the relative volumes of the bodies, but it was evidently traditional in Greek astronomy, for Theon of Smyrna (ed. Hiller p. 197) and Calcidius (ed. Waszink p. 143) quote from Hipparchus' work on sizes and distances the statement that the sun is 1880 times the size of the earth and the earth 27 times the size of the moon; these ratios plainly refer to relative volumes. In his Planetary Hypotheses (ed. Goldstein p. 9) Ptolemy gives the volumes of all the planets relative to the earth.
17. {On the individual parallaxes of sun and moon}\(^{67}\)

With the above as basis, the next problem is to demonstrate, again briefly, how one may calculate the individual parallaxes of sun and moon from the amount of their distances. First [we deal with] the parallaxes with respect to the great circle drawn through the zenith and the body.\(^ {68}\)

[See Fig. 5.13.] In the plane of that great circle, then, let the great circle representing the [surface of the] earth again [as in Fig. 5.10] be AB, the great circle representing the [position of the] sun or moon GD, and the great circle to which the earth bears the ratio of a point EZHΘ. Let K be the centre of all

![Fig. 5.13]

[these circles], and KAGE the diameter through the zenith. Cut off from the zenith point G arc GD; let it be, e.g., 30°, and again draw KDH and ADΘ, from A draw AZ parallel to KH, and drop perpendicular AL on to KH.

Now neither of the luminaries always remains at the same distance. But the resulting difference in the sun's parallaxes will be very small and imperceptible, since the eccentricity of its circle is small, and its distance great. For the moon, however, the resulting difference will be very perceptible, both because of its

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\(^{67}\) See *HAMA* 112-15, Pedersen 213-17.

\(^{68}\) In contrast to the longitudinal and latitudinal components of this 'total' parallax: these are dealt with in V 19.
motion on the epicycle and because of the motion of the epicycle on the
eccentre, each of which produces quite a large difference in the distance.
Therefore we shall demonstrate the solar parallaxes for a single ratio, namely
1210:1, but we shall demonstrate the lunar parallaxes for the four ratios which
will be most convenient for the methods we shall subsequently develop. The
four distances we have chosen are as follows:
The first two are
[1] when the epicycle is at the apogee of the eccentre,
[a] the distance to the apogee of the epicycle, which we concluded from our
previous demonstration [p. 255] to be 64;10 earth-radii;
[b] the distance to the perigee of the epicycle, which we compute to be
[59;0 - 5;10 =] 53;50 earth-radii.
The second two are
[2] when the epicycle is at the perigee of the eccentre,
[a] the distance to the apogee of the epicycle, which we concluded from our
previous demonstration [p. 251] to be [38;43 + 5;10 =] 43;53 earth-
radii;
[b] the distance to the perigee of the epicycle, which we compute as
[38;43 - 5;10 =] 33;33 earth-radii.
Then, since arc GD = 30°, by hypothesis,
\[ \angle GKD = \begin{cases} 30° & \text{where 4 right angles = 360°} \\ 60° & \text{where 2 right angles = 360°} \end{cases} \]
Therefore in the circle about right-angled triangle AKL
arc AL = 60°,
and arc KL = 120° (supplement).
Therefore the corresponding chords
\[ \text{AL = 60°,} \quad \text{and KL = 103;55''} \]
Therefore where AK = 1°, AL = 0;30'' and KL = 0;52''.
And, in the same units,
\[ \begin{align*}
KLD &= \begin{cases} 1210° \text{ for the sun's distance} \\ 64;10° \text{ for the moon's first limit} \end{cases} \\
&= \begin{cases} 53;50° \text{ for the moon's second limit} \\ 43;53° \text{ for the moon's third limit} \end{cases} \\
&= \begin{cases} 33;33° \text{ for the moon's fourth limit} \end{cases}
\end{align*} \]
And, by subtraction, LD (= KLD - KL), which is the same as AD, since the
difference is imperceptible.
\[ \begin{align*}
\therefore AD &= \begin{cases} 1209;8° \text{ for the sun's distance} \\ 63;18° \text{ for the moon's first limit} \end{cases} \\
&= \begin{cases} 52;58° \text{ for the moon's second limit} \\ 43;1° \text{ for the moon's third limit} \end{cases} \\
&= \begin{cases} 32;41° \text{ for the moon's fourth limit} \end{cases}
\end{align*} \]
Therefore, where hypotenuse AD = 120°, then (assuming the same order, to
avoid repetition)
\[ \begin{align*}
[\text{Sun}] &= [1a] \quad [1a] \quad [1b] \quad [2a] \quad [2b] \\
[\text{Moon}] \quad [2a] \quad [2b] \\
AL &= 0;2,59° \quad 0;56,52° \quad 1;7,58° \quad 1;23,41° \quad 1;50,9°.
\end{align*} \]
Therefore in the circle about right-angled triangle DLA\(^69\)

\[
\text{arc AL} = \begin{cases} 
0.25° & 0.54.18° \\
1.45° & \text{about 1.20° about 1.45°.}
\end{cases}
\]

\[
\therefore \angle ADB = \begin{cases} 
0.25° & 0.54.18° \\
1.45° & \text{where 2 right angles = 360°}
\end{cases}
\]

\[
\angle \text{ZAO} = \begin{cases} 
0.125° & 0.27,9° \\
0.32,27° & 0.40° \\
0.52,30° & \text{where 4 right angles = 360°}
\end{cases}
\]

So, since point A is negligibly different from centre K, and arc ZHΘ is negligibly greater than arc HΘ (for the whole earth has the ratio of a point to circle EZHΘ), in circle EZHΘ, the arc of the parallax

\[
\text{arc } \text{HΘ} = \begin{cases} 
0.125° & \text{for the sun’s distance} \\
0.27,9° & \text{for the moon’s first limit} \\
0.32,27° & \text{for the moon’s second limit} \\
0.40° & \text{for the moon’s third limit} \\
0.52,30° & \text{for the moon’s fourth limit.}
\end{cases}
\]

Q.E.D.

In the same way we calculated the parallaxes for the other zenith distances (at intervals of 6° up to the 90° of the quadrant) at each limit, and constructed a table to determine the parallaxes. The table has, again, 45 lines, and 9 columns.

H432 In the first column we put the 90 degrees of the quadrant, tabulating them, obviously, at two-degree intervals; in the second column we put the minutes of solar parallax corresponding to each argument; in the third column the lunar parallax at the first limit; in the fourth column the increment in the [lunar] parallax at the second limit over the first limit; in the fifth column the [lunar] parallax at the third limit; and in the sixth the increment in the [lunar] parallax at the fourth limit over the third limit. Thus, for example, for an argument of 30° we put 0.125° for the sun, then 0.27,9° for the first limit of the moon; next 0.32,18°, which is the increment of the second limit over the first; then 0.40°, for the third limit, and next 0.12,30°, which is the increment of the fourth limit over the third.

We needed to provide a convenient method of calculating the parallax (corresponding to the appropriate argument) for distances [of the moon] at intermediate positions between apogee and perigee [of eccentricity and epicycle] from the parallaxes tabulated at the above four limits, using minutes [of interpolation]. To this end we added the remaining three columns, to account for those differences. We calculated these columns in the following manner.

H433 Let [Fig. 5.14] the moon’s epicycle be ABGD on centre E, and let Z be the centre of the ecliptic and the earth. Join [ZE with line] AEDZ, draw ZGB, join BE, GE, and drop perpendiculars on to AD, BH from B, and GΘ from G. Let us suppose, first, that arc AB, the moon’s distance from A, the true apogee [of the epicycle] as taken with respect to centre Z, is, e.g. 60°.

\[
\therefore \angle \text{BEH} = \begin{cases} 
60° & \text{where 4 right angles = 360°} \\
120° & \text{where 2 right angles = 360°.}
\end{cases}
\]

\(^{69}\)From here on Ptolemy drastically rounds his computations for the moon’s third and fourth limits. His rationale, no doubt, is that in computing solar eclipses (for which the parallax table is principally designed) the moon is by definition near the apogee of the eccentric, and hence there is no use for the third and fourth limits. Cf. p. 264 n.73.

\(^{70}\)Reading o μ (with D,Ar) for o π o (0.40,0) at H431.4 and at H431.13.
Therefore in the circle about right-angled triangle BEH

\[ \text{arc BH} = 120^\circ \]

and \[ \text{arc EH} = 60^\circ \] (supplement).

Therefore the corresponding chords

\[ \begin{align*}
BH &= 103;55^p \\
EH &= 60^p
\end{align*} \]

where diameter \( EB = 120^p \).

But when centre \( E \) of the epicycle is at the apogee of the eccentre,

\[ \text{ZE:EB} = 60 : 5;15. \]

Therefore, where \( \text{EB} = 5;15^p \),

\[ \begin{align*}
\text{BH} &= 4;33^p \\
\text{EH} &= 2;38^p
\end{align*} \]

and, by addition [of \( \text{EH} \) to \( \text{EZ} \)], \( \text{HEZ} = 62;38^p \).

And \( \text{ZB}^2 = \text{ZH}^2 + \text{HB}^2 \).

\[ \therefore \text{ZB} = 62;48^p, \]

where

the distance of the first limit, \( \text{ZA} = 65;15^p \)

the distance of the second limit, \( \text{ZD} = 54;45^p \)

and the difference between the two limits, \( \text{AD} = 10;30^p \).

Therefore the difference at \( B \) with respect to the first limit is \([65;15 - 62;48 =] 2;27^p \) where the total difference is \( 10;30^p \). Therefore where the total
difference is $60^\circ$, the difference at B will be $14;0^\circ$. This $[14;0^\circ]$, then, is the amount which we shall enter in the seventh column on the line [corresponding to the argument] of half of the number 60, namely 30. The reason for this is that the 90 degrees comprised in the first column of the table contain half of the 180 degrees from A to D. 

By the same reasoning, if we suppose arc GD to be the same size [as arc AB above], $60^\circ$, it will be shown that

\[
\begin{align*}
G\Theta &= 4;33^\circ \\
E\Theta &= 2;36^\circ
\end{align*}
\]

where radius $EG = 5;15^\circ$.

Hence, by subtraction [of $E\Theta$ from $ZE$], $Z\Theta = 57;22^\circ$.

By the same reasoning [as above], hypotenuse $ZG = \sqrt{57;22^2 + 4;33^2} = 57;33^\circ$.

We again subtract this from the $65;15^\circ$ of the first limit, and find that the result, $7;42^\circ$, is 44;0 sixtieths of the total difference. This is what we shall enter in the same [seventh] column opposite the argument 60, since arc $ABG = 120^\circ$.

With the same arcs [AB and GD] as basis, let us suppose that centre E is at the perigee of the eccentric, which is the position defining the third and fourth limits. In this position

\[
\frac{ZB}{EB} = 60:8.72
\]

Therefore where $BE = 8^\circ$, and assuming both arc $AB$ and arc $GD$ as $60^\circ$,

\[
\begin{align*}
BH &= G\Theta = 6;56^\circ \\
EH &= E\Theta = 4;0^\circ
\end{align*}
\]

where $ZE = 60^\circ$.

so, by the same reasoning [as above]

\[
\begin{align*}
\text{hypotenuse } ZB &= \sqrt{ZH^2 + BH^2} = 64;23^\circ \\
\text{hypotenuse } ZG &= \sqrt{Z\Theta^2 + G\Theta^2} = 56;26^\circ.
\end{align*}
\]

where the [distance of] the third limit, $ZA = 68^\circ$.

And $68^\circ - 64;23^\circ = 3;37^\circ$, which is 13;33 sixtieths of the total difference, $16^\circ$.

We enter this amount [13;33] in the eighth column opposite the argument 30, in the same way as before.

Also, $68^\circ - 56;26^\circ = 11;34^\circ$, which is 43;24 sixtieths of the total difference, $16^\circ$. This amount we enter, similarly, in the eighth column opposite the argument 60.

That, then, is the way we shall set out the corrections computed for the motion of the moon on the epicycle. The corrections for the motion of the epicycle on the eccentric will be derived as follows.

Let [Fig. 5.15] the moon's eccentric be $ABGD$ on centre E and diameter $AEG$, on which Z is taken as the centre of the ecliptic. Draw $BZD$, and let angles $AZB$ and $GZD$ both, again, be taken as $60^\circ$. These situations occur at elongations of $30^\circ$ (when the centre of the epicycle is at B), and $120^\circ$ (when the centre of the epicycle is at D). Join $BE$, $ED$, and drop perpendicular $EH$ from $E$ on to $BZD$.

\[71\] The main part of Table V 18 (cols. 2 to 6) is a function of the zenith distance, which varies between $0^\circ$ and $90^\circ$. The interpolation columns 7 and 8, however, are a function of the anomaly $\alpha$, which varies between $0^\circ$ and $180^\circ$. In order to use the same argument column for both, Ptolemy tabulates cols. 7 and 8 as a function of $\frac{1}{2} \alpha$.

\[72\] Cf. V 7 p. 235.
Then, since \( \angle BZA = 120^\circ \) where 2 right angles = 360°, in the circle about right-angled triangle EZH,

- arc \( EH = 120^\circ \)
- and arc \( ZH = 60^\circ \) (supplement).

Therefore the corresponding chords

\[
\begin{align*}
EH &= 103:55^\circ \\
HZ &= 60^\circ
\end{align*}
\]

where hypotenuse \( EZ = 120^\circ \).

Therefore where the distance between the centres, \( EZ = 10;19^p \)
and the radius of the eccentre is 49;41°,

- \( EH = 8:56^p \)
- and \( ZH = 5;10^p \).

And since \( BH^2 = BE^2 - EH^2 \),

\( BH = DH = 48;53^p \) in the same units.

Therefore, by addition [of \( ZH \) to \( BH \)], \( ZB = 54;3^p \).

and, by subtraction [of \( ZH \) from \( DH \)], \( ZD = 43;43^p \)

where [the distance for] the first [two] limits, \( ZA = 60^p \)

[the distance for] last [two] limits, \( ZG = 39;22^p \)

and the difference between them = 20;38°.

Now \( 60^p - 54;3^p = 5;57^p \), which is 17;18 sixtieths of the total difference of 20;38°;

and \( 60^p - 43;43^p = 16;17^p \), which is 47;21 sixtieths of the total difference of 20;38°.

Therefore, obviously, we shall enter 17,18 in the ninth column opposite the argument 30° of elongation, and 47,21 opposite 120°, i.e. again opposite 60°;

for, since the perigee [of the eccentre] lies at 90° [of elongation], an elongation of 60° is equivalent in distance to an elongation of 120°.
In the same way we calculated the minutes [of coefficient of interpolation] for the differences over the three intervals in question for the other arcs. We performed the calculation at intervals of 12°, which corresponds to 6° in the arguments in the table, since the 180° from apogee [of the epicycle or eccentre] to perigee correspond to the 90° of [the argument column in] the table. We entered these minutes, calculated geometrically, opposite the appropriate argument. We derived the entries for the intermediate arguments by linear interpolation over the six-degree intervals: for the difference between the results so derived and [accurate] geometrical calculation is negligible over such a short interval, both for the minutes and for the actual parallaxes.

The table is as follows.

18. {Parallax Table}[^1]  
[See p. 265.]

19. {On the determination of the parallaxes}[^2]

So, when we decide to determine the amount of the moon’s parallax at any given [lunar] position, (first) with respect to the great circle drawn through the moon and the zenith, we examine its distance (in equinoctial hours) from the meridian at the latitude in question. With the distance found as argument, we enter the Table of Angles [II 13] for the appropriate latitude and zodiacal sign, and take the amount in degrees in the second column corresponding to the hour, interpolating between integer hours if necessary.[^3] This gives us the distance of the moon from the zenith, measured along the great circle joining the two. With this as argument, we enter the Table of Parallaxes [V 18], determine on which line in the first column the argument is to be found, and taking the numbers corresponding to this in the four columns following the column of solar parallaxes, namely the third, fourth, fifth and sixth columns, write each one down separately. Then we take the corrected anomaly (i.e. with respect to the true apogee [of the epicycle]) at that moment: [if it is less than 180°,] we take the anomaly itself, but if it is greater than 180°, we take (360° minus anomaly); we always halve the amount so obtained, and, entering with this into the same [column of] arguments, determine the number of minutes corresponding to it in both the seventh and eighth columns separately. We take the minutes found from the seventh column, multiply them into the difference

[^1]: As Ptolemy says (pp. 260 and 264), the entries in this table are calculated at every 6° of argument (i.e. every third entry), the intermediate values being derived by linear interpolation. Note that the values for the third and fourth limits (cols. 5 and 6), though tabulated to 3 significant places, are in fact calculated to only 2 places (for the reason see p. 260 n.69): the calculated values (for args. 6°, 12° etc.) always end in 0 or 30. They are therefore rather inaccurate.


[^3]: Literally ‘either in toto, or the amount proportional to the fraction of an hour’.
265

V 18. Table o f total parallax fo r sun and moon
PARALL.AX TABLE
1

3

4

5

6

r

8

c

Moon
Parallaxes
at First
Limit

Moon
Difference
at Second
Limit

Moon
Parallaxes
at T hird
Limit

M oon
Difference
at Fourth
Limit

Sixtieths
for
Epicycle
at Apogee

Sixtieths
for
Epicycle
at Perigee

Sixtieths
for
Eccentre

2

S un’s
A rgu­
ments Parallaxes
2
4
6

0 0 7
0 0 13
0 0 19

0
0
0

1 54
3 48
5 41

0
0
0

0 23
0 45
1 7

0
0
0

0
0
0

0
0
0

0 50
1 40
2 30

0 14
0 28
0 42

0 11
0 22
0 33

0 15
0 30
0 45

8
10
!2

0 0 25
0 0 31
0 0 37

0 7 34
0 9 27
0 !! !9

0
0
0

1 29
1 51
2 !2

0 11 40
0 14 20
n 17 0

0
0
0

3 20
4 10
5 0

1 22
2 2
2 42

I 7
1 41
2 15

1 33
2 21
3 9

14
16
18

0 0 42
0 0 48
0 0 53

0 13 10
0 15 0
0 16 49

0
0
0

2 33
2 54
3 15

0 19 40
0 22 20
0 25 0

0
0
0

5 50
6 40
7 30

3 35
4 28
5 21

3 13
4 11
5 9

4 22
5 35
5 48

20
22
24

0 0 58
0 1 4
0 1 9

0 18 36
0 20 22
0 22 6

0
0
0

3 36
3 57
4 18

0 27 40
0 30 20
0 33 0

0 8 20
0 9 10
0 10 0

6 39
7 57
9 15

6 25
7 41
8 57

8 25
10 2
11 39

26
28
30

0 1 14
0 1 20
0 1 25

0 23 49
0 25 30
0 27 9

0
0
0

4 39
4 59
5 18

0 35 20
0 37 40
0 40 0

0 10 50
0 11 40
0 12 30

10 50
12 25
14 0

10 29
12 1
13 33

13 32
15 25
17 18

32
34
36

0 1 30
0 1 35
0 1 40

0 28 46
0 30 21
0 31 54

0
0
0

5 37
5 55
6 13

0 42 20
0 44 40
0 47 0

0 13 20
0 14 10
0 15 0

15 52
17 44
19 36

15 22
17 11
19 0

19 23
21 28
23 33

38
40
42

0 1 44
0 1 49
0 1 54

0 33 24
0 34 51
0 36 14

0
0
0

6 30
6 47
7 4

0 49
0 51
0 53

0
0
0

0 15 40
0 16 20
0 17 0

21 36
23 36
25 36

20 59
22 58
24 57

25 40
27 47
29 54

44
46
48

0 1 58
0 2 3
0 2 8

0 37 37
0 38 57
0 40 14

0
0
0

7 20
7 35
7 49

0 55
0 57
0 59

0
0
0

0 17 40
0 18 20
0 19 0

27 40
29 44
31 48

27
29
31

32 0
34 6
36 12

50
52
54

0 2 12
0 2 16
0 2 20

0 41 28
0 42 39
0 43 45

0
0
0

8 3
8 16
8 29

1 0 40
1 2 20
1 4 0

0 19 40
0 20 20
0 21 0

33 52
35 56
38 0

33 14
35 19
37 24

38
40
42

56
58
60

0 2 23
0 2 26
0 2 29

0 44 48
0 45 48
0 46 46

0
0
0

8 42
8 53
9 3

1 5 20
1 6 40
1 8 0

0 21 20
0 21 40
0 22 0

40
42
44

0
0
0

39 24
41 24
43 24

43 49
45 35
47 21

62
64
66

0 2 32
0 2 34
0 2 36

9 13
9 22
9 31

1 9 20
1 10 40
1 12 0

0 22 20
0 22 40
0 23 0

45 50
47 40
49 30

45 13
47 2
48 51

48 49
50 17
51 45

68
70
72

0 2 38
0 2 40 '
0 2 42

0 47 40 1
1 ^
0 48 30 ! 0
0 49 15 ' 0
0
0 49 57
0
0 50 36
0
0 51 11

9 39
9 46
9 53

1 13
1 14
1 15

0
0
0

0 23 10
0 23 20
0 23 30

50 56
52 22
53 48

50 24
51 57
53 30

52 57
54 9
55 21

74
76
78

0 2 44
0 2 46
0 2 47

0 51 44
0 52 12
0 52 34

0 9 59
0 10 4
0 10 8

1 15 40
1 16 20
1 17 0

0 23 40
0 23 50
0 24 0

54 57
56 6
57 15

54 41
55 52
57 3

56 12
57 3
57 54

80
82
84

0 2 48
0 2 49
0 2 50

0 52 53
0 53 9
0 53 21

0 10 11
0 10 14
0 10 16

1 17 20
1 17 40
1 18 0

0 24 10
0 24 20
0 24 30

57 57
58 39
59 21

57 47
58 31
59 15

58 26
58 58
59 30

86
88
90

0 2 50
0 2 51
0 2 51

0 53 29
0 53 33
0 53 34

0 10 16
0 10 17
0 10 17

1 18 20
1 18 40
1 19 0

0 24 40
0 24 50
0 25 0

59 34
59 47
60 0

59 30
59 45
60 0

59 40
59 50
60 0

3
6
9

1
5
9

9
6
3


found from the fourth column, and (always) add the result to the parallax from the third column. [Likewise] we take the minutes found from the eighth column, multiply them into the difference found from the sixth column, and again (always) add the result to the parallax from the fifth column. Thus we have obtained two parallaxes; we take the difference between these and write it down. Next we take the mean elongation of the moon from the sun, or else the mean elongation of the moon from the point opposite the [mean] sun, whichever of these two distances is the lesser, and entering with this too into the arguments in the first column, take the minutes corresponding to it in the ninth and last column. We multiply these into the difference between the two parallaxes which we wrote down, and (always) add the result to the smaller (that is, the one derived from the third and fourth columns). This sum will give us the moon's parallax as measured along the great circle through the moon and the zenith.

The sun's parallax for a similar situation [i.e. as measured along an altitude circle] is immediately determined, in a simple fashion, (for solar eclipses), from the number in the second column corresponding to the size of the arc from the zenith [to the sun].

Now, in order to determine the parallax with respect to the ecliptic, in both longitude and latitude, at the given time, we again enter, with the same distance of the moon from the meridian in equinoctial hours [as before], into the same part of the Table of Angles [II 13], and take the number of degrees corresponding to that hour, in the third column if the moon is to the east of the meridian, or in the fourth column if it is to the west of the meridian. We examine the result, and if it is less than 90° we write down the number itself; but if it is greater than 90°, we write down its supplement, since that will be the size in degrees of the lesser of the two angles at the intersection [of ecliptic and altitude circle] in question. We double the number written down, and enter with this [doubled] number, and also with its supplement, into the Table of Chords [I 11]. The ratio of the chord of the doubled number to the chord of the supplement will give the ratio of the latitudinal parallax to the longitudinal parallax (for circular arcs of such small size are not noticeably different from straight lines). So we multiply the amounts of the chords in question by the parallax determined with respect to the altitude circle, and divide the products, each separately, by 120. The results of the division give us the separate components of the parallax. The following general rules apply.

For the latitudinal parallax, when the zenith is to the north of the point of the ecliptic then culminating, on the meridian, the [effect of the] parallax will be towards the south of it [the ecliptic]; but when the zenith is to the south of the culminating point, [the effect of] the parallax in latitude will be towards the north.

For the longitudinal parallax: the angles tabulated in the Table [II 13] represent the northernmost of the two angles cut off to the rear of the intersection

\[ \eta' = \begin{cases} \eta - 180 & \text{if } 270 \leq \eta \leq 360 \\ \eta' = 360 - \eta & \text{if } 0 \leq \eta < 90 \end{cases} \]

266 V 19. Use of parallax table

\[ \eta' = \begin{cases} \eta - 180 & \text{if } 270 \leq \eta \leq 360 \\ \eta' = 360 - \eta & \text{if } 0 \leq \eta < 90 \end{cases} \]

For a parallax computation see Appendix A, Example 10.

266 I.e. (see HAMA 114) we take as argument \( \eta' \) (which cannot exceed 90°), derived from the mean elongation \( \bar{n} \) according to the rules 0 ≤ \( \bar{n} \) ≤ 90: \( \eta' = \bar{n} \); 90 ≤ \( \bar{n} \) ≤ 180: \( \eta' = 180 - \bar{n} \); 180 ≤ \( \bar{n} \) ≤ 270: \( \eta' = 360 - \bar{n} \).
of ecliptic [and altitude circle]. Therefore, when the latitudinal parallax is to the north, if the angle in question is greater than a right angle, the effect of the longitudinal parallax will be in advance [i.e. in reverse order] of the signs, but if the angle is less than a right angle, the effect will be towards the rear [i.e. in the order of the signs]. However, when the latitudinal parallax is to the south, the reverse will be true: if the angle in question is greater than a right angle, the longitudinal parallax will be towards the rear [i.e. in the order] of the signs, but if it is less than a right angle, the longitudinal parallax will be in advance.

Our previous demonstrations concerning the sun proceeded on the assumption that it has no perceptible parallax, though we are well aware that the parallax, which, as we subsequently showed, affects the sun also, will make some difference in them. However, we do not think that the resulting error in [predicting] the phenomena will be of sufficient concern to necessitate changing any of the theorems constructed without taking such a small effect into consideration. Similarly, for lunar parallaxes, we considered it sufficient to use the arcs and angles formed by the great circle through the poles of the horizon [i.e. an altitude circle] at the ecliptic, instead of those at the moon's inclined circle. For we saw that the difference which would result at syzygies in which eclipses occur is imperceptible, and to set out the latter would have been complicated to demonstrate and laborious to calculate; for the distance of the moon from the node is not fixed for a given position of the moon on the ecliptic, but undergoes multiple changes both in amount and in relative position.

In order to make clear what we mean, let [Fig. 5.16] ABG be a segment of the ecliptic, AD a segment of the moon's inclined circle, point A the node, and D the centre of the moon. Draw DB at right angles to the ecliptic. Let E be the pole of the horizon, and draw through E the great circle arcs EDZ through the moon's centre, and EB through B. Let arc DH represent the moon's parallax, and through point H draw HÔ at right angles to BD and HK at right angles to BZ. Thus AB represents the true distance [of the moon] in longitude from the node, and AK the apparent distance, while BD represents the true distance in latitude from the ecliptic, and KH the apparent. Furthermore an arc equal to ÔH represents the longitudinal component of parallax (with respect to the ecliptic) derived from DH, and an arc equal to ÔD represents the latitudinal component of parallax.

From the preceding theorems, [we know that] parallax DH can be found if arc ED is given, and both [components of] parallax, ÔD and ÔH, if Z GZE is given. But what we determined previously was the arcs and angles formed at given points of the ecliptic by the altitude circle; and the only point on the ecliptic which is given in this situation is B. Hence it is clear that we are using arc EB instead of arc ED, and Z GBE instead of Z GZE.

78 Cf. II 10 p. 105.
79 See the last part of Appendix A, Example 10.
80 I.e., nowhere in Bks. III to V were corrections made to the solar position to account for parallax, although in some cases it would theoretically make a difference (e.g. in observations made with the astrolabe in which both sun and moon were sighted, V 3).
81 Reading δυτὶ τὸῦ H (with Ar. δὺ τοῦ D) for δὺ' αὐτοῦ at H449,16. Suggested by Heiberg and adopted by Manitius.
Now Hipparchus attempted to correct this kind [of inaccuracy] too, but it is apparent that he attacked the problem in a very careless and irrational way.\(^{82}\) For firstly, he does it for [just] a single value of the distance \(AD\), instead of all [possible] values, or a number of values, as would have been appropriate in a situation where one has chosen to be nicely accurate about small [errors]. Furthermore, without realising it, he has fallen into a number of [even] stranger errors. Having also [like us] previously demonstrated [the amounts of] the arcs and angles with respect to [intersections of altitude circles with] the ecliptic, and shown that, if \(ED\) is given, \(DH\) can be found (he shows this in Bk. I of his ‘On parallaxes’), in order to get \(ED\) as a given quantity, he assumes that arc \(EZ\) and \(\angle EZG\) are given (in this way, in Bk. II, he calculates \(ZD\) and takes \(ED\) as remainder [of \(EZ-ZD\)]). However he was misled by his failure to notice that the given point of the ecliptic is not \(Z\) but \(B\), and hence the given arc is not \(EZ\), but \(EB\), and the given angle not \(EZG\) but \(EBG\). Yet it is these [arc \(EZ\) and \(\angle EZG\)] which were the [necessary] starting-points for making even such a partial correction. For in many situations there is a quite noticeable difference between the arc \(ED\) and the arc \(EZ\),\(^{83}\) whereas the difference between \(EB\) (which really is

\(^{82}\) No one has given a satisfactory explanation of the procedure of Hipparchus which Ptolemy alludes to here. Pappus devotes a section of his commentary to it (Rome[I] I 151-5), but his reconstruction of Hipparchus’ method seems entirely fictitious (see HAMA 323-5); there are errors in Rome’s text and notes ad loc.

\(^{83}\) At certain situations (cf. Table II 13) the angle between altitude circle and ecliptic (\(\angle EZA\) in Fig. 5.16) can be close to 180°: then the angle between altitude circle and moon’s orbit (\(\angle EDA\)) will also be close to 180°, and hence \(DZ\) will be a large arc, and the error of taking \(EZ\) for \(ED\) can be
given) and ED is, at most, the amount of the arc BD for any given distance [of the moon] from the node.

The logical procedure for making the correction by a [mathematically] sound method can be displayed as follows.

[First, see Fig. 5.17], let ABG be the ecliptic, and DBE at right angles to it. Let the moon be at either D or E, at a latitudinal distance from the ecliptic ABG which is a given arc, e.g. BD or BE. Then the zenith arcs and the angles are given at point B of the ecliptic, and the [corresponding arcs and angles] at D or E are to be found.

![Fig. 5.17](image)

Now if the position of the ecliptic is such that it is at right angles to the great circle drawn through point Z (which we set as the pole of the horizon) and point B, i.e. ZB, it is obvious that this great circle will coincide with arc DE, and the angles at D and E will not differ from that given at B: for [arcs] drawn through these points [from the zenith] are also at right angles to the ecliptic.

And ZD = ZB - BD
ZE = ZB + BE, where both BD and BE are given.

[Second,] let the ecliptic ABG coincide with the great circle through the zenith. Then if [see Fig. 5.18] we take A as the pole of the horizon and draw AD and AE, these [two arcs] will differ from arc AB, and angles BAD and BAE will differ from [the corresponding angle] in the previous case, which was zero.

considerable, whereas the error of taking EB for ED cannot exceed arc BD which (since ZDBA is right) cannot exceed the inclination of the moon’s orbit, 5°. After this I have excised, at H451,12–13, διὰ τὸ πολὺ μᾶλλον ἐκείνων αὐτὰς μὴ δεδόσθαι, ‘because the former [ED] is even farther from being given than the latter [EZ]’, as an interpolation which is a (very lame) explanation of the preceding (in fact it is a consequence, not a cause). Heiberg’s punctuation of this passage makes it unintelligible: remove the stop after EZΓ (line 9) and insert a comma before πολλαχῇ (line 10).

84 Literally ‘which did not exist’. The angle in question is \( \angle BZD \) in Fig. 5.17.
And AD and AE are given from the quantities AB and BD, BE (we speak in terms of straight lines, since the difference [from arcs] is negligible),

since $AB^2 + BD^2 = AD^2$

$AB^2 + BE^2 = AE^2$

And the angles BAD and BAE can thence be derived.

[Third,] let the ecliptic be inclined [to the altitude circle]. If [Fig. 5.19] we take Z as pole of the horizon and draw ZB, ZHD and ZEO, arc ZB and $\angle ABZ$ will be given, and so again, obviously, will be BD and BE. What we need to be given are arcs ZD and ZE, and angles AHZ, AΩZ. These too are given if perpendiculars DK and EL are drawn to ZB.

For since $\angle ABZ$ is given, and $\angle ABE$ is always a right angle, the right-angled triangles BKD and BLE are given, and so is the ratio of ZB to the sides containing the right angle, since [the ratio of ZB] to the hypotenuse DB and BE is given. Hence there will be given ZD, the hypotenuse [of right-angled triangle ZDK, of which sides ZK and KD are given], and ZE, the hypotenuse [of right-angled triangle ZLE, of which sides ZL and LE are given], and also the angles DZK and EZL, which are the differences from the required angles. For

$\angle AHZ = \angle ABZ + \angle DZB$

and $\angle AΩZ = \angle ABZ - \angle EZL$.

It is clear that, for the same latitudinal distance, the greatest difference [with respect to the arcs and angles at B] will occur

[1] for the angles, when point B itself is the zenith. For if the angle [formed by the altitude circle through the moon] at B is zero, the [arcs] through D and E from the zenith form right angles with the ecliptic;
Errors in parallax due to lunar latitude

Fig. 5.19

[2] for the arcs

[a] in the same situation [i.e. when point B is in the zenith]. For when the arc [from the zenith] to B is zero, the arcs to D and E will be equal in size to the moon’s latitude; also

[b] when the circle through the zenith is perpendicular to the ecliptic. For the difference between arc ZB and ZD or ZE will again be equal to the whole amount of the [lunar] latitude.

But in other situations, in which DE is inclined to ZB, the resultant differences between the arcs and angles will be less. Thus, when the moon’s distance in latitude from the ecliptic is 5°, the greatest difference in the parallaxes [as computed at the ecliptic and at the moon’s orbit] will be about 10 minutes. For the 5°, representing the greatest difference between the arcs, produces that number of minutes [when one enters Table V 18] at the least distance and the greatest difference. But when the moon is at the maximum latitude which it can attain at a solar eclipse, which is about 1°, the difference between the parallaxes will be the same number, [i.e.] 1°, of minutes. And this happens rarely.85

85 To verify these figures, take entries at 5° interval in Table V 18, using cols. 5 and 6 (which are chosen because they give the maximum difference). The rate of change is fastest near zero, hence: for arg. 0, 0 + 0 = 0; for arg. 5°, 0:730 + 0:25 = 0:985 ≈ 10°. For eclipses, which occur at conjunction, we have to take the values from cols. 3 and 4. Here, between 0° and 1°, we find: 0 + 0 = 0; 0:125 + 0:18 = 0:305 (which is closer to 1° than 1°°). The maximum latitude of the moon at a solar eclipse is about 1°, the sum of the apparent radii of the bodies (each about 1°) and the maximum parallax at conjunction (about 1°; see VI 6 p. 293). There is no reason to suspect an interpolation here. With Manitius (p. 447); he has misunderstood the passage, notably mistranslating τὰ Ἰου ἔξηκοστά. H455,15-16.
A convenient method for making the above kind of correction of the angles and arcs, if anyone wants to make it when the [differences] involved are so small, would be as follows.

As a general rule, we double the amount of the angle [between altitude circle and ecliptic], and entering with this as argument into the Table of Chords take the chord corresponding to it, and also the chord corresponding to its supplement. We multiply both of the latter separately by the [moon's] latitude, in degrees, divide each of the products by 120, and record the results [separately]. As for the result derived from the first angle, we subtract it from the relevant arc from the zenith [to the ecliptic] when the moon is on the same side [of the ecliptic] as the zenith, but add it when it is on the opposite side [of the ecliptic to the zenith]. We square the result, add that to the result derived from the supplementary angle, also squared, and take the square root of the sum: this will give us the corresponding arc [ZE or ZD in Fig. 5.19] which is required.

Next we take the result which we recorded from the [second,] supplementary angle, multiply it by 120, and divide the result by the arc we found [ZE or ZD]. With the resulting [chord] we enter into the [body of the] Table of Chords [I 11], take the corresponding arc [in the column of argument], and halve it. If the corrected arc [ZE or ZD] is greater than the original [ZB] we add the result to the amount of the original angle, but if [the corrected arc is] less [than the original], we subtract it: the result will be the corrected angle.

To give an example, in the previous figure [5.20] let arc ZB be 45°, \( \angle ABZ \) 30°, and both arc DB and arc BE 5° in latitude.
Now $\text{Crd} (2 \times 30^\circ) = \text{Crd} 60^\circ = 60^\circ$,  
and $\text{Crd} (180 - 60)^\circ = \text{Crd} 120^\circ \approx 104^\circ$.  
$\therefore BL:LE = BK:DK^{86} = 60:104$, where the hypotenuse [BE or BD] = 120°.  
So we multiply each number by the 5° of the hypotenuse and divide by 120.  
$\therefore KB = BL = 2;30^\circ$  
and $DK = EL = 4;20^\circ$.  
First let us suppose the moon to be at E:  
so we subtract the 2;30° from the 45° of arc ZB, since the moon’s distance in 
latitude is in the same direction as the zenith (i.e. they are either both south or 
both north of the ecliptic).  
Thus arc ZL = 42;30°.  
Secondly, suppose the moon to be at point D. Then we add [2;30°] to the 45°, 
since the relative positions are reversed, and  
$\text{ZK} = 47;30^\circ$.  
We form either $\text{ZL}^2 + \text{EL}^2 = 42;30^2 + 4;20^2$  
or $\text{ZK}^2 + \text{DK}^2 = 47;30^2 + 4;20^2$,  
and get either $\text{ZE} \approx 42;46^\circ$  
or $\text{ZD} \approx 47;44^\circ$.  
We multiply 4;20 by 120 and divide by 42;46 and 47;44 separately.  
Then $\text{EL} \approx 12;8^\circ$ where hypotenuse $\text{ZE} = 120^\circ$  
and $\text{DK} \approx 10\frac{1}{2}^\circ$ where hypotenuse $\text{ZD} = 120^\circ$.  
The arc corresponding to the chord $12;8^\circ$ is about $11\frac{1}{2}^\circ$  
and the arc corresponding to the chord $10\frac{1}{2}^\circ$ is about $10\frac{1}{2}^\circ$.  
Taking half of these, we subtract $\angle EZL$, [namely] $5\frac{1}{2}^\circ$, from $\angle ABZ$, i.e. 30°,  
since arc ZE is less than arc ZB.  
Thus $\angle A\Theta Z = 24\frac{1}{2}^\circ$;  
and we add $\angle DZK$, [namely] $5\frac{1}{2}^\circ$, to the same $[\angle ABZ$, i.e.] 30°,  
since arc ZD is greater than arc ZB.  
Thus $\angle A\Theta Z = 35\frac{1}{2}^\circ$.  
Such is the procedure which was required.  

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86 Change the full stop after $\rho\delta$ at H457.7 to a comma.  
87 Although one might expect that, as Neugebauer states (H.A.M.A 116, which gives an incorrect 
account of Ptolemy’s procedure) that this method, which treats the large spherical triangles ZBD 
and ZBE as plane triangles, would lead to great inaccuracy, this is not so (as I have verified by 
taking the worst possible cases): the reason is that the bases of these triangles are small (BD and BE 
cannot exceed 5°, the maximum lunar latitude).
Book VI

1. {On conjunctions and oppositions of sun and moon} \(^1\)

The next subject we have to treat concerns the syzygies of sun and moon at which eclipses occur. The first topic of this, in turn, is the determination of the true conjunctions and oppositions. Now we do indeed think that the periodic and anomalistic motions which we have [already] established for each of the luminaries are sufficient for the first determination of the above; for these [motions] enable one, if he does not shrink from [the labour of] comparing the individual positions of the luminaries at every appropriate occasion,\(^2\) to compute the places and times of the resulting syzygies, both those taken with respect to the mean motions and the true syzygies, [i.e.] taking the anomaly into account. Nevertheless, in order to provide a more convenient way of finding these [syzygies] too, by having set out in a readily available form the times and places of the mean conjunctions and oppositions, together with the position of the moon in anomaly and latitude at [these] mean times (which are the basis for the correction leading to the true syzygies and thence to the ecliptic syzygies), we constructed tables for this purpose. Their structure is as follows.

2. {Construction of the tables of mean syzygies}

First, we want to begin the epoch of the [synodic] months, like all other epochs, from the first year of Nabonassar. So we divided the mean position [of the moon] in elongation at noon, Thoth 1\(^3\) in the Egyptian calendar in that year, which we showed above [IV 8 p. 205] to be 70;37\(^\circ\) by the mean daily motion in elongation, and found 5;47,33\(^d\). Therefore the previous mean conjunction preceded noon on Thoth 1 by that amount. So the next [mean conjunction] occurred about [29;31,50 - 5;47;33 =] 23;44,17\(^d\) after that noon, i.e. 0;44,17\(^d\) after noon on the 24th.

In 23;44,17\(^d\)

\(^1\) On chs. 1 and 2 see *HAMA* 118-21, Pedersen 220-2.

\(^2\) I.e. at every syzygy (whereas Ptolemy’s tables VI 13 enable one to pick out the syzygies at which eclipses are possible with much less labour).

\(^3\) Here (H462.5) and elsewhere in this chapter (H462.9 and 16; H463.3) most Greek mss. and Pappus’ commentary give *vequmvion* (literally ‘new moon’) to express this date. As Manitius notes (338 n. d), the word is appropriate for the first day of the month in Greek luni-solar calendars, but not in the Egyptian calendar, where the months bear no relationship to the phases of the moon. In all but the last of these places D has α (‘1’), which may well have been Ptolemy’s designation.
mean motion of the sun = 23;23,50°
mean motion of the moon in anomaly = 310;8,15°
mean motion of the moon in latitude = 314;2,21°.

And the mean positions at noon on Thoth 1 were:

longitude of sun:  \( \neq 0;45° \)
distance of sun from its apogee (this is convenient to have): 265;15°
anomaly of moon, counted from the apogee of the epicycle: 268;49°
[argument of] latitude of moon, counted from the northern limit on [the moon's] inclined circle: 354;15°.

Therefore, at the above-mentioned moment of the [first] mean conjunction after the first day [of Thoth],
the distance of the sun and moon in mean longitude from the sun's apogee, namely \( \Xi 5;30° \), was 288;38,50°
the distance of the moon in anomaly from the apogee [of the epicycle] was 218;57,15°
the distance of the moon in latitude from the northern limit was 308;17,21°.

So we will set out, first, a table of conjunctions, containing, again, 45 lines, and 5 columns. On the first line we will put, in the first column, year 1 of Nabonassar; in the second column, the days of Thoth. 24:44,17 (for the sixtieths [of a day] are after noon on the 24th); in the third column the distance from the sun's apogee of the mean position [of sun and moon], 288;38,50°; in the fourth column the moon's distance in anomaly from the apogee [of the epicycle], 218;57,15°; and in the fifth column the [moon's] distance in [argument of] latitude from the northern limit, 308;17,21°.

Now half a mean [synodic] month comprises approximately 14;45,55°, 14;33,12° of solar [mean] motion, 192;54,30° of lunar anomaly, and 195;20,6° of [argument of] latitude; we subtract the above amounts from the [corresponding positions] for the conjunction in question, and put the results, arranged in the same way as before, at the beginning of the second table, which has a structure similar [to the first], but will serve for the oppositions.

The entries are:

days: 9;58,22°
distance from the sun's apogee: 274;5,38°
distance in anomaly from the moon's apogee: 26;2,45°
distance in latitude from the northern limit: 112;57,15°.

Now 25 Egyptian years less 0;2,47,5° contain approximately an integer number of [mean synodic] months; and [in 25 years] the mean motions (beyond complete revolutions) are:
sun: 353;52,34,13°
moon, anomaly: 57;21,44,1°
moon, latitude: 117;12,49,54°.

1 Although the conjunction is only 23;44,17° after epoch, Ptolemy tabulates 24;44,17, i.e. he is here using inclusive reckoning for dates. The convenience of this to the user became so obvious that in his Handy Tables he adopted it generally.

2 The relationship 25 Egyptian years \( \approx 309 \) synodic months was probably known in Egypt long before Ptolemy. For an example of its use in Egypt, and the reasons for dating its origin to the fourth century B.C., see HAMA II 563-64. 309 \times 29;31,50,8,20'' = 2,32,4,57,12,55, which is exactly (not approximately, as Ptolemy implies) 0;2,47,5° short of 25 \times 365 = 2,32,5°.
So we will increase [each line in succession of] the first columns of the two tables by 25 years, and decrease [those of] the second columns by 0;2,47,5, and increase [those of] the remaining columns, the third by 353;52,34,13°, the fourth by 57;21,44,1°, and the fifth by 117;12,49,54°.

Following this we construct a table of years, in 24 lines, and then beneath it another table, of months, in 12 lines, each having the same number of columns as the first [two tables]. In the table for months we will enter on the first line, in the first column, the first month; in the second column, the days in one [synodic] month, 29;31,50,8,20; in the third column, the [mean] motion of the sun during that period, 29;6,23,1°; in the fourth column, the motion of the moon in anomaly [in one synodic month], 25;49,0,8°; and in the fifth, the motion in [argument of] latitude, 30;40,14,9°. The [line to line] increments in this table will be the same as the entries in the first line.

In the table for years we will enter on the first line, in the first column, year 1; in the second column, the number of days [beyond 365] contained in 13 synodic months, 18;53,51,48; in the third column, the increment in sun's motion during that period, 18;22,59,18°; in the fourth column, the moon's motion in anomaly, 335;37,1,51°; and in the fifth column, the motion in latitude, 38;43,3,51°. The [line to line] increments in this table will sometimes be the above 13-month increments, and at other times the 12-month increments. The latter come to:

- Days: 354;22,1,40°
- Sun's [mean] motion: 349;16,36,16°
- Moon's anomalistic motion: 309;48,1,42°
- Moon's latitudinal motion: 8;2,49,42°

This [alternation between 12- and 13-month intervals] is in order that what appears in the table will be the first syzygy in each integer Egyptian year.

In the actual tabular entries it will be sufficient to go only as far as the second sexagesimal [fractional] place. The layout of the tables is as follows.

3. [Tables of conjunctions and oppositions]

[See pp. 278-80.]

4. [How to determine the mean and true syzygies]

So when we want to find the mean syzygies for any given year, we calculate the number of the year in question in the era Nabonassar. Then we determine what combination of 25-year periods (taken from the first or second table, as the

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6 Reading 48 for 48 (18;53,52,48) at H465,10, with D,Ar. Corrected by Manitius.
7 For an explanation of how this principle works for the choice of 12- or 13-month increment see HAMA 120.
8 As Ptolemy says, these tables are computed to 3 sexagesimal fractional places, but rounded to 2 in the actual tabulation.
9 The eclipse limits on p. 280 are those derived later, VI 5 pp. 286-7.
10 See HAMA 121-4, Pedersen 223-6.
11 I.e. we enter with the current year. Cf. p. 276 n.4.
### Table of Conjunctions of Sun and Moon

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### [ECLIPSE] LIMITS OF SUN IN MEAN [LATITUDINAL] MOTION:
from 69°19' to 101°22' and from 258°38' to 290°41°

### [ECLIPSE] LIMITS OF MOON IN MEAN [LATITUDINAL] MOTION:
from 74°48' to 105°12' and from 254°48' to 285°12'

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case may be [i.e. for conjunction or opposition] and single years (taken from the third table) adds up to that number of years, take the entries corresponding to those lines [in the table], and add the entries from [each] successive column separately: for conjunctions we add the entries from the first and third tables, and likewise for oppositions we add the entries from the second and third tables. The sum derived from the entries in the second column will give us the moment of syzygy, counted from the beginning of that year; e.g., if the sum is 24;44, [the syzygy will be] 44 sixtieths of a day after noon on Thoth 24; or, again, if it is 34;44, it will be 44 sixtieths of a day after noon on Phaophi 4. The sum derived from the entries in the third column will give us the [mean] position of the sun in degrees counted from the apogee; the fourth column, the anomaly of the moon counted from the apogee [of the epicycle]; the fifth column, the [argument of] latitude counted from the northern limit. At the same time we can readily calculate the subsequent [syzygies of the year in question], either all, or some, as we choose, in logical fashion, by adding the appropriate entries in the fourth, monthly table. For practical purposes we will always convert the time measurements from sixtieths of a day into equinoctial hours. However, the time in hours resulting from the addition [of the entries] will be expressed in mean solar days, whereas the time expressed in seasonal hours is not always identical with that, but is based on true solar days. So we will correct this too, by calculating the difference due to this effect, by the method indicated above: if the amount of time-degrees corresponding to [the rising-time of] the apparent motion is greater [than the interval in mean motion], we subtract the difference from the total [of hours] derived on the basis of mean solar days, but if it is less, we add it to that total.11

Once we have derived, by the above procedure, the time of mean conjunction or opposition, and the position of each luminar’ in anomaly at that time, it will be easy to determine the time and place of the true syzygy, and also the moon’s position in latitude, by comparing the anomalies of the two bodies. For by applying each anomaly in turn, we calculate the true position of sun, moon and moon’s latitude, at the moment defined by the mean syzygy in question, by means of the equation thus found, and examine these positions. If we find that the bodies are still at the same longitude [for conjunction], or exactly opposite [for opposition], then the time of true syzygy will be the same [as that of mean syzygy]. If not, we take the difference between the bodies in longitude, expressed in degrees, and increase it by a twelfth part of itself,12 to account approximately for the additional motion of the sun [between mean and true syzygy]. We then determine how long, in equinoctial hours, the moon in its anomalistic [i.e. true] motion, takes to cover that interval. If the true longitude

11 Ptolemy here echoes III 9 p. 171. There he expressed the rule in the form necessary for going from true to mean time. Here the case (and the rule) are reversed.
12 This rule is justified by a particular example at VI 5 (p. 286); where Ptolemy, assuming the moon to move 13 times as fast as the sun, calculates that the extra distance required is $\frac{1}{12} + \frac{1}{12} \times \frac{1}{12} = \frac{1}{12}$ of the original. Hence Pedersen (224) assumes that Ptolemy found $\frac{1}{12}$ by summing the convergent series $\frac{1}{12} + \left(\frac{1}{12}\right)^2 + \ldots$. Although the passage VI 5 supports him, one can also derive it without summing a series, as follows: if the moon starting from point A and the sun starting from point B meet at point C, and the moon’s speed is 13 times the sun’s, then $AC = 13BC$, hence $AB$ (the original distance between them) is 12 times BC (the extra distance travelled).
of the moon [at mean syzygy] is less than the true longitude of the sun, we add the result to the time of mean syzygy, but if it [the moon’s longitude] is greater, we subtract the result from the time of mean syzygy. Similarly, if the true longitude of the moon at mean syzygy is less than the sun’s [true longitude], we add the interval in degrees (increased, again, by a twelfth) to both the longitude and the argument of latitude [at mean syzygy], but if it is greater we subtract it from both. Thus we get the time of true syzygy, and the approximate true position of the moon on its inclined circle.¹³

The method of finding the moon’s true hourly motion at the syzygy for any given position is as follows. We enter the table of the moon’s anomaly [IV 10] with the anomaly at the moment in question, take the corresponding equation, and then determine the size of the increment in the equation [at that point] corresponding to an increment of 1 degree in anomaly. We multiply this increment by the mean motion in anomaly in 1 hour, 0;32,40°,¹⁴ and, if the anomaly [with which we entered the table] as argument is in the lines above the greatest equation, we subtract the product from the mean hourly motion in longitude, 0;32,56°, but if [the anomaly] is in the lines below [the greatest equation], we add the product to 0;32,56°. The result will be the moon’s true motion in longitude in one equinoctial hour at that position.¹⁵

Now the above procedure will give us the time of true syzygy at Alexandria, since all epochs have been defined in terms of time as expressed in hours [i.e. counted from noon] with respect to the meridian through Alexandria. But it is easy to find the time of a given syzygy for any place whatever from the time of that syzygy at Alexandria.¹⁶ From the difference in position between the two places, we determine the interval, in degrees, between the meridian through the place required and the meridian through Alexandria. If the meridian through the required place is to the east of the meridian through Alexandria, the phenomenon will appear to be observed there that amount (in time-degrees) later, but if it is to the west, that amount earlier. (Obviously, as always, 15 time-degrees represent 1 equinoctial hour.)

Now that we have explained the above methods, it would be appropriate to follow up with the considerations pertinent to the ecliptic limits for both solar and lunar eclipses. The purpose of this is that if we decide to compute, not all

¹³For a year’s series of computed mean and true oppositions see HAMA 121. 123-4. See also Appendix A, Examples 11 and 12.
¹⁴Reading o Λβ πί for o Λβ ιό (0:32.40.0) at H475.2, and similarly o Λβ πί for o Λβ πί ο (0:32.56.0) at H475.5-6. Supported by D,Ar.
¹⁵For a justification of this rule see Pedersen 226. He objects that it is approximately valid only if the lunar deferent has no eccentricity, i.e. if one uses the simple hypothesis of Bk. IV. But Ptolemy advocates its use only ‘at the syzygy’, and he has already shown that there is no significant difference between the two hypotheses at syzygy (V 10).
¹⁶Omitting the clause (H475, 15-17) δοθέντος τοις κατ’ αὐτήν πλήθους τῶν ἱστημένων ἑως τῆς ἀπὸ τοῦ μεσημβρινοῦ ἀποχής (‘once we are given the distance of it [the syzygy] from the meridian, expressed in equinoctial hours’), a clumsy and confusing interpolation found in all mss.
¹⁷See HAMA 125-9, Pedersen 227-30.
mean syzygies [in a given year], but just those which could fall into the category concerning eclipse prognostications,\(^1^8\) we may have a handy method of deciding which these are from the entry for the moon’s mean position in latitude at each mean syzygy.

Now in the preceding book [V 14, p. 254] we have shown that the moon’s diameter subtends an arc which is 0;31,20° of the great circle drawn about the centre of the ecliptic at the moon’s greatest distance. We calculated this by means of two eclipses which occurred near the apogee of the moon’s epicycle. So now too, when we propose to determine the maximum limits of ecliptic syzygies (which limits are determined by the position of the moon at the perigee of the epicycle), we shall, in this situation too, demonstrate in the same way the size of the arc subtended by the moon’s diameter, by means of two eclipses [this time] from among those which have been observed near the perigee [of the epicycle]. For it is safer to demonstrate this kind of parameter from the actual phenomena.

In the seventh year of Philometor, which is the 574th from Nabonassar, on Phamenoth [VII] 27/28 in the Egyptian calendar [-173 May 0/1], from the beginning of the eighth hour till the end of the tenth in Alexandria, there was an eclipse of the moon which reached a maximum obscuration of 7 digits from the north. So mid-eclipse occurred 2\(\frac{1}{2}\) seasonal hours after midnight, which corresponds to 2\(\frac{1}{4}\) equinoctial hours, since the true position of the sun was \(8\ 61^6\).\(^1^9\) And the time from epoch to mid-eclipse is

\[
\begin{align*}
573 \text{ Egyptian years} & \quad 206 \text{ days} \\
& \quad \left\{ \begin{array}{l}
14\frac{1}{2} \text{ equinoctial hours reckoned simply} \\
14 \text{ equinoctial hours reckoned in mean solar days.}
\end{array} \right.
\end{align*}
\]

At this moment the position of the centre of the moon was as follows:

- mean longitude: \(m\ 7;49^6\)
- true longitude: \(m\ 6;16^2\)
- distance [in anomaly] from the apogee of the epicycle: 163;40°
- distance from the northern limit on the inclined circle: 98;20°.

Hence it is clear that when the moon’s centre is 8;20° from the node (measured along the inclined circle), while the moon is near its least distance [at syzygy], and the centre of the shadow is on the great circle drawn through the moon’s centre at right angles to the inclined circle (which is the position of

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\(^{18}\)The word used here, \(\varepsilon\pi\sigma\mu\nu\mu\alpha\iota\nu\varepsilon\), means ‘prognostication [concerning weather]’ or ‘significance in prognostication’ at HII 204,\(\gamma\) and HII 536,21: 537,8; 540,7. This is a traditional meaning (e.g. Ptolemy, \(\text{Phases, Op. Min.}\) 11,4: 20,5), also applying to the verb \(\varepsilon\pi\sigma\mu\nu\mu\alpha\iota\nu\varepsilon\nu\nu\iota\nu\) (ibid. 31,10: cf. \(\text{Apothelesmatica II 14, ed. Boll-Boer 100,17.}\) I therefore assume that meaning wherever it occurs in the Almagest, except in the phrase \(\varepsilon\pi\sigma\mu\nu\mu\alpha\iota\nu\varepsilon\) \(\delta\xi\tau\iota\lambda\sigma\) \(\Pi\lambda\sigma\iota\varepsilon\), HII 188,3, where it means merely ‘deserving note’. There is a good discussion of \(\varepsilon\pi\sigma\mu\nu\mu\alpha\iota\nu\varepsilon\nu\nu\iota\nu\) and related terms in Pfeiffer, \(\text{Studien zum antiken Sternglauben 84-93.}\)

\(^{19}\)Reading \(\zeta\ 5^6\) for \(\zeta\ 5\) (6;4°) at H477,10. The reading is assured by computation (\(\lambda\circ = 8\ 16;13,25^6\) and by the position of the true moon just below. 61 is the reading of AD, Ar and probably all mss. (i.e. the error is Heiberg’s). Corrected by Manitius.

\(^{20}\)This implies an equation of \(-1;33^6\), which agrees fairly well with that derived from an anomaly of 163;40° (below: accurate would be \(-1;32^6\)), if one uses the simple lunar hypothesis. However, if one computes with the full accuracy of the tables V 9, one finds \(\lambda\ 3 = 216;23^6\) (for at true syzygy \(2\pi\) \(\approx 5^1\)), which produces a change in \(a\ of +50^6\, and hence a decrease in the equation of \(4^7\) (precisely the maximum amount by which, according to Ptolemy in V 10 p. 243, the full hypothesis can differ from the simple at syzygy). This also affects the moon’s position on its orbit, which should be 8;22° (rather than 8;20°) from the node.
greatest obscurations), \((\frac{1}{3} + \frac{1}{5})\)th of the moon's diameter is immersed in the shadow.\(^{21}\)

Again, in the thirty-seventh year of the Third Kallippic Cycle, which is the 607th from Nabonassar, Tybi [V] 2/3 in the Egyptian calendar [-140 Jan. 27/28], at the beginning of the fifth hour [of night] in Rhodes, the moon began to be eclipsed; the maximum obscuration was 3 digits from the south.

Here, then, the beginning of the eclipse was 2 seasonal hours before midnight, which corresponds to 2\(\frac{1}{2}\) equinoctial hours in Rhodes and in Alexandria, since the true position of the sun was \(\approx 5;8^\circ\). And mid-eclipse, at which the greatest obscuration occurred, was about 1\(\frac{1}{8}\) equinoctial hours before midnight. The time from epoch to mid-eclipse is 606 Egyptian years 121 days 10\(\frac{1}{2}\) equinoctial hours, whether reckoned simply or in mean solar days.

At this moment the position of the centre of the moon was as follows:

- mean longitude: \(\Omega \ 5;16^\circ\)
- true longitude: \(\Omega \ 5;8^\circ\)\(^{22}\)
- distance [in anomaly] from the apogee of the epicycle: 178;46\(^\circ\)
- distance from the northern limit on the inclined circle: 280;36\(^\circ\).

Hence it is clear that when the moon's centre is 10;36\(^\circ\) (measured along the inclined circle) from the node, while the moon is (as before) near the least distance, and the centre of the shadow is at the intersection of the ecliptic and the great circle drawn through the moon's centre at right angles to the [moon's] inclined circle, then a quarter of the moon's diameter will be immersed in the shadow.\(^{23}\)

But\(^{24}\) when the moon's centre is 8\(\frac{1}{2}\)\(^\circ\) from the node on its inclined circle, it is 43\(\frac{1}{2}\)\(^\circ\), measured along the great circle drawn through the poles of the inclined circle, from the ecliptic; and when it is 10\(\frac{1}{2}\)\(^\circ\) from the node on its inclined circle, it is 54\(\frac{1}{2}\)\(^\circ\), measured along the great circle drawn through the poles of the inclined circle, from the ecliptic. Now the difference [in magnitude] between the two eclipses comprises \(\frac{1}{3}\)rd of the moon's diameter, and the difference in the above two distances of its centre, measured along the same great circle, from the same point of the ecliptic (i.e. the centre of the shadow) is 0;11,47\(^\circ\). So it is clear that the whole diameter of the moon subtends an arc of about 0;35,20\(^\circ\) of the great circle drawn on the centre of the ecliptic at the moon's least distance [at syzygy].

Furthermore, in the second eclipse, in which \(\frac{1}{4}\) of the moon's diameter was

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\(^{21}\) Oppolzer no 1587: mid-eclipse 23;44\(^\circ\) (\(\approx 1;45\) a.m. Alexandria, which is very close to the time of true conjunction one finds from Ptolemy tables), magnitude 7.4 digits.

\(^{22}\) Again (cf. p. 283 n.20) the equation implied, \(-0;8^\circ\), agrees well enough with that derived from the anomaly of 178;46\(^\circ\) according to the simple hypothesis, but application of the full hypothesis produces a significant difference in the true longitude of the moon (\(\Omega \ 5;13^\circ\)) and its position on the orbit (10;42\(^\circ\) from the node instead of 10;36\(^\circ\)).

\(^{23}\) That this eclipse was observed by Hipparchus, as one would expect from the date and place, is confirmed at VI 9 (p. 309). It is Oppolzer no. 1638: time 20;1\(^\circ\) (\(\approx 10\) p.m. Alexandria), magnitude 3.2\(^\circ\), half-duration 58 mins. Ptolemy assumes 30 mins., which is only about half of what he would derive from his own eclipse tables, VI 8. The difficulties associated with the observation and reduction of this eclipse have been much discussed: see Fotheringham [3] 579, with references to older literature, and Britton [1] 94.

\(^{24}\) For the following calculations see \(H.A.M.1\) 105-8, and cf. p. 254 n.61.
obscured, the moon’s centre was 54° from the centre of the shadow and 4\textdegree \text{ from the point at which the line joining the centres [of moon and shadow] intersects the perimeter of the shadow. Hence it is immediately obvious that, by subtraction, the radius of the shadow at the moon’s least distance is 46°. This is negligibly greater than 2\textdegree \times the moon’s radius, which is 17\textdegree. Moreover, the sun’s radius subdends 0;15,40° of the great circle drawn through the sun about the centre of the ecliptic. For, as we demonstrated [V 14], the sun covers the same amount of its circle [i.e. subdends the same angle] as the moon does when it is at its greatest distance at syzygy. Therefore, when the apparent centre of the moon is [0;17,40 + 0;15,40 = ] 0;33,20° from the centre of the sun, [measured orthogonally to the moon’s orbit] on either side of the ecliptic, that is the limiting position in which the moon can just be in apparent contact with the sun.

For example [see Fig. 6.1] let us imagine AB as an arc of the ecliptic and GD as an arc of the moon’s inclined circle. These are sensibly parallel to each other, at least as far as concerns the positions [of the bodies] at the time of eclipses. We
draw the arc of the great circle through the poles of the [moon’s] inclined circle, AEG, and imagine the semi-circle of the sun on centre A, and the semi-circle of the apparent moon on centre E, in such a position that it is just touching the sun at point Z. Then arc AE, which is the distance of E, the apparent centre of the moon, from A, the centre of the sun, can at times be as much as 0;33,20°, as established above. But in the regions stretching from Meroe, where the longest day is 13 equinoctial hours, up to the mouths of the Borysthenes, where the longest day is 16 equinoctial hours, the maximum northward effect of the lunar parallax for the moon at least distance in the syzygies (if we subtract the solar parallax) is about 0;8°, and the maximum southward effect, under the same conditions, is 0;58°. When its [latitudinal] parallax is 0;8° northwards, it has a maximum longitudinal parallax of about 0;30°, round about Leo and Gemini; and when its [latitudinal] parallax is 0;58° southwards, it has a maximum
longitudinal parallax of about 0;15°, round about Scorpius and Pisces. So if we suppose that the true centre of the moon is at D, and draw line DE, which represents the total parallax, DG will (approximately) represent the parallax in longitude, and GE the parallax in latitude.

Therefore, when the moon is to the north of the sun and has a maximum southward parallax, arc DG will be 0;15°, and arc AEG [0;33,20° + 0;58° =] about 1;31°. Now the ratio between the arc from the node to G and the arc GA is about 11 1 : 1 for distances between the eclipse limits: this can easily be seen from our previous demonstration of the inclination of the lunar orbit. So the distance from the node to G will be 17;26°, and GD added to this makes 17;41°.

And when the moon is to the south of the sun and has its maximum northward parallax, arc DG will be 0;30°, and the whole of arc AEG, [0;33,20°+ 0;8° =] 0;41°. By the same kind of calculation as before, the distance from the node to G will be 7;52°, and the total distance, including arc GD, 8;22°.

Therefore, the limiting positions, in which the moon can just be in apparent contact with the sun, for the above regions of our part of the inhabited world, are when the true distance of the centre of the moon from either of the nodes on its inclined circle is 17;41° towards the north, or 8;22° towards the south.

Furthermore, since, as we showed, the maximum equation of anomaly is 2;23° for the sun and 5;1° for the moon near the syzygies, it will at times be possible for the true distance of the moon from the sun at mean syzygies to reach 7;24°. But, in the time the moon takes to traverse the distance [7;24°], the sun will traverse an extra distance of about 1/11th of that amount, i.e. 0;34°; and again, while the moon is traversing that extra 0;34°, the sun will traverse an extra 1/11th of that, or about 0;3° (a 1/11th of the latter is negligible). So if we add the sum, 0;37° (which is 1/11th of the original 7;24°) to the 2;23° of the solar [equation of] anomaly, we get 3°, which is, approximately, the maximum difference in longitude and [argument of] latitude between mean position [of the bodies] at mean syzygy and their true position [at true syzygy]. So the limiting positions in which the moon can just be in apparent contact with the sun, for the above regions, are when the mean distance of the centre of the moon from [either of] the nodes on its inclined circle is 20;41° to the north, or 11;22° to the south. And by the same argument, the above effect can take place in the regions in question only when the amount of the distance of the moon from the northern limit corresponding [in the fifth column of Table VI 3] to the mean syzygy falls between 69;19° and 101;22°, or between 258;38° and 290;41°.

Next, to obtain the moon's ecliptic limits: since, as we showed [p. 284], the moon's radius at its least distance [at syzygy] subtends 0;17,40°, and the

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25 Ptolemy computes the maximum effect of the parallax on ecliptic limits for the region embracing the standard '7 climata' (see Introduction p. 19). There are some serious problems in his (unsupported) statements here, for which see HAMA 127–9.
26 i.e. taking the inclination as 5° (V 12 p. 247), and taking the small spherical triangle formed by the latitude, the ecliptic and the moon's orbit as plane, we compute $\omega : \beta = \text{Crd} 110° : \text{Crd} 10° = 119;32,37 : 10;27,32 = 11.43 : 1 \approx 11 1 : 1$.
27 Cf. p. 281 n.12.
shadow's radius, being about $2\frac{1}{2}$ times that, comes to 0;45,56°, it is clear that when the true distance of the moon's centre is 1;3,36° from the shadow's centre on either side of the ecliptic (as measured along the great circle drawn through the poles of the moon's inclined orbit), or about 12;12° from either of the nodes on its inclined circle (according to the ratio 1 : 11 1/2), that is the limiting position in which the moon can just touch the shadow. And by the same argument as was deduced above from the anomaly, the limiting position for the moon to touch the shadow will be when the distance of the mean moon's centre from the node on its inclined circle is 15;12°. Hence the [mean moon], in distance from the northern limit, must fall within the boundaries 74;48° to 105;12°, or 254;48° to 285;12°.

We will, then, include these numbers for the moon's [argument of] latitude at solar and lunar [eclipse] limits in the preceding table of syzygies, in order to provide a convenient method of determining whether [a given syzygy] could fall into the category of an eclipse.

6. {On the interval of months between eclipses}

In addition to the above, it would also be useful to discuss the problem of the intervals at which, in general, it is possible for ecliptic syzygies to occur, so that, once we have determined a single example of an ecliptic syzygy, we need not apply our examination of the [ecliptic] limits to every succeeding syzygy in turn, but only to those which are separated [from the first] by an interval of months at which it is possible for an eclipse to recur.

Now it is immediately obvious that eclipses of both sun and moon can occur at 6-month intervals, since the increment in the moon's mean motion in [argument of] latitude over 6 months comes to 184;1,25°, and the arcs between the ecliptic limits [at opposite nodes], for both sun and moon, comprise less than the above amount if they are less than a semi-circle, and more than the above amount if they are greater than a semi-circle.

For, in the case of the sun, the ecliptic limits cut off 20;41° (as we showed [p. 286]) to the north of both nodes on the moon's inclined circle, and 11;22° to the south. Thus the arcs on which eclipses cannot occur comprise 138;38° to the north [of the nodes], and 157;16° to the south.

And, in the case of the moon, the ecliptic limits cut off 15;12° [above] of the circle [of the moon's orbit] from the nodes on both sides of the ecliptic. Thus each of the arcs on which eclipses cannot occur comprises 149;36°.

Note that Ptolemy takes precisely $2\frac{1}{2}$ times the moon's radius, instead of the value which he had actually derived from the observations, 0;46°.

See H.1.M.1 129-34. Pedersen 230-1 is too summary to be useful.

For what follows refer to Fig. H, and, for the increments in motion, to Table VI 3. For the moon, $DA = BC = 149;36° < 184;1,25°$, and $AD = CB = 210;24° > 184;1,25°$. For the sun, $BC = 138;38° < 184;1,25°$, $AD = 202;44° > 184;1,25°$, $DA = 157;16° < 184;1,25°$, and $CB = 221;22° > 184;1,25°$. It is necessary that both conditions be fulfilled for it to follow that when the (mean) moon is on one of the ecliptic arcs (AB, CD) at the beginning of the interval it will be on the other (at a distance of 184;1,25°) at the end.

Omitting κατ (with D) at H485,22.
VI 6. Lunar eclipses at 5-month interval possible

On the basis of the theories developed above, it is possible for eclipses of the moon to recur at a 5-month interval which is the longest possible, i.e. an interval in which the sun has the greatest possible motion and the moon the least. We can see that as follows.

In the mean 5-month interval we find the following increments in the motions:

- mean motion in the longitude of both luminaries: 145°32'
- motion of the moon on the epicycle in anomaly: 129°5'.

The sun's 145°32', when its [true] motion is greatest, (i.e. distributed
symmetrically] either side of the perigee, produce an addition to the mean motion of 4;38°. The 129;5° of the moon's anomaly on the epicycle, when its [true] motion is least, [i.e. distributed symmetrically] either side of the apogee, produce a decrement from the mean motion of 8;40°. Therefore over the period of 5 mean synodic months during which the sun has its greatest possible motion and the moon its least, the moon will still be in advance of the sun by the sum of both [above equations of] anomaly, i.e. 13;18°. We take 1\(^{\text{st}}\) of this (for the reasons explained above [p. 286]), and get about 1;6° for the additional motion of the sun before the moon overtakes it. So, since it has an additional 4;38° of motion from its own anomaly, and another 1;6° from the motion needed for overtaking [the sun] at true syzygy, the greatest possible 5-month interval will be greater than the mean by 5;44° of longitude. Hence the moon's additional motion in latitude on its inclined circle will be about the same amount [5;44°] over the mean motion in latitude in 5 months, which comes to about 153;21°. Thus the true motion in latitude over the greatest possible 5-month interval comes to 159;5°.

But the ecliptic limits of the moon for the moon's mean distance enclose about 1° (either side of the ecliptic) of the great circle drawn through the poles of the moon's inclined circle; for at the moon's least distance [the corresponding amount] is 1;3,36°, and at its greatest distance 0;56,24°; thus [the ecliptic limits enclose] 11;30° of the inclined circle either side of the nodes, and hence the anecliptic arc between them comprises 157;0°. This amount is 2;5° less than the 159;5° of the [moon's] inclined circle which is the increment over the greatest possible 5-month interval. From these considerations it is clear that, if one takes the longest possible 5-month interval, the moon can be eclipsed at the opposition at the beginning of that interval, while it is receding from either of the nodes, and then be eclipsed again at the opposition at the end of the interval, while it is approaching the opposite node. The obscuration will take place from the same side of the ecliptic (never from opposite sides) in both eclipses.

Thus we have shown that the longest possible 5-month interval can produce two lunar eclipses. However, it is impossible for this to occur if 7 months intervene, even if we assume the shortest possible 7-month interval, namely that in which the sun has its least motion and the moon its greatest. We can see this by the same method as above.

For in the mean 7-month interval the increments in motion are as follows:

mean motion in longitude of both luminaries: \(203;45°\)

moon's motion on the epicycle: \(180;43°\).

The sun's 203;45°, when its [true] motion is least, [i.e. distributed symmetrically] either side of the apogee, produce a decrement from the mean motion of 4;42°, while the 180;43° of the moon's [anomaly] on the epicycle, when its [true] motion is greatest, [i.e. distributed symmetrically] either side of the perigee, produce an addition to the mean motion of 9;58°. Therefore over the period of 7 months the same result is obtained.

32 I.e. the solar equation is \(-2;19°\) at a solar anomaly of 180°−(145;32±2)°, or 107;14°, and +2;19° at the symmetric position of 252;46°. The corresponding true longitudes are 65;30° greater, or about 20° and \(\pm 20°\), cf. p. 290.

33 See pp. 287 and 254. The amount is the sum of the radii of moon and shadow. At greatest distance this is 0;15,40° + (2\times0;15,40)° = 0;56,24°.
mean synodic months in which the sun has its least possible motion and the moon its greatest, the moon will be beyond the sun by the sum of both [above equations of] anomaly, 14;40°. For the same reason [as before], we take \( \gamma \)\( \frac{\text{th}}{\text{of this}} \) 1;13°, and add it to the decrement due to the sun’s anomaly, 4,42°. The result, 5,55°, gives us the approximate amount by which [the bodies’] motion in longitude over the shortest possible 7-month interval falls short of that over the mean 7-month interval. The moon’s motion in latitude will fall short of that over the mean 7-month interval, 214;42°, by the same amount [5;55°]. So in the least possible 7-month interval the increment in the moon’s latitudinal motion on its inclined circle will be 208;47°. But the total amount of the greatest arc between the [ecliptic] limits of the moon at mean distance, that is the arc between the limit preceding one node and the limit following the other node, is only \( [180° + 2 \times 11;30° = 203°] \). Therefore it is impossible for the moon to be eclipsed at the first opposition of a 7-month interval and then to be eclipsed again, in any way whatever, at the last opposition of that interval, even if it is the shortest possible.

We must now prove that, over the greatest possible 5-month interval, the sun too can be eclipsed twice for observers in the same place, and in all regions of our part of the inhabited world.

In the longest possible 5-month interval, the moon’s increment in [argument of] latitude is, as we have shown [p. 289], 159;5°. And the arc on which solar eclipses cannot occur, for the moon’s mean distance, is 167;36°; for the sun’s ecliptic limits are 0;32,20° from the ecliptic, as measured along the great circle through the poles of the ecliptic, and about 6;12°, as measured along the moon’s inclined circle. So it is clear that, if the moon has no parallax, the event in question [solar eclipses at a 5-month interval] will be impossible, since the anecliptic arc exceeds the motion over the longest possible 5-month interval by 8;31° counted along the [moon’s] inclined circle, which corresponds to about 0;45° on the [great circle] orthogonal to the ecliptic. However, at any place where the moon can attain a parallax so great that the parallax at either of the conjunctions at the two ends [of the interval], or the sum of the parallaxes at both conjunctions combined, exceeds 0;45°, it is possible for the conjunctions at both ends to produce an eclipse at that place.

Now we have shown [p. 289] that, over the period of that mean 5-month interval in which the moon has its least possible motion and the sun its greatest, [which is] from two-thirds through Virgo up to two-thirds through Aquarius, the moon is still in advance of the sun by the sum of both [equations of] anomaly, 13;18°. It takes the moon, in mean motion, 1° 21' to move (13;18° + \( \frac{1}{2} \times 13;18° \)).

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34 The ecliptic limits of the sun are, in latitude, the sum of the radii of the sun (0;15,40°, p. 285) and the moon at mean distance (mean between 0;15,40°, p. 254, and 0;17,40°, p. 285, i.e. 0;16,40°). 0;15,40° + 0;16,40° = 0;32,20°. The corresponding distance from the node is 11\( \frac{1}{2} \) × 0;32,20° = 6;11,50° = 6;12°. So the anecliptic arc is \( (180° - 2 \times 6;12°) = 167;36° \).

35 It is essential to read (with D,Ar) \( \tau\gamma\zeta \mu\gamma\iota\sigma\tau\omega\eta\nu\omega\nu\) at H490,16 for \( \tau\gamma\zeta \mu\gamma\iota\sigma\tau\omega\eta\nu\omega\nu\) ("the greatest 5-month interval"). The meaning is 'the interval of 5 mean synodic months'. The change to \( \mu\gamma\iota\sigma\tau\omega\eta\nu\omega\nu\) was probably made by someone who compared \( \tau\gamma\zeta \mu\gamma\iota\sigma\tau\omega\eta\nu\omega\nu\) (H489,25), where the phrase is in order only because it refers to true synodic months. However, for a purely mechanical confusion between \( \mu\varepsilon\sigma\sigma\nu\mu\gamma\iota\sigma\sigma\tau\nu\upsilon\sigma\upsilon\) compare p. 292 n.43.

36 See p. 289 n.32.

37 In 1° 21' the moon moves 14;24,42° in longitude. 13;18° + 1;6° (p. 289) = 14;24°.
Hence it is clear, since the period of the mean 5-month interval is about 147° 15′, that the period of the longest possible 5-month interval will be 148° 18′. Therefore the last conjunction, which takes place about two-thirds through Aquarius, will be earlier [in the day] than the first conjunction, which takes place about two-thirds through Virgo, by 6 hours (which is the difference [of the above period] from an integer number of days). So we have to search for a place and time at which, if the moon is in Virgo [ca. 20°] and also, 6 hours earlier, in Aquarius [ca. 20°], its parallax exceeds the above-mentioned 0° 45′, that is, either its parallax in one of those signs taken singly, or the combined parallax in both of those signs.

Now we find that the moon's northward parallax never reaches that amount (under the prescribed conditions) in any place in our part of the inhabited world. Hence it is impossible for the sun to be eclipsed twice in the longest possible 5-month interval when the moon's position is to the south of the ecliptic, that is when it is receding from the descending node at the first conjunction and approaching the ascending node at the last. However, it can achieve a southward parallax of this amount, in all regions (beginning almost at the equator, and going northwards), if one takes the combined parallax at both the above signs with a 6-hour difference. This occurs when 20° is at the setting-point at the first conjunction, and 20° in the meridian at the second conjunction. For in those situations we find the following approximate southward parallaxes, for the moon at mean distance (subtracting the solar parallax):

<table>
<thead>
<tr>
<th></th>
<th>D in ( \alpha )</th>
<th>D in ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>at the equator</td>
<td>0° 22′</td>
<td>0° 14′</td>
</tr>
<tr>
<td>where the longest day is 12 hours</td>
<td>0° 27′</td>
<td>0° 20′</td>
</tr>
</tbody>
</table>

Thus already in latter region the combined parallaxes exceed the 0° 45′ in question by 4 minutes. And since the southward parallax increases as one takes regions farther north, it is obvious that there will be an increasing possibility, [as one goes to regions farther north,] for the sun to be eclipsed for the inhabitants of those regions twice in the longest possible 5-month interval. However, this can happen only while the moon's position is to the north of the ecliptic, that is when it is receding from the ascending node at the first eclipse and approaching the descending node at the second.

I say, furthermore, that it is possible for the sun to be eclipsed twice for observers in the same place also in the shortest 7-month interval. For, as we have shown [p. 290], the moon's motion in [argument of] latitude over the shortest 7-month interval is 208° 47′. And the greatest arc of the [moon's] inclined circle intercepted between [two] ecliptic limits (which is the arc between the limit preceding one node and the limit succeeding the opposite node) is, for the sun when the moon is at mean distance, 192° 24′. So it is again clear that, if the moon has no parallax, the event in question cannot take place, since the arc of the [moon's] inclined circle covered in the shortest 7-month

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38 Result of multiplying 29;31;50,8,20′ by 5. More accurate would be 15°.
39 The details of the computation of these are given in the commentary of Pappus (Rome [1] 1 225–9), who finds 0° 29′ instead of 0° 27′.
40 I.e. 180° + 2×6;12′. Cf. p. 290 n.34.
interval exceeds the greatest arc cut off between the sun's ecliptic limits by 16;23°, as measured on the inclined circle, [which corresponds to] 1;25° on the circle through the poles of the ecliptic. But in any place where the moon's parallax is great enough so that the parallax at either of the conjunctions at the two ends [of the interval], or the sum of the parallaxes at both conjunctions combined, exceeds 1;25°, it is possible for the conjunctions at both ends to produce an eclipse at that place.

Now we have shown [p. 290] that, over the period of that mean 7-month interval in which the moon has its greatest [true] motion, and the sun its least, [which is] from the end of Aquarius to the middle of Virgo, the moon, in true motion, has already overtaken the sun by 14;40°. The moon in mean motion traverses (14;40 + \(\frac{2}{3}\) x 14;40)° in 1° 5°. Hence, since the period of the mean 7-month interval comprises about 206° 17°, the period of the shortest possible 7-month interval will be 205° 12°. Therefore, the last conjunction, which takes place about the middle of Virgo, will be 12 hours later [in the day] than the first conjunction, which takes place about the end of Aquarius. So we have to search for a place and time at which the moon's parallax can exceed 1;25°, either at one of those situations singly or at both situations combined, when the two situations are separated by 12 hours, i.e. one sign is setting and the other rising (for otherwise it will be impossible for both eclipses to occur above the horizon).

Now, again, it is impossible for the moon to achieve a northward parallax of that amount for any region in our part of the inhabited world, since, even for those living directly below the equator, the [northward] parallax in latitude at the [moon's] mean distance never exceeds 23 minutes. Hence it is impossible for the sun to be eclipsed twice in the shortest 7-month interval when the moon's position is to the south of the ecliptic. i.e. when it is approaching the ascending node at the first conjunction and receding from the descending node at the last conjunction. But we find that a southward parallax of that amount [i.e. greater than 1;25°] is achieved [for regions north of a latitude which is] approximately the parallel through Rhodes, when the end of Aquarius is rising and the middle of Virgo is setting. For in Rhodes, and those regions beneath the same parallel, at both of the above situations the parallax of the moon at mean distance (with the solar parallax subtracted) is about 0;46° southwards. Thus already in these regions the sum of the parallaxes at both conjunctions is greater than 1;25°. And since for regions yet farther north than this parallel the southward parallax is greater, it is obvious that for the inhabitants of those regions an

41 Cf. p. 289 n.32. Here the longitudes are given by

\[65;30° \mp \{203;45° - 4;42°\} = \{25;58\}\]

\[\mp 15;15°.\]

42 Reading \(\mu\varepsilon\sigma\nu\) (with Ar) for \(\mu\varepsilon\gamma\iota\sigma\sigma\nu\) ('greatest distance') at H494,12. The reading is multiply guaranteed: Ptolemy uses the moon's mean distance throughout this section (cf. pp. 289, 290); taking the greatest distance decreases the parallax (which is in conflict with the argument here). Numerically, from Table V 18, for a zenith distance of 24° (the maximum zenith distance of the ecliptic at the terrestrial equator) the parallax (lunar minus solar) at mean distance is 0;22,6 + \(\frac{1}{3}\) x 0;4,18 = 0;23,6° (likewise at minimum distance it is 0;22,6 + 0;4,18 - 0;1,9 = 0;25,15°, cf. p. 294). Corrected by Manitius.

43 A somewhat unsatisfactory numerical verification of this (using the Handy Tables) is in Pappus' commentary (Rome[1] I 232-4).
eclipse of the sun can be observed twice in the shortest 7-month interval. However, this is, again, possible only when the moon's position is north of the ecliptic, i.e. when it is approaching the descending node at the first eclipse and receding from the ascending node at the second.

It remains for us to prove that it is impossible for the sun to be eclipsed twice at one month's interval in our part of the inhabited world, either [for observers] at the same latitude or at different latitudes, even if one assumes a combination of conditions which could not in fact all hold true at the same time, but which may be lumped together in a vain attempt to provide a possibility of the event in question happening. These assumptions are, that the moon is at least distance (to make its parallax greater); that the month is the shortest possible (so that the amount by which the month's motion in latitude exceeds the distance between the sun's ecliptic limits be as small as possible); and that we use, without analysis [of whether it is a possible situation], those times and zodiacal signs in which the moon's apparent parallax is greatest.

Now in 1 mean synodic month the mean motions of the bodies are as follows:

increment of motion in longitude for both luminaries: 29;6°

moon's [anomaly] on the epicycle: 25;49°.

The 29;6° of the sun's motion, [when distributed symmetrically] either side of the apogee to produce its least [true] motion, result in an equation of -1;8° from the mean. And the 25;49° of the moon's motion, [when distributed symmetrically] either side of the perigee to produce its greatest [true] motion, result in an equation of +2;28° to the mean. In accordance with our previous demonstration, we take the sum of both equations of anomaly, 3;36°, and add 1/ of this, 0;18°, to the amount by which the sun was behind [i.e. 1;8°]. This gives us 1;26° for the amount by which the motion over the shortest month in longitude and [argument of] latitude is exceeded by that in 1 mean synodic month. Therefore, since the motion in latitude during one mean synodic month is 30;40°, that in the shortest month is 29;14°, which corresponds to about 2;33° on the great circle perpendicular to the ecliptic. But the total amount of [the corresponding distance at] the sun's ecliptic limits when the moon is at least distance is 1;6°, which the shortest-month distance exceeds by 1;27°. Therefore, if the sun is to be eclipsed twice at an interval of 1 month, it would be absolutely necessary either for the moon to have no parallax at one conjunction and more than 1;27° at the other, or, secondly, for the parallaxes at both conjunctions to be in the same direction and for the difference between the parallaxes to be greater than 1;27°, or. [thirdly], for the parallax at one conjunction to be towards the north and the parallax at the other to be towards the south, while their sum exceeded that amount [1;27°]. But nowhere on earth does the moon at syzygy, even at its least distance, have a latitudinal parallax of more than 1° (when the solar parallax is subtracted). Therefore it will not be possible for a solar eclipse to occur twice at the interval of the shortest month.

45 As Ptolemy implies, these two conditions cannot both hold: for the moon, to achieve greatest parallax, has to be at the perigee of the epicycle, but to produce the shortest month (see below) has to be at symmetrical positions either side of the perigee.

46 The sum of the radii of sun and moon at least distance is 0;33,20° (p. 285). Ptolemy rounds this to 0;33° and doubles it (since we are dealing with two eclipses).
either when the moon has no parallax at one conjunction or when its parallax is in the same direction at both conjunctions. For the difference between the parallaxes cannot exceed $1^\circ$, and we need $1;27^\circ$. Hence the event in question could occur only under the condition that the two parallaxes are in opposite directions, and that the sum of both exceed $1;27^\circ$. This could happen for parts of the inhabited zones in different [parts of the earth], since it is possible for the southward parallax of the moon in the regions north of the equator, in our part of the inhabited world, and the northward parallax in the regions south of the equator, among the so-called 'antipodes', to reach as much as $1^\circ$ (with subtraction of the solar parallax).  

However, it could never happen in the same part of the inhabited world, since in both [oikoumenai] alike, for those situated directly beneath the equator, the maximum parallax of the moon, both to the north and to the south, does not exceed $25^\circ$, and for those at the extreme north, or extreme south [respectively of their oikoumene] the parallax in the opposite direction does not exceed the above-mentioned $1^\circ$, so that even in this case [i.e. taking the equator and the extreme northern or southern limits] the sum of the parallaxes is still less than $1;27^\circ$. And since both opposite parallaxes become progressively much smaller in regions between the equator and the other extreme [of each oikoumene], the impossibility becomes ever greater for such regions. Therefore it is impossible for the sun to be eclipsed twice in one month for the same observers anywhere on earth, or for different observers in the same part of the inhabited world. This was what we intended to prove.

By means of the above it has become clear to us which intervals between syzygies should be taken into account when we are examining for eclipses. Now, after having determined the times of mid-eclipse at these [syzygies], and computed the moon's positions at that moment, (the apparent positions at conjunctions and the true positions at oppositions), we want to have a convenient means of determining, from the moon's position in latitude, which of those syzygies will definitely produce an eclipse, and the magnitudes and times of obscuration for these eclipses. To solve this problem we have constructed tables, two for solar eclipses and two for lunar eclipses ([in each case] one for the moon's greatest distance and one for its least distance). The interval which we establish [between successive entries in the tables] is

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47 This was already shown by Hipparchus, as is clear from Pliny, NH II 57, a passage which shows that Hipparchus had anticipated Ptolemy in the investigation of the topic of eclipse intervals. Cf. H.A.M.I 322. The word I have translated 'antipodes' is δυτικοί τάχθες ('[people in] the opposite [part of the earth]'). See LSJ s.v. 2. I have excised δόξα o KE at H498.8. This would have to mean 'to be between the limits of $0;25^\circ$ and $1^\circ$', which is nonsense, since the lower limit is zero. The phrase was interpolated by someone who misunderstood this use of μεταξύ, and took the $25^\circ$ (senseless in this context) from just below.

48 Cf. p. 292 n.43.

49 See H.A.M.I 134-41, Pedersen 231-5.
determined by the amount of obscuration, being \( \frac{1}{12} \)th of the diameter of whichever of the luminaries is eclipsed.\(^{50}\)

The first table for solar eclipses, which covers the interval between the limits of eclipses at the moon’s greatest distance, will be arranged on 25 lines in 4 columns. The first two columns will contain the apparent position of the moon in [argument of] latitude on the [moon’s] inclined circle for each [unit of] obscuration. Since the sun’s diameter is 0;31,20°, and, as was proven (p. 254), the moon’s diameter at its greatest distance is also 0;31,20°, it follows that when the moon’s apparent centre is 0;31,20° from the sun’s centre on the great circle through both their centres, (and thus is 6° from the node along its inclined circle, according to the previous ratio, 11:30 : 1), that will be the situation in which the moon just touches the sun. So in the first line of the first column we put ‘84°’, and in the first line of the second column, ‘276°’; again, in the last line of the first column we put ‘96°’, and in the last line of the second column, ‘264°’.

Furthermore, since the amount of the [moon’s] inclined circle which corresponds to \( \frac{1}{12} \)th of the solar diameter is about 0;30°,\(^{51}\) we increase or decrease the entries in the above-mentioned two columns by that amount, beginning from the lines at both ends and going towards the middle. On the middle line we put ‘90°’ and ‘270°’.

The third column will contain the magnitude of the obscuration. On the two lines at top and bottom we put the ‘0’ representing the touching position, on the two lines next to those ‘1 digit’ (representing \( \frac{1}{12} \)th of the diameter), and so forth for the rest, with an increment [from line to line] of 1 digit up to the middle line, which will receive the entry ‘12 digits’.

The fourth column will contain the distance travelled by the centre of the moon corresponding to each [tabulated] obscuration, without however taking into account either the sun’s additional motion [during the phase of the eclipse] or the moon’s epiparallax [i.e. the change in the moon’s parallax].

The second table for solar eclipses, which covers the interval between the limits of eclipses at the moon’s least distance, will be arranged in the same way as the first, except that it will have 27 lines in 4 columns. The moon’s radius at its least distance is, as we have shown (p. 284), 0;17,40° where the sun’s radius is 0;15,40°. So when the moon [at least distance] is just touching the sun, its apparent centre is 0;33,20° from the sun’s centre, and 6;24° from the node along its inclined circle. So\(^{52}\) the entries for the apparent [argument of] latitude in the top and bottom lines are ‘83;36°, 276;24°’, and ‘96;24°, 263;36°’ [respectively],

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50 I.e. the intervals between successive arguments in the tables (cols. 1 and 2 in Table VI 8) is determined by taking integer values of the magnitude (col. 3), in contrast with the normal procedure, in which one takes the argument at purely arbitrary intervals. This is more of a convenience for the compiler of the tables than for the user, but it persisted in eclipse tables of the Handy Tables and in many of the mediaeval tables derived from them (see e.g. Toomer [10] no. 59 p. 88).

51 \( \frac{0;31,20°}{12} \times 11\frac{1}{2} = 0;30,2 = 0;30 \).

52 Heiberg’s text in this paragraph is in disarray. To produce a logical sequence, insert a strong stop at the end of 501,9, begin the next sentence (καὶ) διὰ (with Ar), remove the strong stop at the end of 501,17, and-excite the γάρ (with D,Ar) in 501,18.
and the entry for the digits on the middle line, if we use linear extrapolation, will be $12^\circ$ digits. For this entry there will also be a duration of totality.  

Each of the lunar [eclipse] tables will be arranged in 45 lines and 5 columns. In the first table we will tabulate the [argument of] latitude for greatest distance of the moon. The moon’s radius at its greatest distance is, as we showed [p. 254], $0;15,40^\circ$, and the radius of the shadow, $0;40,44^\circ$. So, when the moon is just touching the shadow, the moon’s centre is $0;56,24^\circ$ from the shadow’s centre along the great circle through both centres, and $10;48^\circ$ from the node along the [moon’s] inclined circle. So we put, on the first line, ‘$79;12^\circ$’ [in the first column] and ‘$280;48^\circ$’ [in the second column], and on the last line ‘$100;48^\circ$’ and ‘$259;12^\circ$’. By the same reasoning as in the first [solar table], we increase or decrease each line by $0;30^\circ$, which corresponds to $\frac{1}{12}$th of the lunar diameter for that distance.

In the second table we will tabulate the [argument of] latitude for the moon at least distance, at which, as was shown [p. 284], its radius is $0;17,40^\circ$, and the radius of the shadow $0;45,56^\circ$. Therefore, when the moon just touches the shadow, its centre is, by the same argument as before, $1;3,36^\circ$ from the centre of the shadow, and $12;12^\circ$ from the node along the moon’s inclined circle. Hence we put, on the first line, ‘$77;48^\circ$’ and ‘$282;12^\circ$’, and, on the last line, ‘$102;12^\circ$’ and ‘$257;48^\circ$’. and again increase or decrease the entries by the amount corresponding to $\frac{1}{12}$th of the lunar diameter for that distance, [namely] $0;34^\circ$.

The third column [in each table], for the digits, will be arranged in the same way as that in the solar tables. So too will be the succeeding columns, which contain the travel of the moon for each [tabulated] obscuration, namely [the fourth column] for both immersion and emersion, and also [the fifth column] for half of totality.

We computed the travel of the moon tabulated for each obscuration geometrically, but as if [the problem were confined to] a single plane and straight lines, since such small arcs do not differ sensibly from the corresponding chords, and furthermore the moon’s motion on its inclined circle is not noticeably different from its motion with respect to the ecliptic.

[I say this] in case anyone should suppose that we do not realise that, in general, the moon’s motion in longitude is affected by the use of arcs of the inclined circle instead of arcs of the ecliptic, and also that it does not follow that the time of syzygy is exactly the same as the time of mid-eclipse. [To illustrate this, see Fig. 6.2], we cut off from the node A two equal arcs of the circles in question [orbit and ecliptic], AB and AG, join BG and from B draw BD perpendicular to AG. Then it is immediately obvious that, if we suppose the moon at B, when we use arc AG of the ecliptic instead of arc AD, then, since motion with respect to the ecliptic is determined by [the great circle] through the poles of the ecliptic, the difference [in longitude] due to the inclination of the lunar orbit will be GD.

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54 The interval of argument corresponding to 1 digit of eclipse magnitude is $0;30^\circ$ elsewhere in the table. Since the interval here is $0;24^\circ$, the corresponding amount in digits is $\frac{3}{4}$. Accurate computation from the radii $0;17,40^\circ$ and $0;15,40^\circ$ gives the magnitude of the maximum solar eclipse as $12;46^\circ$. The amount beyond 12 digits represents the ‘duration of totality’ ($\mu$ovn), as in lunar eclipses. See also p. 305 n.63.
Or again, if we imagine the sun or the centre of the shadow at B, the time of syzygy will occur when the moon is at G (we can say this since the difference due to the two circles [ecliptic and orbit] is negligible), but the time of mid-eclipse when the moon is at D, since, again, the time of mid-eclipse is defined by the circle through the poles of the moon's orbit. And thus the time of syzygy will differ from the time of mid-eclipse by arc GD.

The reason that we did not take these arcs into account in our derivations of the individual [entries] is that the differences they cause are small and imperceptible. While it would be absurd not to recognise any of these effects, on the other hand, when one considers the resulting complication in the methods necessary to deal with each of them, deliberate neglect of effects small enough to be overlooked both in theory and observation evokes [in the reader] a strong feeling of the advantage of greater simplicity, and no regret, or little, for the resulting error in representing the phenomena. In any case, we find that the arc corresponding to GD does not, in general, exceed 0.5°. This can be demonstrated by means of the same theorem which we used [116] to calculate the difference between arcs of the equator and corresponding arcs of the ecliptic, as defined by a [great] circle drawn through the poles of the equator. And in eclipses [the arc corresponding to GD] does not exceed 2'. For, if we take:

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34 I.e. the two arcs are now interchanged, AB being the ecliptic and AG the moon's orbit. Instead of using the same figure, Ptolemy should have drawn another one, in which GB is perpendicular to AB (i.e. AB ≠ AG). Compare Fig. J (taken from Manitius 452-53), which shows that the true syzygy (at G) precedes the eclipse-middle (at D) before the node, but succeeds it after the node.
VI 7. Computation of 'travel' of moon during eclipse

arc $AB = arc AG = 12^\circ$, which is the maximum amount of the moon's distance [from the node] at eclipses, then $BD$ is about $1^\circ$. And hence $AD$ is about $11;58^\circ$, and, by subtraction, $GD$ is $2^\circ$, which corresponds to less than $\frac{1}{12}$th of an equinoctial hour.\(^{55}\) Scrupulous accuracy about such a small amount is a sign of vain conceit rather than love of truth.

For the above reasons we have computed the travel of the moon during the obscurations in question as if the circles [of ecliptic and orbit] were sensibly identical. The method of calculation, to give one or two examples, is as follows.

Let [Fig. 6.3]\(^ {56}\) A be the centre of the sun or the shadow, and $BGD$ the straight line representing the arc of the moon's [inclined] circle. Let the points representing the moon's centre when it is just touching the sun or the shadow be, at the moon's approach [i.e. at first contact] $B$, and at its recession [i.e. at last contact] $D$. Join $AB$ and $AD$, and drop perpendicular $AG$ from $A$ on to $BD$.

![Fig. 6.3](image)

Now it is clear that eclipse middle and greatest obscuration occur when the centre of the moon is at $G$, because [1] $AB$ equals $AD$, and hence the distances travelled, $BG$ and $GD$, are also equal, and because [2] $AG$ is the least of all lines joining the two centres [when the moon is] on $BD$. It is also clear that $AB$ and $AD$ each comprise the sum of the radii of moon and sun or [moon and] shadow, and that each of them exceeds $AG$ by that part of the diameter of the eclipsed body which is cut off by the obscurcation.

This being the case, let the obscuration be, e.g., 3 digits. First let $A$ represent the sun's centre.

Therefore,\(^ {57}\) when the moon is at its greatest distance,

$AB = 31;20$ minutes [p. 295].
\[AB^2 = 981;47.\]

And $AG = 23;30$ minutes, since it is less than $AB$ by $\frac{1}{12}$ths of the sun's diameter, i.e. $7;50$ minutes.

\(^{55}\) Cf. HAMA 83 n.5, estimating a maximum error of 6' as a result of neglecting the inclination of the lunar orbit in computing longitudes. Using the formula $\tan \lambda = \tan \omega \cos t$, I find, for $t = 5^\circ$, the maximum difference between $\lambda$ and $\omega$ is about 6' for $\omega \approx 45;3^\circ$. Using the same formula for $\omega = 12^\circ$, I find $\lambda = 11;57,20^\circ$, hence $GD = 0;2,40^\circ$, which still leads to less than $\frac{1}{12}$th of an hour's difference in the time of mid-eclipse. Ptolemy computes crudely $BD = AB/11 \approx 1$, $AD = \sqrt{12^2 - 11} \approx 11;58$.

\(^{56}\) Figs. 6.3 and 6.4 are elucidated by Figs. K and L respectively, in which the circles representing the sun, moon and shadow are drawn in. These are taken from Manitius, but are also very similar to the alternative diagrams found in ms. D.

\(^{57}\) Reading $\epsilon \tau \mu \nu \delta \rho \alpha$ (with D) for $\epsilon \tau \mu \nu \omega \delta \rho \alpha$ at H507.3.
VI 7. Computation of ‘travel’ of moon during eclipse

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So $AG^2 = 552;15.$

Hence $BG^2 = 429;32.$

and $BG \approx 20;43$ minutes.

This is the amount which we will enter in the fourth column of the first table for solar [eclipses] opposite ‘3 digits’.

For the moon’s least distance, $AB = 33;20$ minutes [p. 295].

$AB^2 = 1111;7.$

And $AG = [0;33.20° - 0;7.50° =] 25;30$ minutes.

so $AG^2 = 650;15.$

And, by subtraction, $BG^2 = 460;52.$

and so $BG = 21;28$ minutes.

This is the amount which we will enter in the fourth column of the second table for solar [eclipses] opposite ‘3 digits’.

Next let A represent the centre of the shadow, and let the obscuration be the same fraction as before, $\frac{1}{3}$, [but now] of the lunar diameter.

Then, for the moon’s greatest distance, $AB = 56;24$ minutes [p. 296],

so $AB^2 = 3180;58.$

and $AG = 48;34$ minutes, since it is less than $AB$ by $\frac{1}{3}$ of the lunar diameter, i.e. (for the moon’s greatest distance) 7;50 minutes.

So $AG^2 = 2358;43.$

Hence, by subtraction, $BG^2 = 822;15.$

and $BG = 28;41$ minutes.

This is the amount which we will enter in the fourth column of the first table for lunar [eclipses] opposite ‘3 digits’. It represents the travel during immersion, which is sensibly equal to that during emersion.

For the [moon’s] least distance,

$AB = 63;36$ minutes [p. 296],

so $AB^2 = 4044;58.$
VI 7. Computation of duration of totality

And $AG = 54;46$ minutes, since the difference [between $AB$ and $AG$], $8;50$ minutes, is, again, $\frac{1}{4}$ of the moon’s diameter, [here] at least distance.

$\therefore AG^2 = 2999;23.$

So, by subtraction, $BG^2 = 1045;35,$

and $BG = 32;20$ minutes.

This is the amount which we will enter opposite ‘3 digits’, as before, in the fourth column of the second table for lunar [eclipses].

Next, to represent those [phases of] the lunar obscurations comprising the duration of totality, let [Fig. 6.4] $A$ be the centre of the shadow, and $BGDEZ$ the straight line standing for the arc of the moon’s inclined circle. Let $B$ represent

![Fig. 6.4](image)

the place of the centre of the moon when it is just externally tangent to the circle of the shadow, at approach, $G$ the place of the centre of the moon when it is just internally tangent to the circle of the shadow at the beginning of totality, $E$ the place of the centre of the moon when it is just internally tangent to the circle of the shadow as [the moon] recedes [at the end of totality], and $Z$ the place of the centre of the moon when it is externally tangent to the shadow at the very end of its emersion [from obscuration]. Again drop perpendicular $AD$ from $A$ on to $BZ$. The same conclusions as before remain valid, and it is furthermore clear

![Fig. L](image)
that AG and AE each comprise the amount by which the radius of the shadow exceeds the radius of the moon. Hence the distance GD is equal to the distance DE, and each represents half of totality, while BG, the remainder [of BD-GD], which represents the immersion, is equal to EZ, the remainder [of DZ-DE], which represents the emersion.

So let us take [for an example] an eclipse for which the entry [in the table] is ‘15 lunar digits’, i.e. one in which D, the moon’s centre [at mid-eclipse], lies $1\frac{1}{2}$ lunar diameters inside the boundary set by the limits of the eclipse. That is to say, when

$$\begin{align*}
(AB - AD) &= (AZ - AD) = 1\frac{1}{2} \text{ lunar diameters} \\
\text{and } (AG - AD) &= (AE - AD) = \frac{1}{2} \text{ lunar diameter}.
\end{align*}$$

Then, for the moon’s greatest distance,
as before [p. 299], $AB = 56;24$ minutes and $AB^2 = 3180;58$.
And $AG = 25;4$ minutes, since the moon’s diameter at greatest distance is $31;20$ minutes.

$$\therefore AG^2 = 628;20,$$
and, by a similar argument, $AD = [56;24 - (31;20 + 7;50) =] 17;14$ minutes and $AD^2 = 296;59$.

So, by subtraction [of $AD^2$ from $AB^2$], $BD^2 = 2883;59$,
and $BD = 53;42$ minutes.

And, by subtraction [of $AD^2$ from $AG^2$], $GD^2 = 331;21$,
and $GD = 18;12$ minutes.

So, by subtraction, $BG = 35;30$ minutes.

So we will put, opposite the entry ‘15 digits’ in the first table for lunar eclipses, in the fourth column ‘35;30 minutes’ for the immersion (which will be the same for the emersion), and, in the fifth column ‘18;12 minutes’ for half the duration of totality.

For the moon’s least distance,
as before [p. 299], $AB = 63;36$ minutes
and $AB^2 = 4044;58$;

$$\begin{align*}
AG &= 28;16 \text{ minutes, since, as was shown, the} \\
\text{moon’s diameter at least distance is } 35;20 \\
\text{minutes,}
\end{align*}$$

and $AG^2 = 799;0$.
And, by similar argument, $AD = [63;36 - (35;20 + 8;50) =] 19;26$ minutes,

$$\therefore AD^2 = 377;39.$$  

Therefore, by subtraction, $BD^2 = 3667;19$,
and $BD = 60;34$ minutes.

And, by subtraction, $GD^2 = 421;21$
and $GD = 20;32$ minutes.

So, by subtraction, $BG = 40;2$ minutes.

Therefore we will put, opposite the entry ‘15 digits’ in the second table for lunar eclipses, in the fourth column ‘40;2 minutes’ for the immersion (which will again be the same for the emersion), and, in the fifth column, ‘20;32 minutes’ for half the duration of totality.

In order to have a convenient way of obtaining the fraction of the difference [between values derived from the first and second tables] for positions of the
moon on the epicycle in between greatest and least distances ([which we do] by the method of sixtieths [of interpolation]), we have drawn up, below the above tables, another little table. This contains, as argument, the position [in anomaly] on the epicycle, and, [as function], the corresponding number of sixtieths to be applied [as interpolation coefficient] in every case to the difference [between values] derived from the first and second eclipse tables.

We have already computed the amounts of these sixtieths for the table of the moon’s parallax [V 18]: they are set out in the seventh column [of that table], since the epicycle has to be taken at the apogee of the eccentre to represent the situation at syzygy.

But most of those who observe the [weather] indications derived from eclipses measure the size of the obscuration, not by the diameters of the disks [of sun and moon], but, on the whole, by [the amount of] the total surface of the disks. Since, when one approaches the problem naively, the eye compares the whole part of the surface which is visible with the whole of that which is invisible. For this reason we have added to the above table yet another little table with 12 lines and 3 columns. In the first column we put the digits from 1 to 12, where each digit represents $\frac{1}{12}$th of the diameter of each luminary, as in the actual eclipse tables. In the other two columns we put twelfths of the whole surface-area corresponding to these [linear digits], those for the sun in the second, and those for the moon in the third. We computed these amounts only for the sizes [of the apparent diameters] for the moon at mean distance, since very nearly the same ratio will result [at other distances], given so small a variation in the diameters. Furthermore, we assumed that the ratio of the circumference to the diameter is $3\frac{1}{2}:1$, since this ratio is about half-way between $3\frac{2}{5}:1$ and $3\frac{3}{5}:1$, which Archimedes used as rough [bounds].

First, to represent solar eclipses, let [Fig. 6.5] the sun’s disk be $ABGD$ on centre $E$, and the disk of the moon at mean distance $AZGH$ on centre $O$, intersecting the sun’s disk at points $A$ and $G$. Join $BE_0H$, and let us suppose that $\frac{1}{16}$ of the sun’s diameter is eclipsed.

Thus $ZD = 3$ where diameter $BD = 12$.

and the moon’s diameter, $ZH \approx 12\frac{20}{24}$ in the same units, according to the ratio $15;40 : 16;40$.

Hence $E\Theta = \left[\frac{1}{16} \times (12 + 12\frac{20}{24}) - 3\right] 9;10$ in the same units.

Therefore the circumferences of the disks are, according to the ratio $1 : 3\frac{1}{2}$.

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58 Reading γινομένων for φαίνομένων ('which appear from') at H512.1. Although found in all Greek mss. and part of the Arabic tradition, the latter is without parallel in the Almagest, and must be replaced by a word like γινομένων (palaeographically close), or συναγομένων. Cf. e.g. H384.21-2, τῶν γινομένων διάφορον εκ τῆς δεύτερας ἀνωμαλίας, H385.5-7, τῶν συναγομένων ὑπεροχήν εκ τῆς ... ἀνωμαλίας. Is has 'allaiti tugraju', which supports my emendation.

59 Although there is no reason to doubt Ptolemy’s statement, I know of no surviving ancient eclipse magnitude which is unambiguously given in these ‘area digits’.


61 The sun’s radius is 0;15.40° (p. 285). The moon’s radius at mean distance is the mean between 0;15.40° and 0;17.40°, i.e. 0;16.40°. But Ptolemy has made a calculating error (cf. Manitius p. 385 n. b) and Pappus, Rome I 11281): 12 × (16;40/15;40) = 12;46, not 12;20. This affects the accuracy of every entry in the second column, but the results are so crudely rounded that it is of little importance.
sun’s circumference: $37;42^\circ$
moon’s circumference: $38;46^\circ$.

Similarly, since the product of the radius and the circumference is twice the area of the circle, the areas of the whole disks are:

- sun’s area: $113;6^p$
- moon’s area: $119;32^p$.

With the above as given quantities, let the problem be to find the area of the surface enclosed by $ADGZ$, where the total area of the sun’s surface is 12 parts.

Join $AE$, $A\Theta$, $GE$, $G\Theta$, and also draw perpendicular $AKG$.

Now, where $E\Theta = 9;10^\circ$, $AE = EG = 6^p$, and $A\Theta = \Theta G = 6;10^\circ$.

Furthermore, the angle at $K$ is right.

Therefore, if we divide $(0A^2 - AE^2)$, or $2;2$, by $E\Theta$, we will get $(K\Theta - EK)$ as $0;13\frac{1}{2}^\circ$.

Hence $EK$ comes out to $4;28^\circ$ and $K\Theta$ to $4;42^\circ$.

Therefore $AK = KG \approx 4^p$.

Accordingly the area of triangle $AEG = 17;52^p$ and the area of triangle $A\Theta G = 18;48^p$.

Furthermore, where diameter $BD = 12^p$ and $ZH = 12;20^\circ$, $AG = 8^p$; so where diameter $BD = 120^p$, $AG = 80^p$, and where diameter $ZH = 120^\circ$, $AG = 77;50^p$.

Therefore the corresponding arcs are:

- arc $ADG = 83;37^\circ$ of circle $ABGD$
- arc $AZG = 80;52^\circ$ of circle $AZGH$.

---

$62$ For $\Theta A^2 - AK^2 = K\Theta^2$, $AE^2 - AK^2 = EK^2$; subtracting, $\Theta A^2 - AE^2 = K\Theta^2 - EK^2 = (K\Theta + EK) (K\Theta - EK) = E\Theta (K\Theta - EK)$.

At H514.20 I read $\gamma^\gamma$ (with $A,D$, $l$s) for $\gamma^{13;3'}$. Corrected by Rome[1] II 262 n. (3), whence Neugebauer in the 2nd edn. of Manitius.
So, since the ratio of a circle to one of its arcs equals the ratio of the area of the whole circle to the area of the sector beneath that arc, 

area of sector AEGD = 26;16\(^{a}\) where area of circle ABGD = 113;6\(^{a}\),
as was shown,
and, in the same units, area of sector ΘGZ = 26;51\(^{a}\)
(for circle AZGH was shown to be 119;32\(^{a}\)).

And, in the same units, we showed that

area of triangle AEG = 17;52\(^{a}\)
and area of triangle ΘG = 18;48\(^{a}\).

Therefore, by subtraction, area of segment ADGK = 8;24\(^{a}\)
and area of segment ΘGZ = 8;3\(^{a}\).

So, by addition, area of AZGD = 15;27\(^{a}\) where area of circle ABGD = 113,6\(^{a}\).

Therefore where the area of the sun's disk equals 12\(^{a}\),
the area of the eclipsed part ≈ 1\(^{a}\).

This is the amount which we will enter in the above-mentioned table in the second column on the line with '3 digits' [as argument].

Again, in the same figure [Fig. 6.5], to represent lunar eclipses, let the moon's disk be ABGD, and the shadow's disk at mean [lunar] distance AZGH, and, as before, let \(\frac{1}{3}\) of the diameter of the moon be eclipsed.

Hence, where diameter BD = 12\(^{a}\), the eclipsed section, ZD = 3\(^{a}\).

And, according to the ratio 2;36 : 1,
the diameter of the shadow, ZH = 31;12\(^{a}\).

Therefore \(EK\) comes to \(\left(\frac{1}{3} \times 12 + 31;12\right) - 3 =\) 18;36\(^{a}\).
So the circumferences are as follows:

- moon's disk: 37;42\(^{a}\)
- shadow's disk: 98;1\(^{a}\)

and the areas are:

- moon's disk: 113;6\(^{a}\)
- shadow's disk: 764;32\(^{a}\).

Here again, where \(E\Theta = 18;36\(^{a}\),
\[\begin{align*}
AE &= EG = 6\(^{a}\), \\
\text{and } \Theta G &= 15;36\(^{a}\)
\end{align*}\]
by assumption.

\[\therefore (K\Theta - EK) = (\Theta A^2 - AE^2) \text{, } E\Theta = 11;8\(^{a}\).\]

H517 So \(EK\) comes out to 3;44\(^{a}\) and \(K\Theta\) to 14;52\(^{a}\).
Hence \(AK = KG = 4;42\(^{a}\).

Accordingly, the area of triangle AEG = 17;33\(^{a}\)
and the area of triangle ΘG = 69;52\(^{a}\).

Furthermore, where diameter BD = 12\(^{a}\) and ZH = 31;12\(^{a}\), AG = 9;24\(^{a}\).

So where diameter BD = 120\(^{a}\), AG = 94\(^{a}\),
and where diameter ZH = 120\(^{a}\), AG = 36;9\(^{a}\).

Therefore the corresponding arcs are:

- arc ADG = 103;8\(^{a}\) of circle ABGD
- and arc AZG = 35;4\(^{a}\) of circle AZGH.

Therefore, by the previous argument,
area of sector AEGD = 32;24\(^{a}\) where, as was shown, area of circle ABGD = 113;6\(^{a}\)
and, in the same units, area of sector AGZ = 74;28\(^{a}\),
since area of circle AZGH was shown to be 764;32\(^{a}\).
And, as we showed, in the same units
area of triangle AEG = 17;33°
and area of triangle A0G = 69;52°.
Therefore, by subtraction, area of segment ADGK = 14;51°
and area of segment AZGK = 4;36°.
So, by addition, the area enclosed by AZGD is 19;27°
where the area of circle ABGD is taken as 113;6°.
Therefore, where the area of the lunar disk is 12°,
the area comprised by its eclipsed section will be about 2
This is the amount which we will enter in the above-mentioned table in the
third, lunar, column, on the line with ‘3 digits’ [as argument].

The layout of the tables is as follows.

8. [Eclipse tables]\(^{93}\)  \[See pp. 306-8.\]

9. [Determination of lunar eclipses]\(^{94}\)

Having set out the above as a preliminary, we can predict lunar eclipses in the
following manner.

We set down the amounts in degrees, computed for the required opposition at
the time of mid-syzygy at Alexandria, of the so-called anomaly, [counted] from
the apogee of the epicycle, and the [argument of] latitude, [counted] from the
northern limit. Having corrected the latter by means of the equation [of
anomaly], we first enter with this corrected [argument of] latitude into the
tables for lunar eclipses. If it falls within the range of the numbers in the first two
columns, we take the amounts corresponding to the argument of latitude in the
columns for the [lunar] travel and the column for the digits [of magnitude] in
both tables, and write them down separately. Then, with the anomaly as

\(^{93}\) There are a number of individual errors in these tables, but it is not always certain which are
due to corruption and which to Ptolemy’s faulty computation. Certain scribal errors (corrected in
the translation) are:

Solar eclipse, least distance col. 4, arg. 90;0. Heiberg ‘H519.20’ prints this ‘following most Greek
ms.’ as κβ 0, i.e. 33;22.0. It was originally two entries, 33;20 (correctly computed) and 2;0,
where the first represents the immersion, and the second the duration of totality (μονή), computed
from the difference between lunar and solar radii, 17;40’ and 15;40’. There is a reference to this on
p. 296 (H501.23), but I suspect both that remark and the entry 2;0 here of being interpolations.
Most Arabic mss. have just 33;20.

Lunar eclipse, least distance col. 5, args. 89;8 and 90;52, read κζ νβ for κζ μβ (27;42) at H521.27 (with
D, Ar) and H521.31 (with Ar). Same col., for arg. 90;0, read κη ξς for κη ξ (28;6) at H521.29, with
D, Ar.

Lunar eclipse, col. 3, for arg. 90;0, text has τέλεια (all mss. except P, which has ‘21’). From the
ratio shadow to moon of 2\(\frac{1}{2}\) : 1 one finds the maximum magnitude of a lunar eclipse as 21;36 digits
in all cases. From Ptolemy’s interpolation method (cf. p. 296 n.53) one finds 21;36 at greatest
distance and about 21;32 at least distance.

\(^{94}\) See HAM.I 138-9 (with computed examples), Pedersen 234-5, and Appendix A, Example 11.
argument, we enter into the correction table, and take the corresponding number of sixtieths. We then take this fraction of the difference between the [two sets of] digits, [derived from] the two tables, which we wrote down, and also of the difference between the [two sets of] minutes of travel, and add the results to the amounts derived from the first table. If, however, it happens that the argument of latitude falls within the range of the second table only, we take [as final result] the appropriate fraction (determined by the number of sixtieths found [from the correction table]) of the digits and minutes [of travel] corresponding [to the argument of latitude] in the second table alone. The number of digits which we find as a result of the above correction will give us the magnitude of the obscuration, in twelfths of the lunar diameter, at mid-eclipse.

As for the minutes [of travel] resulting from the same correction, we always increase them by $\frac{1}{n}$th, to allow for the sun's additional motion [during the phase of the eclipse], and divide the result by the moon's anomalistic [i.e. true] hourly

### TABLE FOR SOLAR ECLIPSES

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### Lunar Eclipses

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motion at that point. The results of the division will give us the duration of each phase of the eclipse in equinoctial hours: the result derived from the fourth column will give the duration of the immersion (and also that of the emersion likewise); and the result derived from the fifth column will give the duration of half of the totality. The times of entry and exit at beginning and end [of the various phases] can be derived immediately by adding or subtracting the individual durations to or from the time of the middle of totality, that is, approximately, the time of true opposition. We can also immediately find the area digits by entering with the digits of the diameter into the final small table.

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55 This will already have been determined in the computation of the time of the true syzygy (cf. p. 282).
and taking the corresponding amount in the third column (and similarly for solar eclipses by taking the corresponding amount in the second column).

Now reason informs us that the time interval from the beginning of an eclipse to its middle is not always equal to the time interval from mid-eclipse to the end, because of solar and lunar anomaly, the effect of which is that equal distances are covered [by the bodies] in unequal times. However, as far as the senses are concerned, no noticeable error with respect to the phenomena would result from supposing these intervals equal in time. For, even when [the luminaries] are near mean speed, where the change [in speed] resulting from an [equal] increment [in the argument] is greater [than elsewhere], the motion over the number of hours represented by the whole duration of [even] the maximum possible eclipse does not exhibit the least noticeable difference [in duration] due to the change [in speed].

Furthermore, we can [now] see, by examining the matter on the above basis, that we were quite right to reject as erroneous the period for the moon's [return in] latitude which Hipparchus demonstrated. [As we saw, p. 207.] the increment [in argument of latitude] between the [two] eclipses which he set out appeared smaller according to his hypothesis, whereas according to our calculations it was found to be greater.\(^6\)

To demonstrate his thesis [of the period of return in latitude], he chose two eclipses with an interval between them of 7160 [synodic] months, in both of which it happened that a quarter of the moon's diameter was eclipsed, at the same distance from the ascending node. The first of these was observed in the second year of Mardokempad and the second in the thirty-seventh year of the Third Kallippic Cycle.\(^6\) In order to demonstrate the return [in latitude], he makes the assumption that each eclipse exhibits the same position in mean argument of latitude,\(^6\) on the grounds that the first eclipse occurred when the moon was at the apogee of the epicycle, and the second when it was at the perigee, and hence, he thought, the anomaly had no effect. However, his first mistake is in this very point, since there indeed was a considerable effect from the anomaly: the mean motion was greater than the true at both eclipses, [and] not by an equal amount, but by about \(1^\circ\) in the first eclipse, and \(\frac{1}{2}^\circ\) in the second eclipse. Thus, in this respect, the period in latitude [between the two eclipses] falls short of an integer number of returns by \(\frac{1}{2}^\circ\) of the moon's orbit. Furthermore, he failed to take into account the effect of the lunar distance on the size of the obscuration, although the difference [due to this effect] was the greatest possible between [precisely] these eclipses, since the first occurred when the moon was at its greatest distance, and the second when it was at its least. For

\(^6\) The increment in argument of latitude over the \(211438^d 23^b\) between the two eclipses mentioned below is, according to Hipparchus' value for the mean motion, only about \(3^\prime\) beyond complete revolutions, but about \(12^\prime\) according to Ptolemy's value.

\(^6\) These are the eclipses of \(-719\) Mar. 8 and \(-140\) Jan. 27, both of which have been used before: see IV 6 p. 191, IV 9 p. 208, and VI 5 p. 284, q.q.v. for details of the anomaly. See also, for the first, Appendix A, Example 11.

\(^6\) Literally 'the same position in latitude is comprised at each of the eclipses, from uniform [motion] (\(\xi\,\delta\mu\alpha\lambda\sigma\omicron\))'. On the assumption that the moon was precisely at apogee and perigee of the epicycle, then (in Hipparchus' simple lunar hypothesis) the true position of the moon coincides with the mean.
the same obscuration, of \( \frac{1}{2} \) [of the diameter], must necessarily result at a lesser distance from the ascending node at the first eclipse, and a greater distance at the second. We have shown that the difference between these distances comes to \( 1^\circ.6^9 \). Hence, in this respect, the period of latitude exceeds an integer number of returns by that amount \([11^2]\). Thus, with respect to the absolute error, the return in latitude would have been out by about two degrees (the sum of the [above] two errors), if it happened that the effect of both had been subtractive or additive. However, since one had the effect of falling short of a return and the other of exceeding a return, by a chance stroke of good luck (perhaps Hipparchus too noticed that these effects counterbalance each other somewhat) it turns out that the [motion in latitude] exceeds an [exact] return by only the difference between the [two] errors, [or] a third of a degree.

10. \( \text{Determination of solar eclipses}^70 \)

Correct prediction of lunar eclipses can be achieved merely by the above, if the computations are carried out accurately in the way described. Solar eclipses, however, with which we deal next, are more complicated to predict because of lunar parallax. We will do it as follows.

We determine the number of equinoctial hours by which the time of true syzygy at Alexandria precedes or follows noon. Then, if the geographical position in question, [i.e.] that of the required place, is different [from that], i.e. if it does not lie beneath the same meridian as Alexandria, we add or subtract the difference in longitude between the two meridians, expressed in equinoctial hours, and [thus] decide how many hours before or after noon the true syzygy occurred at that place too. Then we determine, first, the time of apparent syzygy (which will be approximately the same as mid-eclipse) at the required geographical location, by applying the method of computing parallaxes which we explained previously [V 19], [as follows].

We enter the Table of Angles [II 13] and the Table of Parallaxes [V 18], using [as arguments] the appropriate latitude, distance in hours from the meridian, point on the ecliptic where the conjunction occurred, and also distance of the moon. We thus find, first, the moon's parallax along the great circle drawn through the zenith and the moon's centre. We always subtract from this that solar parallax which is on the same line, and from the result determine, in the way indicated, the component of parallax in longitude by itself, which is computed by means of the angle we found [from the table] between the ecliptic and the great circle through the zenith. We always add to this [longitudinal parallax] the increment of 'epiparallax' corresponding to the number of equinoctial hours represented by the longitudinal parallax. This epiparallax is determined as follows. We take the difference (as determined from the same table) between the parallax corresponding to the original zenith distance and the parallax

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69 From Table VI 8, moon, entries for magnitude 3 digits: greatest distance, \( \omega = 80:42^\circ \), least distance, \( \omega = 79:30^\circ \); difference \( 1:12^\circ \).

70 See Appendix A, Example 12.
corresponding to the zenith distance after the passage of the number of equinoctial hours [represented by the longitudinal parallax]. We take the longitudinal component of this by itself, plus an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax. To the total parallax in longitude, computed in this way, we add \( \frac{1}{10} \) of itself, to account for the additional motion of the sun, and convert the total to equinoctial hours by dividing it by the moon's true hourly motion at the conjunction. If the longitudinal parallax we found is towards the rear [i.e. in the order] of the signs (we explained previously [p. 267] how to determine this), we subtract the amount in degrees which we had converted into equinoctial hours from the moon's position, as previously determined, at the moment of true conjunction, in longitude, latitude and anomaly (each separately): this gives us the [corresponding] true positions of the moon at the moment of apparent conjunction, while the number of hours itself [resulting from the above computation] tells us by how much the apparent conjunction precedes the true. But if the longitudinal parallax we found is in advance [i.e. in the reverse order] of the signs, contrariwise, we add the amount in degrees to the position, as previously determined, at the moment of true conjunction, in longitude, latitude and anomaly (each separately); and the number of hours will give us the amount by which the apparent conjunction is later than the true.

Next, using the same methods, we determine from the distance in equinoctial hours of the apparent conjunction from the meridian, first, what the moon's parallax is measured along the great circle through the moon and the zenith. From the result we subtract the solar parallax for the same argument, and use this result to determine, as before, (by means of the angle formed between the circles [of ecliptic and altitude] at that moment), the latitudinal parallax [i.e. the parallax] along a circle orthogonal to the ecliptic. We convert the result to a distance along [the moon's] inclined circle, i.e. we multiply it by 12. If the effect of the latitudinal parallax is northwards with respect to the ecliptic, we add the result to the previous determined true position in [argument of] latitude at the moment of apparent conjunction when the moon is near the ascending node, but subtract it when the moon is near the descending node. Contrariwise, if the effect of the latitudinal parallax is southwards with respect to the ecliptic, we subtract the distance derived from the parallax from the previously determined position in [argument of] latitude at the moment of apparent conjunction, when the moon is near the ascending node, but add it when the moon is near the descending node.

We thus obtain the amount of apparent [argument of] latitude at the moment of apparent conjunction. With this as argument, we enter the solar eclipse tables, and if our argument falls within the range of the numbers in the

\[^{71}\text{I.e. suppose the original longitudinal parallax to be } l_1; \text{ this gives us a correction to the time of conjunction (for the method of computing which see below), and hence a new zenith distance, which will lead to a new longitudinal parallax } l_2. \text{ Ptolemy's rule is: form } l_2 - l_1 = e. \text{ Then the 'epiparallax' } e' \text{ is given by } e' = e + e (c/l_1), \text{ and the final longitudinal parallax by } l = l_1 + e' = l_1 + (l_2 - l_1) + (l_2 - l_1)^2/l_1.\]

\[^{72}\text{From Ptolemy's earlier practice (e.g. VI 5 p. 286 with n.26) one would expect '11\frac{1}{2}', and this is indeed found in the Arabic tradition (Q, Ger). However, the crudity of the approximation to } 1/\sin 5^\circ \text{ is almost negligible when one considers that the latitudinal parallax is usually small.}\]
first two columns, we can say that there will be a solar eclipse, and that its middle coincides approximately with the moment defining apparent conjunction. So we set down separately the amounts of the [magnitude in] digits and the minutes of immersion and emersion corresponding to the argument of latitude, as derived from each of the two tables, then enter, with the distance of the moon in anomaly from the apogee [of the epicycle] at the apparent conjunction, into the table of correction, take the corresponding number of minutes, and take the corresponding fraction of the difference between each [pair of] results we wrote down. In every case we add the result to the number derived from the first table. The digits found by this procedure give us, again, the amount, in twelfths of the sun's diameter, which will be obscured at approximately mid-eclipse. We increase the minutes of travel [found by this procedure] for both [stretches, i.e. immersion and emersion] by 12th, to account for the sun's additional motion, and convert the result into equinoctial hours [by dividing] by the moon's true [hourly] motion. Thus we have the length of both immersion and emersion: this, however, is on the assumption that the [change in] parallax has no effect on these time-intervals.

Now there is in fact a noticeable inequality in these intervals, due, not to the anomalistic motion of the luminaries, but to the moon's parallax. The effect of this is to make each of the two intervals [immersion and emersion], separately, always greater than the amount derived by the above method, and, generally, unequal to each other. We shall not neglect to take this into account, even if it is small. This phenomenon is due to the fact that the effect of the parallax on the moon's apparent motion is always to produce the appearance of motion which would be in advance (if one were to disregard the moon's proper motion towards the rear). For suppose, first, that the moon's apparent position is before [i.e. to the east of] the meridian: then, as it gradually rises higher [above the horizon], its eastward parallax becomes continually smaller than at the moment preceding, and thus its motion towards the rear appears slower. Or suppose, secondly, that its apparent position is after [i.e. to the west of] the meridian: then, again, as it gradually descends [towards the horizon], its westward parallax becomes continually greater than at the moment preceding, and thus, as before, its motion towards the rear appears slower. For this reason the intervals in question are always greater than those derived by the simple procedure described. Furthermore, the difference between successive parallaxes [at equal intervals of time] becomes greater as one approaches the meridian: hence those intervals [of immersion or emersion] which are nearer the meridian must necessarily become more drawn-out. For this reason. the only situation in which the time of immersion is approximately equal to the time of emersion is when mid-eclipse occurs precisely at noon, for then the appearance of motion in advance resulting from the parallax is about equal on both sides [of mid-eclipse]. But when mid-eclipse occurs before noon, then the interval of emersion is closer to the meridian and [thus] longer, while if mid-eclipse occurs after noon, then the interval of immersion is closer to the meridian and longer.

So in order to correct the time-intervals for this effect, we [first] determine, in
VI 10. Method of correcting solar eclipse phases

the way explained, the uncorrected length of each of the intervals in question, and the zenith distance at mid-eclipse. Suppose, for example, that each interval is 1 equinoctial hour, and the zenith distance 75°. In the Parallax Table [V 18] we look for the minutes of parallax corresponding to the argument 75° (for, e.g., the moon’s greatest distance, for which one takes the entries in the third column). We find, corresponding to 75°, 52’. Since, by hypothesis, the time-intervals of both immersion and emersion, in the mean, is 1 equinoctial hour, or 15 time-degrees, we subtract these 15° from the 75° of the zenith distance, and find the minutes of parallax in the same column corresponding to the resulting 60°, [namely], 47’. Hence the displacement in advance resulting from the parallax at the (average) position nearer the meridian comes to 5’. We also add the [15°] to the 75°, and find the minutes of total parallax corresponding to the resulting 90° in the same column, 53½’. Thus here the displacement in advance resulting from the (parallax at) the position nearer the horizon is 1½’.

We take the longitudinal components of these increments we have found, and convert each [separately] into a fraction of an equinoctial hour by means of the moon’s true motion, as described, and then add each result to the appropriate mean interval, calculated simply, of immersion or emersion; that is, we add the greater to the interval bounded by the position nearer to the meridian, and the lesser to the interval bounded by the position nearer the horizon. It is obvious that the difference between the two intervals in the above example is 3½’, or about ⅓th of an equinoctial hour, which is the time taken by the moon in mean motion to traverse that distance.74

There remains only the readily accomplished task, if we wish, of converting the time in equinoctial hours at each interval into the seasonal hours particular [to the given latitude and date], by the method explained in the earlier part of our treatise [II 9].

11. { On the angles of inclination at eclipses}76

The next topic is the examination of the inclinations77 which are formed at eclipses. This kind of investigation is based both on the inclination of the

74μέσην. If not an interpolation, this must mean, taking the position obtained by applying the 15° of the motion of the heavens in 1 hour directly to the zenith distance. In fact 15° is the maximum possible change in the zenith distance in 1 equinoctial hour. Cf. n.75.

75Ptolemy’s procedure here is, to say the least, crude. Instead of computing the actual zenith distances of the bodies at beginning and end of the eclipse, he simply applies the 15° of one hour’s motion of the heavens to the zenith distance at mid-eclipse. Finding the total parallax from the zenith distance, he applies it as if it were the longitudinal parallax. The procedure is perhaps explicable as illustrating the maximum possible effect of this factor: the longest possible solar eclipse is about 2 hours; to get the maximum parallactic difference between the two intervals we have to take the zenith distance as great as possible. Allowing 15° hourly motion (cf. n.74), 75° is the maximum zenith distance which permits the whole eclipse to be visible. The total parallax is the maximum possible value of the longitudinal parallax. To be consistent, however, Ptolemy should have taken the moon at least distance (for which the difference between parallaxes is greater), i.e. col. 3 + col. 4 in V 18. This would have given him corrections of 6° and 2’, with a difference of 4’ (still only ⅟Ⅲth of an hour).

76On Chs. 11-13 see HAMA 141-4.

77Or "directions", προοεδρος. For other uses of this word see p. 43 n.38 and p. 227 n.19. The purpose of computing these angles was presumably weather prediction; see HAMA II 999.
eclipsed part [of the body] to the ecliptic and on the inclination of the ecliptic itself to the horizon. Both of these angles, during the course of every eclipse phase, undergo great changes as a result of the shift in position [of the bodies], in a way which could not be controlled if one wanted to undertake the task of computing the inclinations throughout the whole of the duration [of the eclipse], a superfluous task, since predictions on such a scale are not in the least necessary or useful. For, since the situation of the ecliptic relative to the horizon is determined from the position on the horizon occupied by its rising or setting points, the angle formed by the ecliptic at the horizon must necessarily change continuously during the course of an eclipse, as those points on the ecliptic which are rising or setting change continuously. Similarly, since the inclination of the eclipsed part [of the body] to the ecliptic is determined from the great circle drawn through the two centres, [i.e.] the centres of moon and shadow or the centres of moon and sun, it is, again, a necessary consequence of the motion of the moon's centre during the course of an eclipse that the circle through the two centres occupy a continuously varying position relative to the ecliptic, and [hence] that the angle formed at their intersection vary continuously. Therefore [the need for this kind of examination will be satisfied if it is carried out only for those points in [the progress of] the eclipse which have some significance, and only roughly for the inclinations with respect to the horizon. [To achieve this kind of accuracy] people who actually observe the eclipse as it occurs could, merely by eye, estimate the important inclinations by looking at the relative positions in both cases [at eclipse and horizon], since, as we said, a rough notion [of the amount] is sufficient in such matters. Nevertheless, not to pass over this topic altogether, we shall try to set out some ways of achieving the kind of result desired as conveniently as possible.

The points in [the progress of] the eclipse which we too take into consideration as deserving to be thought significant are:

1. the point of the start of obscuration, which coincides with the very beginning of the whole eclipse;
2. the point of the completion of obscuration, which coincides with the beginning of the phase of totality;
3. the point of greatest obscuration, which coincides with the middle of totality;\(^\text{78}\)
4. the point of the start of emersion, which coincides with the end of the whole total phase;
5. the point of the completion of emersion, which coincides with the end of the whole eclipse.

The inclinations [with respect to the horizon] which we take into consideration as being more reasonable and more significant are those bounded by the meridian and also those bounded by the rising and setting points of the ecliptic at the equinoxes and at summer and winter solstices. As for the points bounding

\(^{78}\) Reading ἤττες ἐν τῷ μέσῳ χρόνῳ τῆς μονής γίνεται (with D,Ar) for ἤττες ἐν τῷ μέσῳ χρόνῳ τῆς ἐκλάσεως ἄνευ τῆς μονής γίνεται at H537, 12-13. The latter would mean 'which coincides with the middle of the eclipse [for those eclipses] in which there is no total phase'. The interpolation is presumably the remains of a feeble attempt to list all possible cases.
the various 'wind-directions', they may be understood in many different ways by many people; nevertheless, if desired, they can be indicated by means of the angles we set out along the horizon.

Considering the intersections of meridian with horizon, let us make the following definitions:

- the northern intersection is the 'northpoint';
- the southern intersection is the 'southpoint'.

Considering the rising and setting [points of the ecliptic, let us make the following definitions]:

- the intersections of the beginning of Aries or Libra with the horizon are known as 'equinoctial rising' and 'equinoctial setting'; these are always the same distance, [i.e.] a quadrant, from the point where the meridian intersects [the horizon];
- the intersections of the beginning of Cancer [are known] as 'summer rising' and 'summer setting', and the intersections of the beginning of Capricorn as 'winter rising' and 'winter setting'.

The distances [from the meridian intersection] of these last [four] points vary according to the latitude in question. The inclinations are sufficiently characterised by saying that they are at one of the above situations or between some pair of them.

To enable one to determine the position of the ecliptic relative to the horizon for any given situation, we computed, by the method indicated in the first books of our treatise, the distance along the horizon, at rising and setting, of the beginning of each zodiacal sign from the points where the equator intersects [the horizon, computing them] on either side of it [i.e. north or south]. We did this for each of those latitudes from Meroe to Borysthenes for which we [earlier] tabulated the angles [II 13]. To provide a means of readily surveying these, instead of a table, we drew a diagram [Fig 6.7] consisting of 8 concentric circles, conceived as lying in the plane of the horizon, to contain the [various] distances and nomenclature for the 7 climata. Then we drew two lines, at right angles to each other, through all the circles: a horizontal one representing the intersection of the planes of horizon and equator, and another, vertical one representing the intersection of the planes of horizon and meridian. On the innermost circle we wrote, at the ends of the horizontal line 'equinoctial rising' and 'equinoctial setting', and at the ends of the vertical line 'north' and 'south'. Similarly we drew [four] straight lines through all the circles at equal

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79 Greek astronomy sometimes adopted the popular way of indicating the points of the compass by wind-names. These do not occur in the Almagest, except for ἀναφής and ὁλύ in VIII 4 to designate the general directions 'east' and 'west', and in the diagram Fig. 6.7, where they are a later interpolation in the mss., not mentioned in the text (see below n.82). On the systems of wind-names (which do indeed vary) see Rehm, Griechische Windrosen.

80 II 2 p. 77.

81 κατὰ τὸ εὐθεῖας ἔγραφον. One would rather expect διὰ τὸ εὐθεῖας, which is implied by Ishaq's translation.

82 In the figures in the Greek mss. these designations are on the outermost circle; hence Heiberg (at H538.7; cf. ibid. p. VI) emended ἔντος, the reading of all mss., to εὐκτος ('outermost'). But in the Arabic tradition they do appear, all or in part, on the inmost circle, and it seems likely that they were transferred to the outermost circle when the names of the winds were (after Ptolemy) added in the inmost circle (cf. above n.79).
Inclinations either side of the equator [i.e. the horizontal line], and wrote along these, in the seven interlinear spaces, the horizon distance of the solstitial point from the equator which we found for each latitude (in units where one quadrant contains 90°). At the ends where these lines meet the inmost circle we wrote, for the southern ones, ‘winter rising’ and ‘winter setting’, and for the northern ones, ‘summer rising’ and ‘summer setting’. To indicate the signs in between [equinoxes and solstices] we inserted two more lines in each of the four segments, and [wrote] along these the horizon distance from the equator of [the beginning of] the appropriate zodiacal sign, adding the name of each sign on the outermost circle. We also wrote, along the meridian line, for [each] parallel, its name, the length [of the longest day] in hours, and the elevation of the pole. In writing in [the data for all of the above], we began with the largest, outermost circle for the northernmost data, [and so on].

In order to have tabulated the apparent inclinations of the actual phases to the ecliptic, i.e. the angles formed between the ecliptic and the great circle joining the centres in question at each of the significant points mentioned above, we computed these too, for [successive] positions of the moon corresponding to a difference of 1 digit in obscuration. However, we did this only for lunar positions at mean distance (since that is sufficient), and under the assumption that those arcs of the ecliptic and the moon’s inclined circle which we consider for the obscurations are sensibly parallel to each other.

For example, let [Fig. 6.6] line AB represent the arc of the ecliptic, with A as the centre of the sun or the shadow, and let line GDE represent the moon’s inclined circle, with G as the point at which the moon’s centre is at eclipse middle, and D as the point at which the centre is when it is just totally eclipsed or just about to begin emerging from totality (i.e. when the moon is internally tangent to the circle of the shadow). Let E be the point at which the moon’s
centre is when either sun or moon is just beginning to be eclipsed or has just completed emersion (i.e. when the circles are externally tangent). Join AG, AD, AE.

It is obvious that angles BAG and AGE, which correspond to the time of mid-eclipse, are right angles to the senses, and that \( \angle BAE \) represents the angles at the beginning and end of the eclipse, while \( \angle BAD \) represents the angles at the end of [the partial phase of] the eclipse and at the beginning of emersion. And it is immediately clear that AE represents the sum of the radii of both circles, and AD their difference.\(^4\)

Then let us take as an example an eclipse in which half the sun's diameter is obscured at mid-eclipse. Let A be the sun's centre. Then in all cases (since we assume the moon at mean distance) AE comes to [0:15,40° + 0:16,40° =] 0:32,20°, and AG, which is less than that by half the sun's diameter, comes to 0:16,40°.

Therefore, since AG = 16:40° where hypotenuse EA = 32:20°  
(according to the magnitude of obscuration assumed),  
where hypotenuse AE = 120°  
AG = 61:51°,

and, in the circle about right-angled triangle AGE  
arc AG = 62:2°.

\[ \therefore \angle AEG = \angle BAE = \begin{cases} 62:2^\circ \text{ where 2 right angles } = 360^\circ \\ 31:1^\circ \text{ where 4 right angles } = 360^\circ. \end{cases} \]

Again, to take the case of a lunar eclipse, let A be the centre of the shadow. Then, since, as before, we assume the moon at mean distance, AE will always be the same amount, namely [0:43,20° + 0:16,40° =] 60 minutes, and AD, likewise, will always be [0:43,20° - 0:16,40° =] 26:40 minutes. Let the moon be eclipsed in a situation such that the magnitude is 18 digits. Thus AG is again less than AD by half the diameter [of the moon]\(^5\) and, by subtraction [of 16:40° from 26:40°], AG comes to 10:0 minutes.

Then, where hypotenuse AE = 120°. AG = 20:0°,

and, in the circle about right-angled triangle AGE.  
arc AG = 19:12°.  
\[ \therefore \angle AEG = \angle BAE = \begin{cases} 19:12^\circ \text{ where 2 right angles } = 360^\circ \\ 9:36^\circ \text{ where 4 right angles } = 360^\circ. \end{cases} \]

Similarly, where hypotenuse AD = 120°. AG = 45°,

and, in the circle about right-angled triangle AGD.  
arc AG = 44:2°.  
\[ \therefore \angle ADG = \angle BAD = \begin{cases} 44:2^\circ \text{ where 2 right angles } = 360^\circ \\ 22:1^\circ \text{ where 4 right angles } = 360^\circ. \end{cases} \]

In the same way we computed the sizes of the angles for the other [integer] digits [of magnitude], [always taking] that angle which was less than a right angle, in units where one right angle equals 90° (corresponding to the graduation of the quadrant of the horizon). We constructed a table with 22 lines

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\(^4\) Cf. HAM.4 Fig. 124 p. 1244.

\(^5\) See Fig. M (copied from the figure on p. 409 of Manitius). Since the eclipse has a magnitude of 18 digits, by definition \(XY = 6^° = \text{radius of moon.} \) Therefore \(AX = AY - XY = \text{radius of shadow minus radius of moon = AD.} \) Therefore \(AG = AX - XG = \text{AD minus radius of moon.}\)
and 4 columns. The first column contains the digits of actual obscuration, measured along the diameter, found for mid-eclipse; the second contains the angles occurring at solar eclipses at the moment of the beginning of the eclipse and the moment of the end of emersion; the third contains the angles occurring at lunar eclipses at the moments of the beginning of the eclipse and of the end of emersion; and the fourth also contains the angles occurring at lunar eclipses, at the moment of the end of [the partial phase of] the eclipse and the moment of the beginning of emersion. The layout of table and circle [diagram] are as follows.

12. {Display of diagrams for the inclinations\}^{86}

[See pp. 319,320.]

13. {Determination of the inclinations}

Thus, as a preliminary, we determine, by the method explained [VI9-10], the time of each significant point [in the eclipse] listed above, and, from the times, those points on the ecliptic which are rising and setting at those moments, and, from the diagram [Fig. 6.7], the situation [of ecliptic] with respect to the horizon. Then, when the centre of the moon (the apparent centre at solar eclipses and the true centre at lunar eclipses) is exactly on the ecliptic, we get the inclination for a solar eclipse at the beginning of the eclipse, and the inclination for

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86 Corrections to Heiberg:
Arg. 4 digits, col. 3, read \( \varphi \lambda \delta \) for \( \varphi \kappa \zeta \) (54°27') at H544,13. All mss. have the incorrect reading, but it is obviously repeated in error from the line above.
Arg. 14 digits, col. 4, read \( \varphi \beta \kappa \alpha \) for \( \varphi \beta \kappa \delta \) (52°24'), with D,Ar, at H544,23.
A lunar eclipse at the end of the partial phase and also at the end of emersion, from the situation on the horizon of the point of the ecliptic setting at the moment in question; we get the inclination for a solar eclipse at the end of the eclipse, and the inclination for a lunar eclipse at the beginning of the eclipse and the beginning of emersion [i.e. end of totality], from the [horizon situation] of the rising-point of the ecliptic. When the moon’s centre is not exactly on the ecliptic, we take from the table the angles corresponding to the relevant magnitude [of the eclipse] in digits, and apply those angles to the intersection of horizon and ecliptic. If the moon’s centre is north of the ecliptic, we set off the angle to the north of the setting-point for eclipse-beginning in solar eclipses and for the end of the partial phase in lunar eclipses; we set it off to the north of the rising-point for the end of emersion in solar eclipses and the beginning of emersion in lunar eclipses; furthermore we set it off to the south of the rising-point for eclipse-beginning in lunar eclipses, and to the south of the setting-point for eclipse-end in lunar eclipses. If the moon’s centre is south of the ecliptic, we set the angle off to the south of the setting-point for eclipse-beginning in solar eclipses and for end of the partial phase in lunar eclipses; to the south of the rising-point for the eclipse-end in solar eclipses and for the beginning of emersion in lunar eclipses; to the north of the rising-point for

<table>
<thead>
<tr>
<th>Digits</th>
<th>2 Sun Beginning of Eclipse and End of Emersion</th>
<th>3 Moon Beginning of Eclipse and End of Emersion</th>
<th>4 [Moon] End of Partial Phase and Beginning of Emersion</th>
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</thead>
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<tr>
<td>0</td>
<td>90° 0</td>
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<td>1</td>
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<tr>
<td>21</td>
<td></td>
<td>1 36</td>
<td>3 35</td>
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</table>
eclipse-beginning in lunar eclipses; and to the north of the setting-point for eclipse-end in lunar eclipses. The result of this procedure will give us the point on the horizon towards which (speaking roughly, as we said), are inclined those points of the luminaries comprising the significant [moments of the phases], namely the beginning and end of eclipse and of total phase.\(^{87}\)

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\(^{87}\) Literally 'the beginnings and ends of the eclipse and emersion', i.e. beginning of eclipse, end of partial phase (= beginning of totality), beginning of emersion (= end of totality), end of emersion.
1. *That the fixed stars always maintain the same position relative to each other*¹

In the preceding part of this treatise, Syrus, we discussed the phenomena associated with *sphaera recta* and *sphaera obliqua*, and also the details of the hypotheses for the motions of sun and moon and the combinations of positions which are seen to result from them. Now, to deal with the next part of the theory, we shall begin discussing the stars, and first, in accordance with the logical order, the so-called fixed stars.

First of all we must make the following introductory point. Concerning the terminology we use, in as much as the stars themselves patently maintain the formations [of their constellations] unchanged and their distances from each other the same, we are quite right to call them 'fixed'; but in as much as their sphere, taken as a whole, to which they are attached, as it were, as they are carried around, also [like the other spheres] has a regular motion of its own towards the rear and the east with respect to the first [daily] motion,² it would not be appropriate to call this [sphere] too 'fixed'. For we find that both these statements are true, at least on the [observational] basis afforded by the amount of time [preceding us]: even before this Hipparchus conceived of both these notions on the basis of the phenomena available to him, but under conditions which forced him, as far as concerns the effect over a long period, to conjecture rather than to predict, since he had found very few observations of fixed stars before his own time, in fact practically none besides those recorded by Aristylllos and Timocharis, and even these were neither free from uncertainty nor carefully worked out; but we too come to the same conclusions by comparing present phenomena with those of that time, but with more assurance, both because our examination is conducted [with material taken] from a longer time-interval, and because the fixed-star observations recorded by Hipparchus, which are our chief source for comparisons, have been handed down to us in a thoroughly satisfactory form.

First, then, no change has taken place in the relative positions of the stars even up to the present time. On the contrary, the configurations observed in

¹ On. chs. 1 and 2 see Pedersen 237–45.
² Note that the motion which in modern terminology is 'precession of the equinoxes' (i.e. a motion in the direction of decreasing longitudes of the tropical points with respect to the fixed stars) is described by Ptolemy as a motion of the fixed stars with respect to the tropical points in the direction of increasing longitudes. This accords with his taking the tropical points as the primary reference points (III 1 p. 132). Hipparchus, however, seems at times to have adopted the modern convention, to judge from the title of his work 'On the displacement of the solstitial and equinoctial points' (III 1 p. 132 and VII 2 pp. 327 and 329).
Hipparchus' time are seen to be absolutely identical now too. This is true not only of the positions of the stars in the zodiac relative to each other, or of the stars outside the zodiac relative to other stars outside the zodiac (which would [still] be the case if only stars in the vicinity of the zodiac had a rearward motion, as Hipparchus proposes in the first hypothesis he puts forward); but it is also true of the positions of stars in the zodiac relative to those outside it, even those at considerable distances. This can easily be seen by anyone who is willing to make an inspection of the matter and examine, in the spirit of love of truth, whether present phenomena agree with those recorded for Hipparchus' time.

In any case, to provide a convenient test of the matter, we too will adduce here a few of his observations, [namely] those which are most suitable for easy comprehension and also for giving an overview of the whole method of comparison, by showing that the configurations formed by stars outside the zodiac, both with each other and with stars in the zodiac, have been preserved unchanged.

Stars in Cancer. [Hipparchus] records that the star in the southern claw of Cancer (α Cnc), the bright star which is in advance of the latter and of the head of Hydra (β Cnc), and the bright star in Procyon (α CMi) lie almost on a straight line. For the one in the middle lies 1 ½ digits to the north and east of the straight line joining the two end ones, and the distances [from it to each of them] are equal.

Stars in Leo. [He records] that the easternmost two (μ, ε Leo) of the four stars in the head of Leo (μ, ε, κ, λ), and the star in the place where the neck joins [the head] of Hydra (α Hya), lie on a straight line. Also, that the line drawn through the tail of Leo (β) and the star in the end of the tail of Ursa Major (η UMa) cuts off the bright star under the tail of Ursa Major (α CVn) 1 digit to the west [i.e. passes 1 digit to the east of it]. Similarly, [he records] that the line through the star under the tail of Ursa Major and the tail of Leo passes through the more advanced of the stars in Coma (Berenices).

In the following lists I give in brackets the modern designation of the stars in question, when the identification is reasonably certain, and, in footnotes, the equivalent in Ptolemy's catalogue. Several of the stars mentioned by Hipparchus are not recorded in that catalogue, and his descriptions of those that are often differ from Ptolemy's. In Ptolemy's own alignments which follow, the descriptions also vary somewhat from the catalogue. The alignments are discussed in detail by Manitius, 'Fixsternbeobachtungen'.

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2 Catalogue XXXV 6 and 9 and XXXIX 2. Like Manitius, I do not understand 'to the north and east'. In the given situation, the only possible deviation is to the north-west or the south-east. I calculate that in Hipparchus' time it was about 5° to the north and west.
3 The 'digit' (δίκτυακος) and 'cubit' (πινχυς, see p. 323) as astronomical measurements were taken by Hipparchus from Babylonian astronomy (in the Almagest they are found only in the Babylonian observations IX 7, pp. 452-3, and XI 7, p. 541, and in passages derived from Hipparchus). The cubit in Babylonian astronomy can represent either 21° or 2° (the latter normal in the Hellenistic period: see H.A.M.A II 591-93). Strabo, 2.1.18, quotes data from Hipparchus in which the 2° norm is certain. It is also found in Hipparchus' commentary on Aratus, where Vogt, 'Wiederherstellung', col. 30, argued for the 21° norm. In the passage below, a 2° cubit produces a smaller error in the estimated distance (inaccurate in either case). The 'digit' in Babylonian astronomy is 5' of the 2° cubit or 5' of the 21° cubit, 5° in either case.
4 Reading ττξ for ττν (misprint in Heiberg) at H4, 14.
5 Catalogue XXVI 3 and 4 and XLI 6.
6 Catalogue XXVI 127, II 27 and II 28. By my calculation, the line passed more like half a degree to the east of α CVn.
7 The latter are probably catalogue XXVI 33 and 34, doubtfully identified as 15 and 7 Com.
VII 1. Hipparchus' star alignments

Stars in Virgo. [He records] that between the northern foot of Virgo (μ Vir) and the right foot of Bootes (ζ Boo) lie two stars; the southern one of these (109 Boo), which is equally bright as the [right] foot of Bootes, lies to the east of the line joining the feet, while the northern one (31 Boo), which is half-bright, lies on a straight line with the feet. Furthermore, of these two stars, the half-bright one is preceded by two bright stars, which form, together with the half-bright one, an isosceles triangle of which the half-bright one is the apex. These [two bright stars] lie on a straight line with Arcturus (α Boo) and the southern foot of Virgo (λ Vir). Also, that between Spica (α Vir) and the second star from the end of the tail in Hydra (γ Hya) lie three stars, all on one straight line (57, 63, 69 Vir). The middle one of these (63) lies on a straight line with Spica and the second star from the end of the tail in Hydra.

Stars in Libra. [He records] that the star (μ Ser) which is very nearly on a straight line towards the north with the [two] bright stars in the claws (α, β Lib) is bright and triple: for on both sides of it lie single small stars (36, 30 Ser).

Stars in Scorpius. [He records] that the straight line drawn through the rearmost of the stars in the sting of Scorpius (λ Sco) and through the right knee of Ophiuchus (η Oph) bisects the interval between the two advance stars in the right foot of Ophiuchus (36, θ Oph) and that the fifth and seventh joints [in the tail of Scorpius, θ, κ Sco] lie on a straight line with the bright star in the middle of Ara (α Ara). Furthermore, that the northernmost star (σ) of the two in the base of Ara (σ, θ) lies between and almost on a straight line with the fifth joint and the star in the middle of Ara, being almost equidistant from both.

Stars in Sagittarius. [He records] that to the east and south of the Circle under Sagittarius [i.e. of Corona Australis] lie two bright stars (α, β Sgr), quite some distance (about 3 cubits) from each other. The southernmost and brighter of these (β), which is on the foot of Sagittarius, lies very nearly on a straight line with the midmost (α CrA) of the three bright stars in the Circle (which lies furthest towards the east in that [constellation]) (γ, α, β CrA), and with the rearmost (ζ Sgr) of the [two] bright stars (ζ, σ Sgr) at opposite angles of the Quadrilateral [in Sagittarius, ζ, τ, σ, φ]; the two intervals [between these three stars] are equal. The northernmost [of the two stars to the east of the Circle, α Sgr] lies to the east of this straight line, but is on a straight line with the [two] bright stars (ζ, σ) at opposite angles of the Quadrilateral.

10 Catalogue XXVII 26 and V 19.
11 Manitius identifies these two stars as nos. 43 and 46 of Bootes in the catalogue of Heis (Köln, 1872). I have not tracked these down in a more recent catalogue, since any identification seems utterly uncertain.
12 Catalogue V 23 and XXVII 25.
13 Catalogue XXVII 14 and XL1 24.
14 This seems preferable to Manitius' identification (61, 63, 69).
15 The first three are catalogue XIV 11 and XXVIII 1 and 3. My identification of the 'triple star' is far more likely than Manitius' α Ser plus λ, 29 Ser.
16 Catalogue XXIX 20 and XIII 12, 14 and 15.
17 Catalogue XXIX 17 and 19 and XLVI 3.
18 Catalogue XLVI 1 and 2.
19 Catalogue XXX 24 and 23. On the cubit see p. 322 n.5.
20 The equivalents in Ptolemy's catalogue are: α, β Sgr: XXX 24, 23; γ, α, β CrA: XLVII 8, 7, 6; ζ, τ, σ, φ Sgr: XXX 22, 21, 6, 7 (not described as a quadrilateral).
Stars in Aquarius. [He records] that the two stars close together in the head of Pegasus [θ, ν Peg] and the rear shoulder of Aquarius [α Aqr] are almost on a straight line,\(^{21}\) to which the line from the advance shoulder of Aquarius [β Aqr] to the star in the cheek of Pegasus [ε Peg] is parallel.\(^{22}\) Also, that the advance shoulder of Aquarius [β], the bright star [ζ Peg] of the two in the neck of Pegasus [ζ, ξ], and the star in the navel of Pegasus [α And] lie on a straight line, with equal intervals between them.\(^{23}\) Furthermore, that the line through the muzzle [ε] of Pegasus and the easternmost [η Aqr] of the four stars in the vessel [of Aquarius, η, ζ, π, γ]\(^{24}\) bisects, almost at right angles, the line through the two stars [θ, ν] close together in the head of Pegasus.

Stars in Pisces. [He records] that the star [β Psc] in the snout of the southernmost fish [of Pisces], the bright star in the shoulders of Pegasus [α Peg], and the bright star in the chest of Pegasus [β Peg] lie on a straight line.\(^{25}\)

Stars in Aries. [He records] that the advance star [β Tri] in the base of Triangulum lies 1 digit to the east of the straight line drawn through the star in the muzzle of Aries [α Ari] and the left foot of Andromeda [γ And].\(^{26}\) Also, that the most advanced of the stars in the head of Aries [β, γ Ari] and the midpoint of the base of Triangulum [i.e. halfway between β and γ Tri] lie on a straight line.\(^{27}\)

Stars in Taurus. [He records] that the [two] easternmost stars of the Hyades [α, ε Tau] and that star [π¹ Ori] in the pelt held in Orion's left hand which is sixth, counted from the south, lie on a straight line.\(^{28}\) And that the line drawn through the advance eye of Taurus [ε Tau] and the seventh star from the south in the pelt [υ² Ori] cuts off the bright star in the Hyades [α Tau] 1 digit to the north.\(^{29}\)

Stars in Gemini. [He records] that the heads of Gemini [α, β Gem] lie on a straight line with a certain star [ζ Cnc] which lies to the rear of the rearmost head by a distance three times that between the heads, and that the same star also lies on a straight line with the [two] southernmost [θ, δ Cnc] of the four stars [θ, δ, γ, η] round the nebula [Praesepe].\(^{30}\)

In these alignments, and similar alignments which enable us to carry out

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\(^{21}\) Catalogue XIX 15 and 16 and XXXII 2.

\(^{22}\) Catalogue XXXII 4 and XIX 17.

\(^{23}\) Catalogue XIX 11, 12 and 1.

\(^{24}\) Catalogue XXXII 12, 11, 10, 9.

\(^{25}\) Catalogue XXXIII 1 and XIX 4 and 3.

\(^{26}\) Catalogue XXI 2, XXII 14 and XX 15. Using the coordinates for these 3 stars computed by Peters-Knobel (pp. 81-2) for the time of Hipparchus, I find β Tri well over a degree to the east of the line connecting α Ari and γ And. There is no doubt about the identification of the stars.

\(^{27}\) Catalogue XXII 2 and 1 and XXI 2 and 4. I have dubious adopted Manitius' identifications here. However, it seems possible that by 'the midpoint of the base of the triangle' Hipparchus may have been referring to the star δ Tri. This lies approximately on a straight line with λ and β Ari. While γ Ari is 'more advanced' than either of these, Hipparchus may, like Ptolemy, have put that 'on the horn' rather than 'in the head'. λ Ari is not included in Ptolemy's catalogue.

\(^{28}\) Catalogue XXIII 14 and 15 and XXXV 20. Ptolemy counts the stars in the pelt from the opposite direction, the north.

\(^{29}\) Catalogue XXIII 15, XXXV 19 and XXIII 14. Manitius identifies the first star with δ Tau, but not only is this discrepant from Ptolemy's catalogue, but it produces a deviation from the line of about 1° to the north, whereas, if one takes the line from ε Tau to υ² Ori, δ Tau lies about 8' to the north, in good agreement with the equivalence, 1 digit = 5'.

\(^{30}\) Catalogue XXIV 1 and 2; XXIV 25; XXV 3, 5, 4, 2; and XXV 1.
comparisons practically throughout the sphere [of the fixed stars], we see that no change has occurred up to the present time. Yet very noticeable changes would have occurred in the 260 or so years between [Hipparchus and now] if the stars near the ecliptic were the only ones to perform an eastward motion.

But, in order to provide those who come after us with a means of comparison over a longer interval [than was possible for us], from an even larger number of alignments of the above kind, we shall add the most easily recognisable from among those which we have observed but which were not previously recorded. We begin from the

*Stars in Aries.* The two northernmost [α, β Ari] of the three stars in the head of Aries [α, β, γ] and the bright star in the southern knee of Perseus [ε Per] and the star called Capella [α Aur] lie on a straight line.31

*Stars in Taurus.* The line drawn through the star called Capella [α Aur] and the bright star in the Hyades [α Tau] cuts off the star in the advance leg of Auriga [τ Aur] a little to the east.32 Also, the star called Capella [α Aur], the star which is common to the rearmost foot of Auriga and the tip of the northern horn of Taurus [β Tau], and the star in the advance shoulder of Orion [γ Ori] lie on a straight line.33

*Stars in Gemini.* Furthermore, the [two] bright stars in the heads of Gemini [α, β Gem] and the bright star in the neck of Hydra [θ Hya] lie very nearly on a straight line.34

*Stars in Cancer.* Furthermore, the two stars close together in the front leg of Ursa Major [ι, κ UMa], the star on the tip of the northern claw of Cancer [ι Cnc], and the northernmost of the [two] 'Aselli' [γ Cnc] lie on a straight line.35 Similarly, the southern Asellus [δ Cnc], the bright star in Procyon [α CMi], and the bright star between them (which is in advance of the head of Hydra) [β Cnc], lie almost on a straight line.36

*Stars in Leo.* Furthermore, the straight line drawn from the midmost star [γ Leo] of the [three] bright stars in the neck of Leo [ζ, γ, η] to the bright star in Hydra [α Hya] cuts off the star on the heart of Leo [α Leo] a little to the east.37 The [line] from the bright star in the rump of Leo [δ Leo] to the bright star [γ UMa] in the back of the thigh of Ursa Major (which is the southernmost star on the rear side of the quadrilateral), cuts off, a little to the west, the two stars which are close together in the rear paw of Ursa Major [ν, ξ UMa].38

*Stars in Virgo.* Furthermore, the line from the star in the back of the thigh of

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31 Catalogue XXII 14. 2. 1; XI 23: and XII 3.
33 Catalogue XII 3, XXIII 14 and XII 10.
34 Catalogue XII 3, XII 11 and XXXV 3.
35 Catalogue XXIV 1 and 2 and XLI 7.
36 Catalogue II 12 and 13 and XXV 7 and 4. The identifications are certain, but the line through ι and UMa passes far to the east of γ and η Cnc, both now and (according to the coordinates of Peters-Knobel) in Ptolemy's time. I have not computed whether modern proper motions suffice to account for this discrepancy. If Ptolemy had written 'the northernmost of the two stars close together' the alignment would be more plausible.
37 Catalogue XXV 5, XXXIX 2 and XXV 9.
38 Catalogue XXVI 5, 6, 7 (ζ, γ, η Leonis); XLI 12; and XXVI 8.
Virgo [ζ Vir] to the second star from the tip of Hydra's tail [γ Hya] cuts off the star called Spica [α Vir] a little to the west. The line from Spica to the star in the head of Bootes [β Boo] cuts off Arcturus [α Boo] a little to the east. Spica and the two stars on the wings of Corvus [δ, γ Crv] lie on a straight line. Spica, the star in the back of Virgo's thigh [ζ Vir], and the northernmost, bright star [η Boo] of the three in the advance knee of Bootes [η, τ, u] lie on a straight line. 40

[Stars in Libra.] Furthermore, the two bright stars in the claws [α, β Lib] and the star on the tip of Hydra's tail [π Hya] are very nearly on a straight line. The bright star in the southern claw [α Lib], Arcturus [α Boo], and the midmost [ζ UMa] of the three stars in the tail of Ursa Major [ε, ζ, η] lie on a straight line. The bright star in the northern claw [β Lib], Arcturus [α Boo], and the star in the back of the thigh of Ursa Major [γ UMa] lie on a straight line. 41

[Stars in Scorpion.] Furthermore, the star on the rear shin of Ophiuchus [ξ Oph], the star in the fifth tail-joint of Scorpius [θ Sco], and the more advanced [υ] of the two stars close together in its sting [λ, υ] lie on a straight line. The most advanced [σ] of the three stars in the breast of Scorpius [σ, α, τ], and the two stars in the knees of Ophiuchus [η, ζ Oph], form an isosceles triangle, the apex of which is the most advanced of the three stars in the breast. 42

[Stars in Sagittarius.] Furthermore, the star on the front, southern hock of Sagittarius (which is of second magnitude) [β Sgr], the star on the arrow-head [γ Sgr], and the star in the rear knee of Ophiuchus [η Oph] lie on a straight line. The star [α Sgr] in the knee of the same [front] leg of Sagittarius (which lies near Corona Australis), the star on the arrow-head [γ Sgr], and the star in the advance knee of Ophiuchus [ζ Oph] lie on a straight line. 43

[Stars in Capricorn.] Furthermore, the line drawn from the bright star in Lyra [α Lyr] to the stars44 in the horns of Capricorn [α, β, v, ξ Cap] cuts off the bright star in Aquila [α Aql] a little to the east. The line from the bright star in Aquila to the first-magnitude star in the mouth of Piscis Austrinus [α PsA] bisects, approximately, the interval between the two bright stars on the tail of Capricorn [γ, δ Cap]. 45

[Stars in Aquarius.] Furthermore, the line from the first-magnitude star in the mouth of Piscis Austrinus [α PsA] to the star in the muzzle of Pegasus [ε Peg] cuts off the bright star in the rear shoulder of Aquarius [α Aqr], a little to the east. 46

[Stars in Pisces.] Furthermore, the stars in the mouths of Piscis Austrinus [α

40 Catalogue XXVII 15 (ζ Vir), XLI 24 (γ Hya), XXVII 14 (α Vir), V 6 and 23 (β, α Boo), XLI 5 and 4 (δ, γ Cor); and V 20, 21, 22 (η, τ, u Boo).
41 Catalogue XXVIII 1 and 3 (α, β Lib); XLI 25 (π Hya); V 23 (α Boo); II 25, 26, 27 (ε, ζ, η UMa); and II 19 (γ UMa).
42 Catalogue XIII 13 (ξ Oph); XXIX 17, 20, 21 (θ, λ, υ Sco); XXIX 7, 8, 9 (σ, α, τ Sco) and XIII 12 and 19 (η, ζ Oph).
43 Catalogue XXX 23 and 1 (β, γ Sgr); XIII 12 (η Oph); XXX 24 (α Sgr); and XIII 19 (ζ Oph).
44 Reading τοομις, with D, Ar (other Greek mss. τοοις) for Heiberg's emendation τοοις 'the star' at H11, 10. Corrected by Manitius, who supposes the stars to be α and β Cap. But these would not give the correct alignment, and in the catalogue Ptolemy puts both these stars on the same horn. I therefore suppose that he is referring to the general direction from Vega of the group of stars.
45 Catalogue VIII 1 (α Lyr); XXXI 1, 2, 3, 4 (α, β, ζ Cap); XVI 3 (α Aql); XLVIII 1 (α PsA); and XXXI 23, 24 (γ, δ Cap).
46 Catalogue XLVIII 1, XIX 17 and XXXII 2.
PsA] and the southern fish [of Pisces, β Psc] and the [two] advance stars of the quadrilateral in Pegasus [α, β Peg] lie on a straight line.  

If one were to match the above alignments too against the diagrams forming the constellations on Hipparchus’ celestial globe, he would find that the positions of the [relevant stars] on the globe resulting from the observations made at that time [of Hipparchus], according to what he recorded, are very nearly the same as at present.

2. *That the sphere of the fixed stars, too, performs a rearward motion along the ecliptic*

From these considerations, and others like these, we can be assured that absolutely all the so-called fixed stars maintain one and the same position relative [to each other], and share one and the same motion. But the sphere of the fixed stars also performs a motion of its own in the opposite direction to the revolution of the universe, that is, [the motion of] the great circle through both poles, that of the equator and that of the ecliptic. We can see this mainly from the fact that the same stars do not maintain the same distances with respect to the solstitial and equinoctial points in our times as they had in former times: rather, the distance [of a given star] towards the rear with respect to [one of] those same points is found to be greater in proportion as the time [of observation] is later.

For Hipparchus too, in his work ‘On the displacement of the solstitial and equinoctial points’, adducing lunar eclipses from among those accurately observed by himself, and from those observed earlier by Timocharis, computes that the distance by which Spica is in advance of the autumnal [equinoctial] point is about 6° in his own time, but was about 8° in Timocharis’ time. For his final conclusion is expressed as follows: ‘If, then, Spica, for example, was formerly 8°, in zodiacal longitude, in advance of the autumnal [equinoctial] point, but is now 6° in advance’, and so forth. Furthermore he shows that in the case of almost all the other fixed stars for which he carried out the comparison, the rearward motion was of the same amount. And we also, comparing the distances of fixed stars from the solstitial and equinoctial points as they appear in our time with those observed and recorded by Hipparchus, find that their motion towards the rear with respect to the ecliptic is, proportionally, similar to the above amount. We conducted this type of investigation by means of the instrument which we constructed previously [see V I] for the observations of

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47 Catalogue XLVII 1, XXXIII 1, and XIX 4 and 3. The ‘quadrilateral’ in Pegasus (not mentioned in the catalogue) is formed by the stars α Peg, β Peg, α And and γ Peg.

48 I interpret this to mean that Hipparchus published a description of the constellations to be drawn on a celestial globe (literally ‘solid sphere’, στερεά σφαίρα, cf. VIII 3). What relationship, if any, this had to Hipparchus’ putative ‘Catalogue’ is obscure. On the general problem see HAMA 284–92.

49 Reference back to I 8 pp. 46–7. This makes it obvious that we must delete έλε γα ἐπόμενα (omitted by al-Hajjaj) at H12:12: it is senseless to talk about a motion ‘towards the rear’ with respect to a circle which is itself in motion. The motive for the interpolation was to gloss ‘in the opposite direction’.

50 Cf. III 1 p. 135 with n.14 for the lunar eclipses involved.
individual moon-sun distances. [In this case] we set one of the astrolabe rings to the apparent position of the moon (computed for the moment of observation), then adjusted the other astrolabe ring to align it with the star being sighted, so that both moon and star would be sighted simultaneously in the proper positions. Thus we obtained the position of every one of the bright stars from its distance from the moon.\textsuperscript{51}

To [illustrate this procedure] by a single example. In the second year of Antoninus, on Pharmouthi [VIII] 9 in the Egyptian calendar [139 Feb. 23], when the sun was just about to set in Alexandria, and the last degree of Taurus was culminating, i.e. $5^{\frac{1}{2}}$ equinoctial hours after noon on the ninth, we observed the apparent distance of the moon from the sun (which was sighted at about $30^\circ$) as $92^{\frac{1}{2}}\circ$. Half an hour later, the sun now having set, and the [first] quarter of Gemini [i.e. $7^{\frac{1}{2}}30^\circ$] culminating, the apparent moon was sighted in the same position [with respect to the astrolabe ring], and the star on the heart of Leo [$\alpha$ Leo, Regulus] had an apparent distance from the moon, [as measured] by means of the other astrolabe [ring], of $57^{\frac{1}{2}}\circ$ towards the rear along the ecliptic.

Now at the first [observation] the true position of the sun was very nearly $30^\circ$. Hence the apparent position of the moon, since it was $92^{\frac{1}{2}}\circ$ towards the rear [of the sun], was approximately $11^{\frac{1}{2}}\circ$, which is also the position it ought to occupy according to our hypotheses. Half an hour later the moon should have moved about $1^{\circ}$ towards the rear, and have a parallax in advance, relative to the first situation, of about $1^{\frac{1}{2}}\circ$. Therefore the apparent position of the moon half an hour later was $11^{\frac{1}{2}}\circ$. Hence the star on the heart, since its apparent distance from the moon was $57^{\frac{1}{2}}\circ$ to the rear, had a position of $2^{\frac{1}{2}}30^\circ$, and its distance from the summer solstice was $32^{\frac{1}{2}}30^\circ$.\textsuperscript{52}

But in the 50th year of the Third Kalliopic Cycle [-128/7], as Hipparchus records from his own observations, [that star] had a distance to the rear of the summer solstice of $29^{\frac{1}{2}}30^\circ$. Therefore the star on the heart of Leo has moved $2^{\frac{1}{2}}30^\circ$ towards the rear along the ecliptic in the 265 or so years from the observation of Hipparchus to the beginning [of the reign] of Antoninus [137/8], which was when we made the majority of our observations of the positions of the fixed stars. From this we find that $1^\circ$ rearward motion takes place in approximately 100 years, as Hipparchus too seems to have suspected, according to the following quotation from his work 'On the length of the year': 'For if the solstices and equinoxes were moving, from that cause, not less than $1^\circ$ of a degree in advance [i.e. in the reverse order] of the signs, in the 300 years they should have moved not less than $3^\circ$.\textsuperscript{53}

In the same way we took sightings of Spica and the brightest among those stars near the ecliptic, from the moon, and then [having done that], were in a

\textsuperscript{51}See V 1, with notes, for a detailed explanation of the use of the instrument. Ptolemy's procedure explains why the mean error in the longitudes of his star catalogue, about $1^\circ$, is the same as the mean error of his lunar and solar positions, derived from his faulty equinox (see III 1 p. 138 with n.21).

\textsuperscript{52}This observation is discussed in some detail by Pedersen, 240-5, with a computation of the parallax. Unfortunately he has made errors, notably in the angle between ecliptic and hour-circle in the first observation (see Toomer [3] p. 143).

\textsuperscript{53}The '300 years' is a reference to the interval between the solstice observation of Meton (-431, cf. III 1 p. 138) and Hipparchus' own time. This was obviously one of the comparisons which Hipparchus made.
better position to use those stars to take sightings of the rest. We [thus] find that their distances relative to each other are, again, very nearly the same as those observed by Hipparchus, but their individual distances from the solsticial or equinoctial points are in each case about $2^\circ$ farther to the rear than those derivable from what Hipparchus recorded.

3. \textit{That the rearward motion of the sphere of the fixed stars, too, takes place about the poles of the ecliptic} \footnote{See Pedersen 246-9.}

From the above it has become clear to us that the sphere of the fixed stars, too, performs a rearward motion along the ecliptic, of approximately the amount indicated. Our next task is to determine the type of this motion, that is to say, whether it takes place about the poles of the equator or about the poles of the inclined circle of the ecliptic. Since great circles drawn through the poles of either one of the above [equator or ecliptic] cut off unequal arcs on the other, [the answer to] the above [question] would become apparent merely from the motion in longitude, were it not for the fact that the motion in longitude over the time available [for comparison of observations] is so extremely small that the difference due to the above effect would be, as yet, imperceptible. The easiest way to detect [the answer] is through [comparison of] the positions [of the stars] in latitude\footnote{\textit{latitude} is ambiguous here and below. It means \textit{direction orthogonal to the circle in question}, i.e. \textit{latitude} (in the modern sense) with respect to the ecliptic, and \textit{declination} with respect to the equator. Cf. Introduction p. 21 and p. 63 n.74.} in ancient times and now. For it is obvious that whichever of the two circles, equator and ecliptic, it is from which they can be shown to maintain a constant distance in latitude, that is the circle about the poles of which the motion of their sphere will take place.

Now Hipparchus agrees with \lbrack the idea of\rbra; the motion taking place about the poles of the ecliptic. For in `On the displacement of the solsticial and equinoctial points' he deduces from the observations of Timocharis and himself that Spica (again) has maintained the same distance in latitude, not with respect to the equator but with respect to the ecliptic, being $2^\circ$ south of the ecliptic at both earlier and later periods. That is why in `On the length of the year' he assumes only the motion which takes place about the poles of the ecliptic, although he is still dubious, as he himself declares, both because the observations of the school of Timocharis are not trustworthy, having been made very crudely, and because the difference in time between [Timocharis and himself] is not sufficient to provide a secure result. We, however, find the [latitudinal distances with respect to the ecliptic] preserved over the much longer interval [down to our times], and that for practically all fixed stars. We can therefore with good reason consider the motion about the poles of the ecliptic as now more firmly established. For when we observe the latitudinal distance of any star with respect to the ecliptic, as measured along the great circle through the poles of the ecliptic, we find that it is practically the same as that computed from the
records of Hipparchus, or if there is a discrepancy, it is of very small size, such as can be accounted for by small observational errors. But when we consider the distances [of the stars] from the equator, as measured along great circles through the poles of the equator, we find [1] that those observed by us do not agree with those recorded in the same way by Hipparchus, and [2] that the latter do not agree with those recorded even earlier by Timocharis and his associates; rather, the constancy of their latitudes with respect to the ecliptic is confirmed even more by these very observations, since the distances from the equator of the stars located on the hemisphere from the winter solstice through the spring equinox to the summer solstice are found to be ever more northerly compared to those [of the same stars] in earlier periods, while for stars located on the opposite hemisphere they are ever more southerly. Furthermore the differences [between earlier and later observations] are greater for stars near the equinoctial points, and less for stars near the solstices, and these differences are just about the same as the amount by which that section of the ecliptic to the rear [of the earliest longitude of any particular star] defined by the corresponding motion in longitude [during the period in question] produces a displacement to the north or south of the equator.

In order to illustrate this point for a few easily recognisable stars we will set out, for each of the two hemispheres mentioned, their vertical distances from the equator, as measured along the great circle through the poles of the equator, as recorded by the school of Timocharis, as recorded by Hipparchus, and also as determined in the same fashion by ourselves. [See p. 331.]

In the case of all the above stars, which are located (to speak of their longitudinal position) on that one of the above-defined hemispheres which contains the spring equinox, the vertical distances from the equator which are later in time are all more northerly than the earlier, and for those stars very near the solsticial points [the difference] is very small, while for those near the equinoxes it is quite considerable: this accords with a rearward motion about the poles of the ecliptic, for if one takes successive sections of this semi-circle [of the ecliptic] going towards the rear, each is more northerly than the one in advance of it, and the difference [between successive equal sections] is again greater near the equinoxes and less near the solstices. [See p. 332.]

56 ταύς κατά τόν Ἰππαρχον ἀναγεγραμμένας καὶ συναγωμένας, literally 'those recorded and computed according to Hipparchus'. I take this to mean that Hipparchus recorded certain stellar positions (mainly declinations), from which Ptolemy computed the latitudes. All the evidence (including this passage) is in favour of the hypothesis that Hipparchus did not record stellar positions in latitude and longitude (except for a few special cases like that of Spica mentioned above, for the specific purpose of determining the precession). Otherwise it is impossible to explain why Ptolemy went through the cumbersome process of comparing declinations (pp. 331-2), instead of simply comparing latitudes observed by Hipparchus and himself.

57 These stars are listed in Ptolemy's catalogue as follows, 1. XVI 3:2, not listed, but cf. XXIII 30-2; 3, XXXII 14; 4, XII 3; 5, XXXV 3; 6, XXXV 27, XXXVIII 1; 8, XXIV 1; 9, XXIV 2. I have followed Manitius in arranging Ptolemy's continuous text in tabular form.

58 Sic (plural, although only the spring equinox is involved). The inaccuracy is probably Ptolemy's, caused by his thinking of the general situation (differences large near either equinox, small near either solstice).
<table>
<thead>
<tr>
<th></th>
<th>North or south of equator</th>
<th>As recorded by Aristyllos or Timocharis</th>
<th>As recorded by Hipparchus</th>
<th>As found by us</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] The bright star in Aquila</td>
<td>north</td>
<td>5°</td>
<td>5°</td>
<td>5°</td>
</tr>
<tr>
<td>[2] The middle of the Pleiades</td>
<td>north</td>
<td>14°</td>
<td>16°</td>
<td>16°</td>
</tr>
<tr>
<td>[3] The bright star in the Hyades</td>
<td>north</td>
<td>8°</td>
<td>11°</td>
<td></td>
</tr>
<tr>
<td>[4] The brightest star in Auriga, called Capella</td>
<td>north</td>
<td>40° (Aristyllos)</td>
<td>41°</td>
<td></td>
</tr>
<tr>
<td>[5] The star in the advance shoulder of Orion</td>
<td>north</td>
<td>1°</td>
<td>2°</td>
<td></td>
</tr>
<tr>
<td>[7] The bright star in the mouth of Canis Major</td>
<td>south</td>
<td>16°</td>
<td>15°</td>
<td></td>
</tr>
<tr>
<td>[8] The more advanced of the [two] bright stars in the heads of Gemini</td>
<td>north</td>
<td>33° (Aristyllos)</td>
<td>33°</td>
<td></td>
</tr>
</tbody>
</table>
In the opposite hemisphere:\[^{59}\]

<table>
<thead>
<tr>
<th>Star Description</th>
<th>North or south of equator</th>
<th>As recorded by [Aristyllus or Timocharis]</th>
<th>As recorded by Hipparchus</th>
<th>As found by us</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] The star on the heart of Leo</td>
<td>north</td>
<td>21°</td>
<td>20°</td>
<td>19°</td>
</tr>
<tr>
<td>[2] The star called Spica</td>
<td>north</td>
<td>1°</td>
<td>only ½° north</td>
<td>1° south</td>
</tr>
<tr>
<td>Of the 3 stars in the tail of Ursa Major:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[3] the one at the tip</td>
<td>north</td>
<td>61° (Aristyllus)</td>
<td>60°</td>
<td>59°</td>
</tr>
<tr>
<td>[4] the second from the end, in the middle of the tail</td>
<td>north</td>
<td>67° (Aristyllus)</td>
<td>66°</td>
<td>66°</td>
</tr>
<tr>
<td>[5] the third from the end, about where the tail joins [the body]</td>
<td>north</td>
<td>68° (Aristyllus)</td>
<td>67°</td>
<td>66°</td>
</tr>
<tr>
<td>Of the bright stars in the claws of Scorpius [i.e. in Libra]:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[7] the one in the tip of the southern claw</td>
<td>south</td>
<td>5°</td>
<td>5°</td>
<td>7°</td>
</tr>
<tr>
<td>[8] the one in the tip of the northern claw</td>
<td>north</td>
<td>1°</td>
<td>only ½° north</td>
<td>1° south</td>
</tr>
<tr>
<td>[9] The bright star in the chest of Scorpius, called Antares</td>
<td>south</td>
<td>18°</td>
<td>18°</td>
<td>20°</td>
</tr>
</tbody>
</table>

\[^{59}\] These stars are listed in Ptolemy's catalogue as follows: 1, XXVII 8; 2, XXVII 14; 3, II 27; 4, II 26; 5, II 25; 6, V 23; 7, XXVIII 1; 8, XXVIII 3; 9, XXIX 8. \[^{60}\] \(\Omega\) and \(\lambda\) have 67°, which may be correct.
Thus in the case of all these stars, the reverse [of the above] is true, as one would logically expect: the later vertical distances from the equator are more southerly than the earlier, in proportion to the time intervals and locations.

Furthermore one can conclude from these data that the rearward motion in longitude of the sphere of the fixed stars is, as we said previously [p. 328], 1° in about 100 years, or 2° in the 265 years between Hipparchus' and our observations. It is particularly [easy to do this] from the differences in declination found for those stars near the equinoctial points.

For the middle of the Pleiades, which was found to be 15° north of the equator in Hipparchus' time, and 16° in our time, has [thus] moved 1° northward in the interval between us: this is nearly the same as the difference in declination from the equator between the two ends of the 2° of the ecliptic near the end of Aries which represents the rearward motion in longitude over that interval. And the star called Capella, which was found to be 40° north of the equator in Hipparchus' time, and 41° in our time, has [thus] moved northward 1°: this is, again, the same as the difference in declination from the equator of [the ends of] the 2° of the ecliptic near the middle of Taurus. Also, the star on the advance shoulder of Orion, which was found to be 1° north of the equator in Hipparchus' time, and 2° in our time, has [thus] moved northward about 1°, which is nearly the same as the difference in declination from the equator of [the ends of] the 2° of the ecliptic two-thirds through Taurus.

The situation is similar on the opposite hemisphere. Spica, which was found to be 5° north of the equator in Hipparchus' time, but 1° south in our time, has [thus] moved southwards 1°, which is, again, the amount of the difference in declination from the equator of the 2° of the ecliptic near the end of Virgo. And the star in the tip of the tail of Ursa Major, which was found to be 60° north of the equator in Hipparchus' time, but 59° in our time, has [thus] moved southwards 1°, which is the amount of the difference in declination from the equator of the 2° of the ecliptic near the beginning of the sign of Libra. Also, Arcturus, which was found to be 31° north of the equator in Hipparchus' time, but 29° in our time, has [thus] moved southward 1°, which is, likewise, approximately the amount of the difference in declination from the equator of the 2° of the ecliptic near the beginning of Libra.

1° From Table I 15.

\[
\delta (30^\circ) = 11;39.59 \\
\delta (32^\circ) = 12;36.29 \quad \Delta = 0;57.30^\circ.
\]

which is considerably less than Ptolemy's 1°. Perhaps he has carelessly computed \(\delta (30^\circ) = 11;40^\circ, 2^\circ 30 \times 11;40^\circ = 1;2^\circ\).

2° In the catalogue these two stars have very nearly the same longitude. Capella being placed in 8 25 (XII 3) and the star in Orion in 8 24 (XXXV 3). Yet here they are placed 'in the middle of Taurus' and 'two-thirds through Taurus' respectively, and this is the basis of Ptolemy's calculations. For, from table I 15, the difference in declination of 2° near 45° is about 49', and near 55° is about 41'. Thus the statement regarding Capella seems to rest on an error.

3° Sir! The longitude of the star in question is 29;2 in the catalogue (II 27), so one would expect 'the beginning of Virgo' here. But the mss. are unanimous, and I hesitate to emend, both because of the other gross inaccuracies in this passage, and because a difference in declination of 1° is too great for the beginning of Virgo (from Table I 15 one finds about 57' for an argument of 30°). However, Ptolemy gives the same amount, 1°, for the 'end of Aries' (above, with n.61).
The point in question will become even clearer to us from the following observations."®

[Firstly] Timocharis, who observed at Alexandria, records the following. In the 47th year of the First Kallippic 76-year period, on the eighth of Anthesterion, which is Athyr 29 in the Egyptian calendar, towards the end of the third hour [of night], the southern half of the moon was seen to cover exactly either the rearmost third or [the rearmost] half® of the Pleiades. That moment is in the 465th year from Nabonassar, Athyr [III] 29/30 in the Egyptian calendar [-282 Jan. 29/30], 3 seasonal hours before midnight, or 3 $\frac{1}{2}$ equinoctial hours (since the sun was in about $\approx 7^\circ$). The interval reckoned in mean solar days comes to about the same number of equinoctial hours [$3\frac{1}{2}$] before midnight. At that moment, according to the hypotheses we demonstrated previously, the position of the moon was as follows:

true longitude: 8 0:20°
(i.e. distance from the spring equinox: 30;20°)

[latitude]: 3;45° north of the ecliptic
apparent longitude 29;20°

apparent [latitude] 3;35° north of the ecliptic®

(for the culminating point was $\&$ through Gemini).

Therefore at that time the rearmost end of the Pleiades was about 29$\frac{1}{2}$° towards the rear from the spring equinox (for the moon’s centre was still in advance of it), and was about 3$\frac{1}{2}$° north of the ecliptic (for, again, it was a little north of the moon’s centre).

H26

[Secondly] Agrippa, who observed in Bithynia, records that in the twelfth year of Domitian, on the seventh of Metroos according to the calendar of that region,® at the beginning of the third hour of night, the moon occulted the rearmost, southern part of the Pleiades with its southern horn. That moment is in the 840th year from Nabonassar, Tybi [V] 2/3 in the Egyptian calendar [92, Nov. 29/30], 4 seasonal hours before midnight, or 5 equinoctial hours (since the sun was in about $\eta 6^\circ$).® Therefore, reduced to the meridian of

VI 7. Precession: the Pleiades

There are numerous difficulties connected with the following observations of occultations, Ptolemy’s interpretations of them, and his calculations. To deal with them here would require too lengthy a discussion. Although they have been much discussed (e.g. by Schjellerup, Recherches’ III, Fotheringham [1] and Fotheringham [2]), the only satisfactory treatment is in Britton [1], 107-28, to which the reader interested in Ptolemy’s (often strange) interpretation of the data is referred. However, Britton does not consider the aspect of the errors resulting from Ptolemy’s misconceptions on the basis of his own theory. The more gross of these are noted below. These only reinforce Britton’s conclusion that the observations could not have been selected at random.

These and similar dates (pp. 335, 336 and 337) attributed to Timocharis must be dates in the artificial Metonic/Kallippic calendar. See Introduction p. 12.

It is most unclear what is meant here. Were there discrepancies in Timocharis’ report (or in the mss. of it available to Ptolemy)? Or does this represent variations in the Almagest ms. tradition? The translation of al-Hajjaj has ‘a half’ only.

Computed from Ptolemy’s tables: $\lambda, \phi = 7;8^\circ, \lambda, \phi = 30;11^\circ, \beta, \phi = 3;45^\circ$. Apparent longitude and latitude at Alexandria 29;0° and +3;38°.

Metroos is the month of the Bithynian calendar. See Introduction p. 14. Agrippa is unknown apart from this passage.

This implies that the longest day was about that of Clima V (Hellespont), which is approximately correct for Bithynia. But Ptolemy’s correction of -20 mins. for reduction to the
Alexandria, the observation occurred 5½ equinoctial hours before midnight, or 5½ hours with respect to mean solar days. At this moment the positions of the centre of the moon were as follows:

- true longitude: 8° 3;7°
- [latitude]: 4° 50' north of the ecliptic
- apparent longitude 8° 3;15° (in Bithynia)
- apparent [latitude] 4° north of the ecliptic

(for the culminating point was two-thirds through Pisces). Therefore at that time the rearmost section of the Pleiades was, in longitude, 33½° towards the rear from the spring equinox, and, [in latitude], 3½° north of the ecliptic.

Hence it is clear that the rearmost part of the Pleiades was, both then and now, the same distance in latitude, 3½°, north of the ecliptic, as measured along the great circle through the poles of the ecliptic, while in longitude it has moved 3;45° towards the rear from the spring equinox (since it was 29½° from the equinox at the first observation and 33½° at the second) in the interval of 375 years comprised between the two observations. Therefore in 100 years the rearmost part of the Pleiades has moved 1° towards the rear.

Again, [firstly] Timocharis, who observed at Alexandria, records that in the 36th year of the First Kallippic Cycle, on Elaphebolion 15, which is Tybi 5, at the beginning of the third hour, the moon covered Spica with the middle of the eastwards edge of its disk which is towards the equinoctial rising-point [i.e. the east], and that Spica, in passing through, cut off exactly the northern third of [the moon's] diameter.

This moment is in the 454th year from Nabonassar, Tybi [V] 5/6 in the Egyptian calendar [-293 Mar. 9/10], 4 seasonal hours before midnight, which is also 4 equinoctial hours approximately, since the sun was in about Δ 15°, and reckoning with respect to mean solar days leads to about the same number of hours before [midnight]. At that moment the positions of the moon's centre were as follows:

- true longitude: 21° 21°

(i.e. distance from the summer solstice was 81° 21° towards the rear)

meridian of Alexandria implies that Agrippa was observing at a place 5° to the east; in fact no place in Bithynia was more than 3° to the east of Alexandria; moreover, in the Geography (8.17.3-7) Ptolemy puts all the cities in Bithynia west of Alexandria.

There are some gross errors here. Computed (for 6:15 p.m. Alexandria): λ (x = 32;13° (0;54° less than the text!), β (y = +4;53°. One might think that Ptolemy computed for 8 p.m., i.e. took 'at the beginning of the third hour' as if it were equinoctial hours at Alexandria, were it not that the culminating point he gives is approximately correct (for 7 p.m. local time Bithynia I find Δ 18½°). His parallax corrections are also inaccurate (I find pλ = +0;19°, pβ = -0;38°, and hence, for the apparent position of the moon, λ = 32;32°, β = +4;15°. One need hardly say that this error is disastrous for the verification of Ptolemy's precession constant.

As Manitius points out (p. 402), in his catalogue (XXIII 32) Ptolemy assigns a latitude of +31° to the rearmost end of the Pleiades. But the discrepancy can easily be explained by the fact that he is referring, not to a specific star, but to part of the general mass.

From Nabonassar 465 to Nabonassar 840.
VII.3. Precession: Spica

true [latitude]: 1° 50' south of the ecliptic
apparent longitude: 82° 15' from the summer solstice
apparent latitude: about 2° south of the ecliptic

(for the middle of Cancer was culminating).

Therefore, from the above, we conclude that Spica was at that moment 82° 15' in longitude from the summer solstice, and just about 2° south of the ecliptic.

Likewise, secondly, in the 48th year of the same [First Kallippic] Cycle, he says that on the sixth day from the end of the last third of Pyanepsion,74 which is Thoth 7, when as much as half an hour of the tenth hour had gone by, and the moon had risen above the horizon, Spica appeared exactly touching the northern point on [the moon].

This moment is in the 466th year from Nabonassar, Thoth [I] 7/8 in the Egyptian calendar [-282 Nov. 8/9]; [the hour is], according to Timocharis himself, 3 1/2 seasonal hours after midnight, or approximately 3 1/4 equinoctial hours,75 since the sun was near the middle of Scorpius; but, according to logical reasoning, it must have been [2 1/2 hours] after midnight. For that is the time when Π 22° is culminating, and Ω 22° (approximately) is rising;76 and that Ω 22° was the longitude of the moon at that moment when, as he says, it was rising. Reckoning with respect to mean solar days, we find that only 2 equinoctial hours had passed since midnight. At this time the positions of the centre of the moon were as follows:

true [longitude]: distance from the summer solstice: 81° 30'
true [latitude]: 2° 45' south of the ecliptic
apparent longitude: 82° 15' [from the summer solstice]
apparent latitude: 2° south [of the ecliptic].

Therefore, according to this observation too, Spica was the same distance of about 2° south of the ecliptic, and was 82° 15' from the summer solstice. So in the 12 years between the two observations it moved about 2° towards the rear from the summer solstice.

Thirdly, the geometer Menelaus says that the following observation was made [by him] in Rome. In the first year of Trajan, Mechir 15 16, when the tenth hour [of night] was completed. Spica had been occulted by the moon (for it could not be seen), but towards the end of the eleventh hour it was seen in

74 Reading ΠΒ (πι' (with A'BCD) for ΠΒ (82:12°), the reading of Ar) at H29.7. In the circumstances of the observation this seems more likely to lead to the position of 82° 15' which Ptolemy deduces for Spica [below]. It is also closer to my computation (λ ζ, apparent, 172° 7'). though this is no argument. Corrected by Manitius.

75 Since the length of 1 seasonal night-hour was 16° 38', the length of 3½ hours was 58° 13', or about 3½ equinoctial hours. Hence I considered emending the text at H29.21 to ά λειπτυφας η (4 - 1). However, it seems more probable that Ptolemy simply made the error of computing day-hours instead of night-hours, which does indeed lead to 3½ equinoctial hours. The error has no consequences, since Ptolemy takes a quite different time.

76 For calculations of these see Appendix A Examples 4 and 5.

77 Calculated (cf. Appendix A Examples 9 and 10): λ ζ = 171° 39', β ζ = -2° 7'. Apparent positions: λ = 173° 1', β = -2° 20'.

In the circumstances of the observation this seems more likely to lead to the position of 82° 15' which Ptolemy deduces for Spica [below]. It is also closer to my computation (λ ζ, apparent, 172° 7'), though this is no argument. Corrected by Manitius.

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For calculations of these see Appendix A Examples 4 and 5.

Calculated (cf. Appendix A Examples 9 and 10): λ ζ = 171° 39', β ζ = -2° 7'. Apparent positions: λ = 173° 1', β = -2° 20'.
advance of the moon's centre, equidistant from the [two] horns by an amount less than the moon's diameter.

This moment is in the 845th year from Nabonassar, Mehir [VI] 15/16 in the Egyptian calendar [98 Jan. 10/11], 4 seasonal hours after midnight when the moon's centre was approximately covering Spica, which corresponds to 5 equinoctial hours, since the sun was in about $\Omega 20^\circ$; when reduced to the meridian through Alexandria this is $6\frac{1}{2}$ equinoctial hours, and [this], with respect to mean solar days, is $6\frac{1}{2}$ hours (or a little more). At this moment the positions of the centre of the moon were as follows:

true [longitude]: $85\frac{3}{4}$° from the summer solstice
true [latitude]: about $1\frac{3}{4}$° south of the ecliptic
apparent longitude: $86\frac{1}{4}$° from the summer solstice
apparent [latitude]: $2^\circ$ south of the ecliptic

(for the culminating point was about a quarter of the way through Libra).

Therefore that was the position of Spica too at that moment.

It is clear that Spica was, again, the same amount south of the ecliptic, namely $2^\circ$, both in Timocharis' time and in our time, and that its movement towards the rear in longitude is

$3;55^\circ$ in the 391 years from the observation in the 36th year [of the First Kallippic Cycle to the observation of Menelaus], and

$3;45^\circ$ in the 379 years from the observation in the 48th year.

Hence from these data too we conclude that the motion of Spica towards the rear in 100 years is about $1^\circ$.

Again, Timocharis, who observed in Alexandria, says that in the 36th year of the First Kallippic Cycle, on Poseideon 25, which is Phaophi 16, at the beginning of the tenth hour, the moon appeared to occult the northernmost of the stars in the forehead of Scorpius very precisely with its northern rim.

This moment is in the 454th year from Nabonassar, Phaophi [II] 16/17 in the Egyptian calendar [-294 Dec. 20/21], 3 seasonal hours after midnight, or $3\frac{3}{4}$ equinoctial hours, since the sun was in about $\Phi 26^\circ$. Reduced to mean solar days this is $3\frac{1}{2}$ hours. At this moment the position of the centre of the moon was as follows:

in true [longitude]: $31\frac{1}{4}$° from the autumnal equinox [towards the rear]
in [true latitude]: $1\frac{1}{9}$° north of the ecliptic

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78 I.e. the longitudinal difference between Rome and Alexandria is taken as about $20^\circ$. In fact it is about $17\frac{1}{2}$°. In the Geography the error is even more exaggerated. There: 8.3.3 Nobbe. Ptolemy states that Rome is $15^\circ$ to the west of Alexandria, in accordance with the assigned longitudes of $36^\circ$ and $60^\circ$ (ibid. 3.1.61 and 4.5.9). Heron, Dioptara, took the difference as 2 hours (Neugebauer [3], 22).

79 Here too my computations show significant discrepancies: $\lambda \xi 175;27^\circ, \beta -1;19,30^\circ$. Apparent positions at Rome, $\lambda 175;39^\circ, \beta -2;10^\circ$. Ptolemy's parallaxes, $+30'$ in longitude and $-40'$ in latitude, imply a total parallax of $50'$, which is approximately correct, and an angle between altitude circle and ecliptic of $c. 140^\circ$, which is impossible at the situation in question (moon roughly $\frac{3}{4}$ west of meridian, as his culminating point shows). Could he have taken the eastern angle in error?

80 In the catalogue (XXVII 14) Spica has coordinates of $26\frac{1}{4}$° and $-2^\circ$, in agreement with the data here (allowing for a movement of $25'$ in longitude in about forty years).

81 Reading $\zeta 0\beta$ (with D,Ar) for $\zeta 375'$ at H32.1. Corrected by Manitius, and by Heiberg, Op. Min. p. XIV.

82 Computed: $\lambda \xi 211;23^\circ, \beta \xi +1;17^\circ$. 
in apparent longitude: \(32^\circ\) [from the autumnal equinox]

in apparent [latitude]: \(1\frac{1}{2}^\circ\) north of the ecliptic

(for the culminating point was the middle of Leo).

Therefore at that moment the northernmost of the stars in the forehead of Scorpius was the same amount, \(32^\circ\), from the autumnal equinox in longitude, and about \(1\frac{1}{2}^\circ\) north of the ecliptic [in latitude].

Similarly, Menelaus, who observed in Rome, says that in the first year of Trajan, Mechir 18/19, towards the end of the eleventh hour, the southern horn of the moon appeared on a straight line with the middle and the southernmost of the stars in the forehead of Scorpius, and its centre was to the rear of that straight line, and was the same distance from the middle star as the middle star was from the southernmost; it appeared to have occulted the northernmost of the stars in the forehead, since [this star] was nowhere to be seen.

This moment is, again, in the 845th year from Nabonassar, Mechir [VI] 18/19 in the Egyptian calendar [98 Jan. 13/14], 5 seasonal hours after midnight, or \(6\frac{1}{4}\) equinoctial hours, since the sun was in about \(\lambda \varphi 23^\circ\). Reduced to the meridian of Alexandria this is \(7\frac{1}{4}\) equinoctial hours, and it is about the same with respect to mean solar days. At this moment the position of the centre of the moon was as follows:

- true [longitude]: \(35\frac{1}{3}^\circ\) from the autumnal equinox [towards the rear]
- true [latitude]: \(2\frac{1}{2}^\circ\) north of the ecliptic
- apparent longitude: \(35;55^\circ\) [from the autumnal equinox]
- apparent [latitude]: \(1\frac{1}{2}^\circ\) north [of the ecliptic]

(for the culminating point was the end of Libra).

Therefore the northernmost of the stars in the forehead of Scorpius had approximately the same position at the moment.

Hence it is clear that for this star too its distance in latitude from the ecliptic has been observed to be the same in former times and in our times, while its position in longitude has moved away from the autumnal equinox towards the rear by an amount of \(3;55^\circ\) in the time between the observations, which comprise 391 years, from which it follows that in 100 years the motion of the star towards the rear amounts to \(1^\circ\).

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81 Reading άπεχον (with D, Ar) for ἐπεχον here (H32,18) and at the similar place H33,20. Corrected by Manitius.

82 Reading \(\delta\) \(\delta\) (with Ar) for \(\delta\) \(\delta\) (1;12°) at H32,19. This gives better agreement with the observational data if a latitude of \(11^\circ\) is to be deduced (below). Corrected by Manitius.

Computed apparent position: \(\lambda \zeta 212;30^\circ\), \(\beta \zeta +1;1^\circ\).

83 Computed: \(\lambda \zeta 215;21^\circ\), \(\beta \zeta +2;5^\circ\).

84 Neugebauer has displayed all the computations leading up to this in various places in HAMA I, culminating in his remarks on pp. 117-18 about the impossibility of assigning a specific cause to the error in the final result. He also suggests (117 n.7) that one should read \(2;6^\circ\) and \(1;3^\circ\) for the true and apparent latitude. Although these numbers agree better with the calculation, \(11^\circ\) is certainly the correct reading, for it agrees with the latitude found from Timocharis' observation, and also with that assigned to this star in the catalogue (XXIX 1).
Thus, from our observations and comparisons of the above stars, from similar observations and comparisons of the other bright stars, and from the fact that we found the distances of the other stars with respect to the [bright stars] which we had established to be in agreement [with the results of our predecessors], we have confirmed that the sphere of the fixed stars, too, has a movement towards the rear with respect to the solstitial and equinoctial points of the amount determined (insofar as the time [for which observations are] available allows); furthermore, [we have confirmed] that this motion of theirs takes place about the poles of the ecliptic, and not those of the equator (i.e. the poles of the first motion). So we thought it appropriate, in making our observations and records of each of the above fixed stars, and of the others too, to give their positions, as observed in our time, in terms of longitude and latitude, not with respect to the equator, but with respect to the ecliptic, [i.e.] as determined by the great circle drawn through the poles of the ecliptic and each individual star. In this way, in accordance with the hypothesis of their motion established above, their positions in latitude with respect to the ecliptic must necessarily remain the same, while their positions in longitude must always traverse equal arcs towards the rear in equal times.

Hence, again using the same instrument [as we did for the moon, V 1], (because the astrolabe rings in it are constructed to rotate about the poles of the ecliptic), we observed as many stars as we could sight down to the sixth magnitude. [We proceeded as follows.] We always arranged the first of the above-mentioned astrolabe rings [Fig. F,5] [to sight] one of the bright stars whose position we had previously determined by means of the moon, setting the ring to the proper graduation on the ecliptic [ring (Fig. F,3) for that star], then set the other ring [Fig. F,2], which was graduated along its entire length and could also be rotated in latitude towards the poles of the ecliptic, to the required star, so that at the same time as the control star was sighted [in its proper position], this star too was sighted through the hole on its own ring. For when these conditions were met, we could readily obtain both coordinates of the required star at the same time by means of its astrolabe ring [Fig. F,2]: the position in longitude was defined by the intersection of that ring and the ecliptic [ring], and the position in latitude by the arc of the astrolabe ring cut off between the same intersection and the upper sighting-hole.

In order to display the arrangement of stars on the solid globe according to the above method, we have set it out below in the form of a table in four sections. For each star (taken by constellation), we give, in the first section, its description as a part of the constellation; in the second section, its position in longitude, as

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87 If the text is sound, Ptolemy is speaking carelessly here. As is clear from the description at V 1, ring no. 2 is indeed graduated, but cannot perform a latitudinal movement; that is done by ring no. 1, which fits inside no. 2 and has the sighting-holes attached to it.

88 Literally 'above the earth'. Cf. p. 219 n.6.

89 For a description of this instrument see VIII 3.

90 Literally 'the shapes' (γενειωνωνυς), i.e. its position as a part of the mythological figure (animal, anthropomorphic or inanimate) which was delineated on the globe and (notionally) in the heavens.
derived from observation, for the beginning of the reign of Antoninus (the position is given) within a sign of the zodiac, the beginning of each quadrant of the zodiac being, as before, established at one of the solstitial or equinoctial points; in the third section we give its distance from the ecliptic in latitude, to the north or south as the case may be for the particular star; and in the fourth, the class to which it belongs in magnitude. The latitudinal distances will remain always unchanged, and the positions in longitude can provide a ready means of determining the [corresponding] longitude at other points in time, if we [calculate] the distance in degrees between the epoch and the time in question on the basis of a motion of 1° in 100 years, [and] subtract it from the epoch position for earlier times, but add it to the epoch position for later times.

For the same reasons, our indications [of relative positions] in the descriptions must also be understood to accord with the above kind of hypothesis about the arrangement of the stars, and with the definition [of position] by [circles drawn] through the poles of the ecliptic. Thus, when we speak of a star as 'in advance of' or 'to the rear of' another, we mean that it occupies the relative position in question as defined by the ecliptic position [of the two stars, 'in advance of'] referring to the section of the ecliptic which is in advance, and ['to the rear'] referring to the section of the ecliptic which is towards the rear; and by 'more to the south' or 'more to the north', we mean nearer to the pole of the ecliptic (southern or northern as the case may be). Furthermore, the descriptions which we have applied to the individual stars as parts of the constellation are not in every case the same as those of our predecessors (just as their descriptions differ from their predecessors): in many cases our descriptions are different because they seemed to be more natural and to give a better proportioned outline to the figures described. Thus, for instance, those stars which Hipparchus places 'on the shoulders of Virgo' we describe as 'on her sides', since their distance from the stars in her head appears greater than their distance from the stars in her hands, and that situation fits [a location] 'on her side', but is totally inappropriate to [a location] 'on her shoulders'. However, one has a ready means of identifying those stars which are described differently [by others]; this can be done immediately simply by comparing the recorded positions.

The layout of the catalogue is as follows.

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91 I.e. according to the Canon Basileon (see Introduction p.11), Thoth 1 of Nabonassar 885 (= 137 July 20).
92 Reading τας της ἐποχής ἐπί τοῦ μεταγενεστέρου (with D,Ar) at H37,2 for τας τοῦ μεταγενεστέρου. Corrected by Manitius.
93 Although this is in general true, there appear to be exceptions. See Introduction p. 20, p.344 n.110 (on catalogue III 15-18) and p. 377 n.35 (on catalogue XXXII 23-4).
94 Thus 5 Vir is described by Hipparchus (Comm. in Hist. 2.5.3., ed. Manitius p.190.10) as 'the northern shoulder of Virgo', and by Ptolemy (catalogue XXVII 10) as 'the star in the right side under the girdle'.
### VII. Constellation I: Ursa Minor

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The star on the end of the tail</td>
<td>II 0 1/4</td>
<td>+66</td>
<td>3</td>
<td>α UMi</td>
</tr>
<tr>
<td>2</td>
<td>The one next to it on the tail</td>
<td>II 2 1/4</td>
<td>+70</td>
<td>4</td>
<td>δ UMi</td>
</tr>
<tr>
<td>3</td>
<td>The one next to that, before the place where the tail joins the body</td>
<td>11 10 26 1/4</td>
<td>+74 1/2</td>
<td>4</td>
<td>ε UMi</td>
</tr>
<tr>
<td>4</td>
<td>The southernmost of the stars in the advance side of the rectangle</td>
<td>11 29 1/4</td>
<td>+75 1/2</td>
<td>4</td>
<td>ζ UMi</td>
</tr>
<tr>
<td>5</td>
<td>The northernmost of those in the same side</td>
<td>12 3 1/4</td>
<td>+77</td>
<td>4</td>
<td>η UMi</td>
</tr>
<tr>
<td>6</td>
<td>The southern star in the rear side</td>
<td>12 17 17 1/4</td>
<td>+72 1/2</td>
<td>2</td>
<td>ζ UMi</td>
</tr>
<tr>
<td>7</td>
<td>The northern one in the same side</td>
<td>12 26 1/4</td>
<td>+74 1/2</td>
<td>2</td>
<td>γ UMi</td>
</tr>
<tr>
<td>8</td>
<td>Nearby star outside the constellation: The star lying on a straight line with the stars in the rear side of the rectangle and south of them</td>
<td>13 13 1/4</td>
<td>+71 1/2</td>
<td>4</td>
<td>5 UMi</td>
</tr>
</tbody>
</table>

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95 On the principles on which my translation of the star catalogue is arranged see Introduction pp. 14-17. Here I note only that I have followed Magni in adding, as the first and sixth columns, running numbers within each constellation, and the identification of the star, and that an asterisk next to any element indicates that there is some uncertainty about its correctness. For an idea of the arrangement in the Greek mss. see Peters and Knobel pls. II-IV.

Abbreviations:

S (plus number) — the list of variants in the various Arabic versions according to ibn ay-Salih, 83-96 of Kunzsch's edn.
P-K — Peters and Knobel, *Ptolemy's Catalogue of Stars*
BSC — Yale Catalogue of Bright Stars
CGal — Gama MaGalacticus (Galactic cluster)
CGLo — Gama MaGlobalis (Global cluster).

96 Reading ζ′ with B. According to S 1, 10 1/4 was in the Syriac and al-Hasan versions. Heiberg (139,6) prints ζ (16), which is also the reading of the rest of the Arabic tradition.

97 The reading 17 1/2 is found in some Greek mss. (not recorded by Heiberg) and the Arabic tradition (I, T, E, F), and is adopted by P-K.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The star on the end of the snout</td>
<td>□ 25°</td>
<td>+39°</td>
<td>4</td>
<td>o UMa</td>
</tr>
<tr>
<td>2</td>
<td>The more advanced of the stars in the two eyes</td>
<td>□ 25°</td>
<td>+43°</td>
<td>5</td>
<td>2(A) UMa</td>
</tr>
<tr>
<td>3</td>
<td>The one to the rear</td>
<td>□ 26°</td>
<td>+43°</td>
<td>5</td>
<td>π² UMa</td>
</tr>
<tr>
<td>4</td>
<td>The more advanced of the two stars in the forehead</td>
<td>□ 26°</td>
<td>+47°</td>
<td>5</td>
<td>ρ UMa</td>
</tr>
<tr>
<td>5</td>
<td>The one to the rear</td>
<td>*□ 26°</td>
<td>+47°</td>
<td>5</td>
<td>σ² UMa</td>
</tr>
<tr>
<td>6</td>
<td>The star on the tip of the advance ear</td>
<td>□ 28°</td>
<td>+50°</td>
<td>5</td>
<td>24(d) UMa</td>
</tr>
<tr>
<td>7</td>
<td>The more advanced of the two stars in the neck</td>
<td>□ 0°</td>
<td>+43°</td>
<td>4</td>
<td>τ UMa</td>
</tr>
<tr>
<td>8</td>
<td>The one to the rear</td>
<td>□ 2°</td>
<td>+43°</td>
<td>4</td>
<td>ν UMa</td>
</tr>
<tr>
<td>9</td>
<td>The northernmost of the two stars in the chest</td>
<td>□ 9°</td>
<td>+42°</td>
<td>4</td>
<td>23(h) UMa</td>
</tr>
<tr>
<td>10</td>
<td>The southernmost of them</td>
<td>□ 11°</td>
<td>+44°</td>
<td>4</td>
<td>θ UMa</td>
</tr>
<tr>
<td>11</td>
<td>The star on the left knee</td>
<td>□ 10°</td>
<td>+35°</td>
<td>3</td>
<td>θ UMa</td>
</tr>
<tr>
<td>12</td>
<td>The northernmost of the [two] in the front left paw</td>
<td>□ 5°</td>
<td>+29°</td>
<td>3</td>
<td>τ UMa</td>
</tr>
<tr>
<td>13</td>
<td>The southernmost of them</td>
<td>□ 6°</td>
<td>+28°</td>
<td>3</td>
<td>κ UMa</td>
</tr>
<tr>
<td>14</td>
<td>The star above the right knee</td>
<td>□ 5°</td>
<td>+36°</td>
<td>4</td>
<td>18(e) UMa</td>
</tr>
<tr>
<td>15</td>
<td>The star below the right knee</td>
<td>□ 5°</td>
<td>+33°</td>
<td>4</td>
<td>15(f) UMa</td>
</tr>
<tr>
<td>16–19</td>
<td>The stars in the quadrilateral:</td>
<td>□ 17°</td>
<td>+49°</td>
<td>2</td>
<td>α UMa</td>
</tr>
<tr>
<td>16</td>
<td>the one on the back</td>
<td>□ 17°</td>
<td>+49°</td>
<td>2</td>
<td>α UMa</td>
</tr>
<tr>
<td>17</td>
<td>the one on the flank</td>
<td>□ 22°</td>
<td>+44°</td>
<td>2</td>
<td>β UMa</td>
</tr>
<tr>
<td>18</td>
<td>the one on the place where the tail joins [the body]</td>
<td>□ 3°</td>
<td>+51°</td>
<td>3</td>
<td>δ UMa</td>
</tr>
<tr>
<td>19</td>
<td>the remaining one, on the left hind thigh</td>
<td>□ 3°</td>
<td>+46°</td>
<td>2</td>
<td>γ UMa</td>
</tr>
<tr>
<td>20</td>
<td>The more advanced of the [two stars] in the left hind paw</td>
<td>□ 22°</td>
<td>+29°</td>
<td>3</td>
<td>λ UMa</td>
</tr>
<tr>
<td>21</td>
<td>The one to the rear of it</td>
<td>□ 24°</td>
<td>+28°</td>
<td>3</td>
<td>μ UMa</td>
</tr>
</tbody>
</table>

98 P–K adopt 27° on very poor authority.
99 The Greek and Arabic ms. tradition is solid for 44, which is much too great. According to S 2 both the Islāq and Thābit versions had 41 (the latter is not borne out by extant ms.) Independently of each other ḥāṣṣālā (p. 48) and Peters (p. 96 no. 18) correct to 37 and 37°, but the corrections are palaeographically improbable.
100 ἀκρόπος. Following Hellenistic practice, Ptolemy normally uses πός and χεῖρ to denote ‘leg’ and ‘arm’. (But not always, cf. e.g. XIII 14 and 24, or XIV 11 and 12). Hence for ‘foot’ and ‘hand’ he has to use terms like ἀκρόπος (ἀκροπόδον) and ἀκροχεῖρον. Translators have often misrepresented the latter by expressions such as ‘tip of the foot’ and ‘end of the hand’.
101 The variant 30° is found in part of both Greek (A'BD) and Arabic traditions (S 3).
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>'The star on the left knee-bend</td>
<td>3Ω 1 1</td>
<td>+35 1</td>
<td>&gt;4</td>
<td>ψ UMa</td>
</tr>
<tr>
<td>23</td>
<td>The northernmost of the [two stars] in the right hind paw</td>
<td>3Ω 9 1</td>
<td>+25 1</td>
<td>3</td>
<td>ι UMa</td>
</tr>
<tr>
<td>24</td>
<td>The southernmost of them</td>
<td>3Ω 10 102</td>
<td>+25</td>
<td>3</td>
<td>ξ UMa</td>
</tr>
<tr>
<td>25</td>
<td>The first of the three stars on the tail next to the place where it joins [the body]</td>
<td>3Ω 12 1</td>
<td>+53 1</td>
<td>2</td>
<td>ε UMa</td>
</tr>
<tr>
<td>26</td>
<td>The middle one</td>
<td>4Ω 18</td>
<td>+55 1</td>
<td>2</td>
<td>ζ UMa</td>
</tr>
<tr>
<td>27</td>
<td>The third, on the end of the tail</td>
<td>4Ω 29 1</td>
<td>+54</td>
<td>2</td>
<td>η UMa</td>
</tr>
</tbody>
</table>

[27 stars, 6 of the second magnitude, 8 of the third, 8 of the fourth, 5 of the fifth]

Stars under [Ursa Major] outside the constellation:

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>The star under the tail, at some distance towards the south</td>
<td>5Ω 27 1</td>
<td>+33 1</td>
<td>3</td>
<td>α CVn</td>
</tr>
<tr>
<td>29</td>
<td>The rather faint star in advance of it</td>
<td>5Ω 20 103</td>
<td>+41 1</td>
<td>5</td>
<td>β CVn</td>
</tr>
<tr>
<td>30</td>
<td>The southernmost of the [two] stars between the front legs of Ursa Major and the head of Leo</td>
<td>5Ω 15</td>
<td>+17 1</td>
<td>4</td>
<td>40 Lyn</td>
</tr>
<tr>
<td>31</td>
<td>The one north of it</td>
<td>5Ω 13 1</td>
<td>+19 1</td>
<td>4</td>
<td>38 Lyn</td>
</tr>
<tr>
<td>32</td>
<td>The rearmost of the remaining three faint stars</td>
<td>5Ω 16 1</td>
<td>+20</td>
<td>6</td>
<td>*10 LMi</td>
</tr>
<tr>
<td>33</td>
<td>The one in advance of this</td>
<td>5Ω 12 1</td>
<td>+25 1</td>
<td>6</td>
<td>*BSC 3809105</td>
</tr>
<tr>
<td>34</td>
<td>The one in advance again of the latter</td>
<td>5Ω 11 1</td>
<td>+20 1</td>
<td>6</td>
<td>*BSC 3612107</td>
</tr>
<tr>
<td>35</td>
<td>The star between the front legs of Ursa Major and Gemini</td>
<td>5Ω 0</td>
<td>+22 1</td>
<td>6</td>
<td>31 Lyn</td>
</tr>
</tbody>
</table>

[8 stars outside the constellation, 1 of the third magnitude, 2 of the fourth, 1 of the fifth, 4 faint]

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102 The variant 13 occurs in part of the Arabic tradition according to S 4.
103 The variant 26 occurs in part of the Greek tradition (G) and, according to S 6, part of the Arabic.
104 Reading κβ ζ' (with BC) at H43,14 for κβ ζ' β ζ (22 + f 1), which is impossible. Corrected by Manitius. Part of the Arabic tradition (L,T,F) has 22 1, which is adopted by P-K.
105 Identification highly uncertain. Mine is that of P-K (Piazzi IX 115), who however also emend the longitude to 15 1.
106 Reading κ ν' at H43,15 for κφ (23). The reading adopted is that of the Greek ms. B and part of the Arabic tradition (see S 7).
107 The identification corresponds to that of P-K (Piazzi VIII 245).
However, the object carried by ljx,tes is called by Ptolemy (no. 10). The object in Thiele 1-ig. 22 p. 96 resembles a shepherd's crook.

translated as shepherd’s stall or club. The former would be more appropriate to the herdsman Bootes, the latter more plausible for the hunter Orion. It seems unlikely that he describes it as ‘to the south’ because no. 5 has a lesser declination than no. 4 (50° to the rear of no. 1). It seems unlikely that he describes it as ‘to the south’ because no. 5 has a lesser declination than no. 4.

\[\text{Description} \quad \text{Longitude in degrees} \quad \text{Latitude in degrees} \quad \text{Magnitude} \quad \text{[Modern designation]}\]

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>The one under that elbow, which also touches it</td>
<td>Ξ 10</td>
<td>+74</td>
<td>4</td>
<td>Θ Cep</td>
</tr>
<tr>
<td>8</td>
<td>The star in the chest</td>
<td>Ψ 29</td>
<td>+635</td>
<td>5</td>
<td>Τ Cep</td>
</tr>
<tr>
<td>9</td>
<td>The southernmost of the 3 stars on the tiara(^1)</td>
<td>Π 71</td>
<td>+621</td>
<td>&gt;4</td>
<td>Ψ Cep</td>
</tr>
<tr>
<td>10</td>
<td>The middle one of the three</td>
<td>Ω 16</td>
<td>+601</td>
<td>5</td>
<td>Ψ Cep</td>
</tr>
<tr>
<td>11</td>
<td>The northernmost of the three</td>
<td>Ξ 17</td>
<td>+61^2</td>
<td>4</td>
<td>Ψ Cep</td>
</tr>
<tr>
<td></td>
<td>[11 stars: 1, the third magnitude, 7, the fourth, 3 of the fifth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Stars around Cepheus outside the constellation:

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>The one in advance of the tiara</td>
<td>Υ 13</td>
<td>+64</td>
<td>5</td>
<td>Ψ Cep</td>
</tr>
<tr>
<td>13</td>
<td>The one to the rear of the tiara</td>
<td>Χ 21</td>
<td>+591</td>
<td>4</td>
<td>Ψ Cep</td>
</tr>
</tbody>
</table>

\[^1\] Cepheus was represented wearing the tiara, the high head-dress of the Persian king, because in many versions of the myth (involving Perseus, Andromeda and her father Cepheus) he was said to be an oriental ruler. See Bull-Gundel, 'Sternbilder' cols. 884-5, with illustration from Vat. Gr. 1087.

\[^2\] The variant ντιρ occurs in the earlier Arabic tradition according to S 9.

\[^3\] The star is to the north only of no. 7, not of no. 6. Hence Manitius endeavours ντιρ at H48.18 to ντιρ, 'of this'. However, it seems probable that Ptolemy was careless, being misled by the fact that the declination of no. 8 is greater than that of both the other stars.

\[^4\] Κόλλαςόπολος, a kind of curved stick traditionally applied to the object held by Bootes, the latter more plausible for the hunter Orion. However, the object carried by Bootes is called by Ptolemy (no. 10) a κόλλαςόπολος (ῥύπαλος), and that is what is represented on the Farnese globe (Thiele Pl. VI top). The object in Thiele Fig. 22 p. 96 resembles a shepherd’s crook.
Manitius would be the sole of the ecliptic, in which the normal rule may not apply. Indeed, on Ptolemy's star globe, the declinations of nos. 15-18 longitude than no. 18, and thus would normally be 'to the rear' of it; for stars with extreme northern latitudes, their declinations may be greater than that of the pole of the ecliptic (90° + c.), in which case the normal rule may not apply. Indeed, on Ptolemy's star globe the equatorial coordinates of nos. 15-18 would be

<table>
<thead>
<tr>
<th>Number in constellation</th>
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<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H44</td>
<td>[II] Constellation of Draco</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The star on the tongue</td>
<td>264</td>
<td>76</td>
<td>4</td>
<td>µ Dra</td>
</tr>
<tr>
<td>2</td>
<td>The star in the month</td>
<td>114</td>
<td>78</td>
<td>&gt;4</td>
<td>π Dra</td>
</tr>
<tr>
<td>3</td>
<td>The star above the eye</td>
<td>133</td>
<td>75</td>
<td>3</td>
<td>β Dra</td>
</tr>
<tr>
<td>4</td>
<td>The star on the joint</td>
<td>271</td>
<td>80</td>
<td>4</td>
<td>ζ Dra</td>
</tr>
<tr>
<td>5</td>
<td>The star above the head</td>
<td>291</td>
<td>73</td>
<td>3</td>
<td>η Dra</td>
</tr>
<tr>
<td>6</td>
<td>The northwestern of the 3 stars in a straight line in the first bend of the neck</td>
<td>241</td>
<td>82</td>
<td>3</td>
<td>θ Dra</td>
</tr>
<tr>
<td>7</td>
<td>The southwestern of these</td>
<td>21</td>
<td>78</td>
<td>4</td>
<td>γ Dra</td>
</tr>
<tr>
<td>8</td>
<td>The middle one</td>
<td>282</td>
<td>80</td>
<td>4</td>
<td>δ Dra</td>
</tr>
<tr>
<td>9</td>
<td>The star to the rear and due east of the latter</td>
<td>191</td>
<td>81</td>
<td>4</td>
<td>ε Dra</td>
</tr>
<tr>
<td>10</td>
<td>The southern star of the two forming the advance side of the quadrilateral in the next bend</td>
<td>8</td>
<td>81</td>
<td>4</td>
<td>π Dra</td>
</tr>
<tr>
<td>11</td>
<td>The more northerly star of the advance side</td>
<td>204</td>
<td>83</td>
<td>4</td>
<td>δ Dra</td>
</tr>
<tr>
<td>12</td>
<td>The northern star of the rear side [of the quadrilateral]</td>
<td>71</td>
<td>88</td>
<td>4</td>
<td>ε Dra</td>
</tr>
<tr>
<td>13</td>
<td>The northern star of the rear side</td>
<td>222</td>
<td>77</td>
<td>4</td>
<td>π Dra</td>
</tr>
<tr>
<td>14</td>
<td>The southern star of [those forming] the triangle in the next bend</td>
<td>101</td>
<td>80</td>
<td>5</td>
<td>α Dra</td>
</tr>
<tr>
<td>15</td>
<td>The more advanced of the other two stars of the triangle</td>
<td>21</td>
<td>81</td>
<td>5</td>
<td>α Dra</td>
</tr>
<tr>
<td>16</td>
<td>The one to the rear</td>
<td>261</td>
<td>80</td>
<td>5</td>
<td>τ Dra</td>
</tr>
<tr>
<td>17</td>
<td>The most advanced of the three stars in the next triangle, which is in advance of the last</td>
<td>133</td>
<td>84</td>
<td>4</td>
<td>ψ Dra</td>
</tr>
</tbody>
</table>

Thus 16 is 'to the rear' of 15, but 17 is 'in advance of' 18, and 17 and 18 are 'in advance of' 15 and 16, in agreement with the text I adopt. Corrected by Manitius.

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H46</td>
<td>[IV] Constellation of Cepheus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The star on the right leg</td>
<td>5</td>
<td>75</td>
<td>4</td>
<td>π Cep</td>
</tr>
<tr>
<td>2</td>
<td>The one on the left leg</td>
<td>3</td>
<td>64</td>
<td>4</td>
<td>γ Cep</td>
</tr>
<tr>
<td>3</td>
<td>The star under the belt on the right side</td>
<td>71</td>
<td>71</td>
<td>4</td>
<td>δ Cep</td>
</tr>
<tr>
<td>4</td>
<td>The star over the right shoulder, which touches it</td>
<td>161</td>
<td>69</td>
<td>3</td>
<td>ω Cep</td>
</tr>
<tr>
<td>5</td>
<td>The star over the right elbow, which touches it</td>
<td>91</td>
<td>72</td>
<td>4</td>
<td>η Cep</td>
</tr>
</tbody>
</table>

Reading γ3 (with Ar) for γ2 (87) at H45.20. Corrected by P-K. 81 fits both Ptolemy's description and the actual location of Dracon much better.

Reading γ3 (with Ar) for γ2 (80) at H47.4. 80 must be wrong, since Ptolemy's description ensures that the latitude of no. 23 lies between that of no. 22 (B1) and that of 24 (H4). Corrected by Manitius and P-K.

Reading γ3 (with Ar, adopted by P-K) for γ2 (10) at H47.7. According to S II the Arabic tradition of no. 27 is unanimous for 135°, the context makes it clear that he has mistakenly attributed the coordinates of no. 26 to no. 27. '135° is probably a scribal error in Ibn as-Salih for '15°'.

Deleting 600 λα ("31 altogether"), with D, at H46.13.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>The southernmost of them</td>
<td>21° 20'</td>
<td>61°</td>
<td>4</td>
<td>ζ Cyg</td>
</tr>
<tr>
<td>4</td>
<td>The one to the rear of these, in between the points where the horns of the lyre are attached</td>
<td>23° 14'</td>
<td>60°</td>
<td>4</td>
<td>η Cyg</td>
</tr>
<tr>
<td>5</td>
<td>The northernmost of the 2 stars close together in the region to the east of the shell</td>
<td>2° 17'</td>
<td>61°</td>
<td>4</td>
<td>η Cyg</td>
</tr>
<tr>
<td>6</td>
<td>The southernmost of them</td>
<td>11° 40'</td>
<td>60°</td>
<td>4</td>
<td>θ Cyg</td>
</tr>
<tr>
<td>7</td>
<td>The northernmost of the two advance stars in the bridge</td>
<td>21° 21'</td>
<td>56°</td>
<td>3</td>
<td>β Cyg</td>
</tr>
<tr>
<td>8</td>
<td>The southernmost of them</td>
<td>20° 24'</td>
<td>55°</td>
<td>3</td>
<td>γ Cyg</td>
</tr>
<tr>
<td>9</td>
<td>The northernmost of the two rear stars in the bridge</td>
<td>21° 24'</td>
<td>55°</td>
<td>3</td>
<td>γ Cyg</td>
</tr>
<tr>
<td>10</td>
<td>The southernmost of them</td>
<td>24° 11'</td>
<td>54°</td>
<td>4</td>
<td>λ Cyg</td>
</tr>
</tbody>
</table>

[1] Conceivably a reference to the version of the myth in which Hermes used the horns of the cattle he stole from Apollo to make this part of the lyre (scholion on Germanicus, ed. Breysig 84). Cf the depiction in Vit. Gr. 1087, reproduced in Bol-Böndel ed. 904, and Thiele Fig. 38 p. 114.

[2] Κύγμα, the 'cross-bar' of the lyre.

[3] Reading κάθα (with D) at H58.3. Heiberg has κάθα (24), which is the reading of Ar. But all other Greek ms. have κάθα (21).

[4] ὅδερ, literally 'bird'. It is not identified with a swan (Cygnus) or any particular bird in the earlier Greek tradition (e.g. Aratus 278), but the extant pictorial representations (e.g. Thiele Fig. 39 p. 114) mostly resemble a swan. For the origin of the appellation 'swan' see Gundel, art. 'Kyknos'. RE 11.2, 1942-3.

[5] Reading αὐτός (with ls) for αὐτόν ('of these') at H58.16. The change is necessary, since the star is north only of no. 10.

<table>
<thead>
<tr>
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<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>The star on the head</td>
<td>7° 31'</td>
<td>45°</td>
<td>3</td>
<td>ζ Cyg</td>
</tr>
<tr>
<td>13</td>
<td>The star on the leg</td>
<td>10° 40'</td>
<td>47°</td>
<td>2</td>
<td>ω Cyg</td>
</tr>
<tr>
<td>14</td>
<td>The star on the left leg</td>
<td>14° 11'</td>
<td>56°</td>
<td>3</td>
<td>ξ Cyg</td>
</tr>
<tr>
<td>15</td>
<td>The more advanced of the 2 stars in the right leg</td>
<td>12° 21'</td>
<td>61°</td>
<td>5</td>
<td>α Cyg</td>
</tr>
<tr>
<td>16</td>
<td>The one to the rear</td>
<td>12° 21'</td>
<td>61°</td>
<td>5</td>
<td>α Cyg</td>
</tr>
<tr>
<td>17</td>
<td>The nebulous star on the right knee</td>
<td>17° 22'</td>
<td>61°</td>
<td>5</td>
<td>α Cyg</td>
</tr>
</tbody>
</table>

Stars around [Cygnus] outside the constellation

| 18                      | The southernmost of the 2 stars under the left wing | 10° 11' | 49° | 4 | τ Cyg |
| 19                      | The northernmost of them | 13° 2' | 51° | 4 | ο Cyg |

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The one north of this, in the middle of the same wing</td>
<td>1° 31'</td>
<td>52°</td>
<td>4</td>
<td>ο Cyg</td>
</tr>
</tbody>
</table>

[1] Conceived as a reference to the version of the myth in which Hermes dictated the lyre to Apollo. See also Gundel, art. 'Kyknos'. RE 11.2, 1942-3.

[2] The variant occurs in both Greek (D) and the later Arabic traditions (see S 17).
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>The one to the rear again of these [8 stars, 1 of the second magnitude, 5 of the fourth, 1 of the fifth, 1 of the sixth]</td>
<td>ο 211</td>
<td>+46</td>
<td>4</td>
<td>α Gru</td>
</tr>
<tr>
<td>8</td>
<td>The star to the rear of all [the others] in the crown</td>
<td>ο 211</td>
<td>+49</td>
<td>4</td>
<td>υ Gru</td>
</tr>
<tr>
<td>1</td>
<td>The star on the head</td>
<td>m 17</td>
<td>+37</td>
<td>3</td>
<td>α Her</td>
</tr>
<tr>
<td>2</td>
<td>The star on the right shoulder by the armpit</td>
<td>m 31</td>
<td>+43</td>
<td>3</td>
<td>β Her</td>
</tr>
<tr>
<td>3</td>
<td>The star on the right upper arm</td>
<td>m 11</td>
<td>+40</td>
<td>3</td>
<td>γ Her</td>
</tr>
<tr>
<td>4</td>
<td>The star on the right elbow</td>
<td>m 10</td>
<td>+37</td>
<td>4</td>
<td>ξ Her</td>
</tr>
<tr>
<td>5</td>
<td>The star on the left shoulder</td>
<td>m 10</td>
<td>+68</td>
<td>3</td>
<td>δ Her</td>
</tr>
<tr>
<td>6</td>
<td>The star on the left upper arm</td>
<td>m 22</td>
<td>+49</td>
<td>&gt;4</td>
<td>λ Her</td>
</tr>
<tr>
<td>7</td>
<td>The star on the left elbow</td>
<td>m 27</td>
<td>+52</td>
<td>&gt;4</td>
<td>μ Her</td>
</tr>
<tr>
<td>8</td>
<td>The rearmost of the 3 stars in the left wrist</td>
<td>t 51</td>
<td>+52</td>
<td>&gt;4</td>
<td>ο Her</td>
</tr>
<tr>
<td>9</td>
<td>The northernmost of the other 2</td>
<td>t 11</td>
<td>+54</td>
<td>&gt;4</td>
<td>ϵ Her</td>
</tr>
<tr>
<td>10</td>
<td>The southermost of them</td>
<td>m 10</td>
<td>+53</td>
<td>4</td>
<td>ζ Her</td>
</tr>
<tr>
<td>11</td>
<td>The star in the right side</td>
<td>m 10</td>
<td>*53</td>
<td>3</td>
<td>η Her</td>
</tr>
<tr>
<td>12</td>
<td>The star in the left side</td>
<td>m 10</td>
<td>*53</td>
<td>3</td>
<td>η Her</td>
</tr>
<tr>
<td>13</td>
<td>The one north of the latter, on the left buttock</td>
<td>m 10</td>
<td>+56</td>
<td>5</td>
<td>59(e) Her</td>
</tr>
<tr>
<td>14</td>
<td>The one on the place where the thigh joins the same [buttock]</td>
<td>m 11</td>
<td>+50</td>
<td>3</td>
<td>61(c) Her</td>
</tr>
<tr>
<td>15</td>
<td>The most advanced of the 3 in the left thigh</td>
<td>m 14</td>
<td>+50</td>
<td>4</td>
<td>π Her</td>
</tr>
</tbody>
</table>

1 Literally 'the [figure] on its knees'. Cf. Aratus 63-7. The figure is not identified with any mythological personage in the earlier Greek tradition, or by Germanicus or Ptolemy. For various late identifications with Hercules and other figures see pseudo-Eratosthenes, e.g. Robert, 62-6, Avenien, *Aratus* 175-94, and Boll-Gundel, *Sculpture*, cols. 900-3. 2 The variant 61 occurs in the Greek tradition (A'BC, written '61 + '61), and, according to S 10, the earlier Arabic tradition.

3 Reading γ γ γ, with Is (confirmed by S 110), found as a variant in L, for v (50i) at H55,3 (D) and al Hijājī have 50i, derived from the correct reading by a common scribal error. P-K also adopt 50i (from αγ-50i).

4 The variant 16 occurs in the tradition of both Greek (AVD) and Arabic (see S 11).

5 P-K have the magnitude >4, in better agreement with modern estimates of the magnitude of η Her (3.9). As Manitius (p. 401) says, adopting this would upset the partial and complete totals of 4th and 5th magnitude stars. But since these are probably later accretions, they indicate only that this was counted as a 5th magnitude star in the late Alexandria tradition.

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>The one to the rear of this</td>
<td>m 15</td>
<td>+60</td>
<td>4</td>
<td>69(c) Her</td>
</tr>
<tr>
<td>17</td>
<td>The one yet further to the rear of this</td>
<td>m 16</td>
<td>+61</td>
<td>&gt;4</td>
<td>p Her</td>
</tr>
<tr>
<td>18</td>
<td>The star on the left knee</td>
<td>η 02</td>
<td>+61</td>
<td>4</td>
<td>θ Her</td>
</tr>
<tr>
<td>19</td>
<td>The star on the left shin</td>
<td>m 22</td>
<td>+69</td>
<td>4</td>
<td>ι Her</td>
</tr>
<tr>
<td>20</td>
<td>The most advanced of the 3 stars in the left foot</td>
<td>m 15</td>
<td>+70</td>
<td>6</td>
<td>74(ε) Her</td>
</tr>
<tr>
<td>21</td>
<td>The middle one of the three</td>
<td>m 16</td>
<td>+71</td>
<td>6</td>
<td>77(γ) Her</td>
</tr>
<tr>
<td>22</td>
<td>The rearmost of them</td>
<td>m 19</td>
<td>+72</td>
<td>6</td>
<td>82(ε) Her</td>
</tr>
<tr>
<td>23</td>
<td>The star on the place where the right thigh joins [the buttock]</td>
<td>m 0</td>
<td>+68</td>
<td>&gt;4</td>
<td>η Her</td>
</tr>
<tr>
<td>24</td>
<td>The star north of it in the same thigh</td>
<td>m 15</td>
<td>+65</td>
<td>&gt;4</td>
<td>η Her</td>
</tr>
<tr>
<td>25</td>
<td>The star on the right knee</td>
<td>η 13</td>
<td>+65</td>
<td>4</td>
<td>ψ Her</td>
</tr>
<tr>
<td>26</td>
<td>The southernmost of the 2 stars under the right knee</td>
<td>η 13</td>
<td>+64</td>
<td>4</td>
<td>υ Her</td>
</tr>
<tr>
<td>27</td>
<td>The northernmost of them</td>
<td>η 14</td>
<td>+60</td>
<td>4</td>
<td>χ Her</td>
</tr>
<tr>
<td>28</td>
<td>The star in the right lower leg</td>
<td>η 10</td>
<td>+60</td>
<td>4</td>
<td>χ Her</td>
</tr>
</tbody>
</table>

The star on the end of the right leg is the same as the one on the tip of the staff [of Boötes, V 9].

<table>
<thead>
<tr>
<th>Number in constellation</th>
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<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>The star south of the one in the right upper arm [1 star of the fifth magnitude]</td>
<td>m 2</td>
<td>+38</td>
<td>5</td>
<td>α Lyra</td>
</tr>
<tr>
<td></td>
<td>Star outside this constellation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The bright star on the shell, also called Lyra</td>
<td>η 17</td>
<td>+62</td>
<td>1</td>
<td>α Lyra</td>
</tr>
<tr>
<td>2</td>
<td>The southernmost of the 2 stars lying near the latter, close together</td>
<td>η 20</td>
<td>+62</td>
<td>&gt;4</td>
<td>α Lyra</td>
</tr>
</tbody>
</table>

6 Most Greek mss. have ξ (65). Heiberg adopted 60i from Bode's conjecture. It is in fact the reading of most of the Arabic tradition, according to S 12, and is found in all Arabic mss. examined by Kunitzsch.

7 The variant 74 is found in the earlier Arabic tradition according to S 13.

8 All Greek mss. have ξ (64). Heiberg adopted 60i by conjecture, but it is in fact the reading of almost all the later Arabic tradition (see S 14).

9 The shell of the tortoise from which, in Greek myth, the infant Hermes constructed the first lyre. See e.g. the Homeric Hymn to Hermes 33, Aratus 268-9, and (for other ancient references) Boll-Gundel cols. 904-5. The modern name for the star is Vega.
Consequently the identifications are uncertain.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Number in constellation} & \text{Description} & \text{Longitude in degrees} & \text{Latitude in degrees} & \text{Magnitude} & \text{Modern designation} \\
\hline
14 & The small star over the left foot & *8 201 & +101 & 6 & *14 Aur \\
\hline
1 & The star on the head & m241 & +36 & >3 & \alpha \text{ Oph} \\
2 & The more advanced of the 2 stars on the right shoulder & m281 & +27l & >4 & \beta \text{ Oph} \\
3 & The rearmost of them & m291 & +261 & 4 & \gamma \text{ Oph} \\
4 & The more advanced of the 2 stars on the left shoulder & m131 & +33 & >3 & \kappa \text{ Oph} \\
5 & The rearmost of them & m141 & +311 & 4 & \lambda \text{ Oph} \\
6 & The star on the left elbow & m81 & +241 & 4 & \mu \text{ Oph} \\
7 & The more advanced of the 2 stars in the left hand & m51 & +17 & 3 & \delta \text{ Oph} \\
8 & The rearmost of them & m61 & +161 & 3 & \zeta \text{ Oph} \\
9 & The star on the right elbow & m261 & +15 & 4 & \nu \text{ Oph} \\
10 & The more advanced of the 2 stars in the right hand & m211 & +131 & >4 & \tau \text{ Oph} \\
11 & The rearmost of them & m311 & +141 & 3 & \eta \text{ Oph} \\
12 & The star on the right knee & m211 & +71 & 3 & \xi \text{ Oph} \\
13 & The star on the right lower leg & *m1231 & +211 & >4 & \zeta \text{ Oph} \\
14 & The most advanced of the 4 stars on the right foot & m123 & +21 & >4 & *36(\text{A}) \text{ Oph} \\
15 & The one to the rear of this & m241 & +11 & >4 & *6 \text{ Oph} \\
16 & The one to the rear again of that & m251 & +12 & >4 & *44(\text{b}) \text{ Oph} \\
17 & The last and rearmost of the 4 & m352 & +01 & 5 & *51(\text{c}) \text{ Oph} \\
\hline
\end{array}
\]

\[16 \text{ P-K adopt the reading 23 from the late Greek ms. Par. 2394. There is no good authority for it.} \]
\[16 \text{ Reading } \gamma \text{ (with } \text{A' and part of the Arabic tradition, see S 20) for } \zeta \text{ (16) at H67,19. The related variant } \gamma \text{ (13) is also found, in D and the later Arabic tradition (ibid.). P-K adopt 101.} \]
\[16 \text{ The identification is very uncertain and depends on the coordinates adopted. Kunitch (ibid.} \text{x24}\text{A} \text{G} \text{B} \text{H} \text{86 n.d.) suggests 5 Aur, adopting the coordinates } \beta 201, +16. I retain that of P-K, 14 Aur, which is supported by the location with respect to the Milky Way, described in VIII 2 p. 402 (this virtually excludes Manitius' identification, 2 Aur).} \]
\[16 \text{ P-K is the reading of DL, adopted by Heiberg. Most Greek ms. have } \delta \text{, P-K also adopt } 231, \text{ claiming that it is the reading of some Greek and one Arabic ms. (it appears to be that of T).} \]
\[16 \text{ Reading } \chi \gamma \text{ (with } \text{A'DAr} \text{) for } \kappa \gamma \text{ (26j), at H69,13. The same correction was made by Manitius and P-K.} \]
\[16 \text{ The uncertainty connected with nos. 14 to 17 is whether the latitudes are south or north (for details of the variations see P-K p. 186 nos. 247–50). Consequently the identifications are uncertain (pace P-K, n. on p. 99).} \]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Number in constellation} & \text{Description} & \text{Longitude in degrees} & \text{Latitude in degrees} & \text{Magnitude} & \text{Modern designation} \\
\hline
18 & The star to the rear of these, which touches the heel & m271 & +1 & 5 & \*\text{1} \text{ Oph} \\
19 & The star in the left knee & m121 & +111 & 3 & \zeta \text{ Oph} \\
20 & The northernmost of the 3 stars in a straight line in the left lower leg & m111 & +31 & >5 & \psi \text{ Oph} \\
21 & The middle one of these & m101 & +31 & 5 & \chi \text{ Oph} \\
22 & The southernmost of these & m91 & +11 & >5 & \psi \text{ Oph} \\
23 & The star on the left heel & m121 & +01 & 5 & \omega \text{ Oph} \\
24 & The star touching the hollow of the left foot & m101 & +01 & 4 & \rho \text{ Oph} \\
\hline
\end{array}
\]

\[\text{Stars around Ophiuchus outside the constellation:}\]
\[\text{25} \text{ The northernmost of the 3 to the cast of the right shoulder} \quad m2 & +281 & 4 & 66(\text{n}) \text{ Oph} \\
\text{26} \text{ The middle one of the three} \quad m21 & +261 & 4 & 67 \text{ Oph} \\
\text{27} \text{ The southernmost of them} \quad m3 & +25 & 4 & 68 \text{ Oph} \\
\text{28} \text{ The star to the rear of these 3, approximately over the middle one} \quad m3 & +27 & 4 & 70 \text{ Oph} \\
\text{29} \text{ The lone star north of [these] 4 (nos 25–28) [5 stars of the fourth magnitude]} \quad m41 & +33 & 4 & 72 \text{ Oph} \\
\hline
\end{array}
\]

\[\text{[XIV] Constellation of Serpens}\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Number in constellation} & \text{Description} & \text{Longitude in degrees} & \text{Latitude in degrees} & \text{Magnitude} & \text{Modern designation} \\
\hline
1-5 & \text{Stars on the quadrilateral in the head: } & \pm & +38 & 4 & \text{Ser} \\
1 & \text{the one on the end of the jaw} & 1 & +40 & 4 & \rho \text{ Ser} \\
2 & \text{the one touching the nostrils} & 21 & +341 & 3 & \beta \text{ Ser} \\
3 & \text{the one in the temple} & 24 & +361 & 3 & \gamma \text{ Ser} \\
4 & \text{the one where the neck joins [the head]} & 22 & +371 & 4 & \kappa \text{ Ser} \\
5 & \text{the one in the middle of the quadrilateral, in the mouth} & 21 & +421 & 4 & \pi \text{ Ser} \\
6 & \text{The star outside the head, to the north of it} & 23 & +291 & 3 & \delta \text{ Ser} \\
\hline
\end{array}
\]

\[16 \text{ The later Arabic tradition is solid for the variant } \theta \text{ (see S 20).} \]
\[16 \text{ Literally 'of the snake of the snake-holder [Ophiuchus]'. This is to distinguish it from Draco and Hydra (the big snake and the water-snake).} \]
\[16 \text{ The Greek tradition is uniform for 211. Heiberg adopted 241 as an emendation by Bode. However, it is well-attested in the Arabic tradition (see S 22).} \]
\[16 \text{ Reading } \gamma \chi \text{ (with } \text{A'DAr} \text{) for } \zeta \gamma \text{ (26j), at H71,18.} \]
**[XII] Constellation of Auriga**

<table>
<thead>
<tr>
<th>Number in [XII] Constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The one to the rear of this, on the left foot</td>
<td>Ν 61.50</td>
<td>+11.00</td>
<td>&gt;3</td>
<td>ζ Per</td>
</tr>
<tr>
<td>2</td>
<td>The one to the north of the one 161 in the right knee [no. 16]</td>
<td>Π 24.50</td>
<td>+20.00</td>
<td>&lt;3</td>
<td>Τaur</td>
</tr>
<tr>
<td>3</td>
<td>The one in advance of those in the Gorgon-head [noa. 12-15]</td>
<td>[3 stars, 2 of the fifth magnitude, 1 faint]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The one to the north of the latter, in the lower hem [of the garment]</td>
<td>Ν 26.40</td>
<td>+41.00</td>
<td>5</td>
<td>Χ Aur</td>
</tr>
<tr>
<td>5</td>
<td>The one north again of this, on the buttock</td>
<td>Ν 26.40</td>
<td>+42.00</td>
<td>5</td>
<td>Φ Aur</td>
</tr>
</tbody>
</table>

---

**[XII] Constellation of Perseus**

<table>
<thead>
<tr>
<th>Number in [XII] Constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The nebulus mass on the right hand</td>
<td>Π 26</td>
<td>+40</td>
<td>neb.</td>
<td>CGal 884 + 869</td>
</tr>
<tr>
<td>2</td>
<td>The star on the right elbow</td>
<td>Ν 14</td>
<td>+37</td>
<td>4</td>
<td>η Per</td>
</tr>
<tr>
<td>3</td>
<td>The star on the right shoulder</td>
<td>Ν 21</td>
<td>+34</td>
<td>&lt;3</td>
<td>γ Per</td>
</tr>
<tr>
<td>4</td>
<td>The star on the left shoulder</td>
<td>Ν 21</td>
<td>+34</td>
<td>4</td>
<td>θ Per</td>
</tr>
<tr>
<td>5</td>
<td>The star on the head</td>
<td>Ν 01</td>
<td>+34</td>
<td>4</td>
<td>Ψ Per</td>
</tr>
<tr>
<td>6</td>
<td>The star on the place between the shoulders 16</td>
<td>Ν 11</td>
<td>+31</td>
<td>4</td>
<td>ι Per</td>
</tr>
<tr>
<td>7</td>
<td>The most advanced of the 3 stars next to the one in the side.</td>
<td>Π 27</td>
<td>+32</td>
<td>3</td>
<td>κ Per</td>
</tr>
<tr>
<td>8</td>
<td>The middle one of the three</td>
<td>Ν 7</td>
<td>+27</td>
<td>4</td>
<td>Ψ Per</td>
</tr>
<tr>
<td>9</td>
<td>The remnant of them</td>
<td>Ν 7</td>
<td>+27</td>
<td>3</td>
<td>δ Per</td>
</tr>
<tr>
<td>10</td>
<td>The star on the left elbow</td>
<td>Ν 01</td>
<td>+27</td>
<td>4</td>
<td>κ Per</td>
</tr>
<tr>
<td>11</td>
<td>The bright one</td>
<td>Π 29</td>
<td>+23</td>
<td>2</td>
<td>β Per</td>
</tr>
<tr>
<td>12</td>
<td>The one to the rear of this</td>
<td>Π 29</td>
<td>+21</td>
<td>4</td>
<td>ω Per</td>
</tr>
<tr>
<td>13</td>
<td>The one in advance of the bright star</td>
<td>Π 27</td>
<td>+21</td>
<td>4</td>
<td>ρ Per</td>
</tr>
<tr>
<td>14</td>
<td>The remaining one, yet again in advance of this</td>
<td>Π 26</td>
<td>+22</td>
<td>4</td>
<td>π Per</td>
</tr>
<tr>
<td>15</td>
<td>The star in the right knee</td>
<td>Ν 14</td>
<td>+26</td>
<td>4</td>
<td>λ Per</td>
</tr>
<tr>
<td>16</td>
<td>The one in advance of this, over the knee</td>
<td>Ν 13</td>
<td>+26</td>
<td>5</td>
<td>53(α) Per</td>
</tr>
<tr>
<td>17</td>
<td>The more advanced of the 2 stars above the bend in the knee</td>
<td>Π 12</td>
<td>+25</td>
<td>4</td>
<td>49(β) Per</td>
</tr>
<tr>
<td>18</td>
<td>The remnant of them, just over the bend in the knee</td>
<td>Π 14</td>
<td>+26</td>
<td>4</td>
<td>μ Per</td>
</tr>
<tr>
<td>19</td>
<td>The star on the right calf</td>
<td>Ν 14</td>
<td>+24</td>
<td>5</td>
<td>56(c) Per</td>
</tr>
<tr>
<td>20</td>
<td>The star on the right ankle</td>
<td>Ν 16</td>
<td>+18</td>
<td>5</td>
<td>λ Per</td>
</tr>
<tr>
<td>21</td>
<td>The star in the right thigh</td>
<td>Ν 61</td>
<td>+21</td>
<td>5</td>
<td>ν Per</td>
</tr>
<tr>
<td>22</td>
<td>The star on the left knee</td>
<td>Ν 61</td>
<td>+21</td>
<td>3</td>
<td>η Per</td>
</tr>
<tr>
<td>23</td>
<td>The star on the left lower leg</td>
<td>Ν 61</td>
<td>+21</td>
<td>4</td>
<td>ξ Per</td>
</tr>
<tr>
<td>24</td>
<td>The one in advance of this, over the knee</td>
<td>Π 12</td>
<td>+20</td>
<td>3</td>
<td>η Per</td>
</tr>
<tr>
<td>25</td>
<td>The star on the left foot</td>
<td>Π 24</td>
<td>+18</td>
<td>5</td>
<td>ι Per</td>
</tr>
<tr>
<td>26</td>
<td>The one to the rear of this, on the left foot</td>
<td>[26 stars, 2 of the second magnitude, 5 of the third, 16 of the fourth, 2 of the fifth, 1 nebulus]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

12 Manilius identifies this as Ρκ Per, P-K as χ Per. These are, respectively, the Galactic Clusters 869 and 884, which appear as a single hazy patch to the naked eye (see Burnham III 1438).

13 μεδόντακος. See p. 356 n.159. Here Perseus may be envisioned as partly turned to the side, so that some of his back is visible.

14 The head of Medusa, carried in Perseus' left hand (see the depiction in Boll-Gundel col. 914).

15 28 is the reading of all Greek ms., 281 that of some Arabic ms. (A,E,F), adopted by P-K.

---

**[XII] Constellation XI. Pictus**

<table>
<thead>
<tr>
<th>Number in [XII] Constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The one to the rear of this, on the left foot</td>
<td>Ν 61</td>
<td>+11</td>
<td>&gt;3</td>
<td>ζ Per</td>
</tr>
<tr>
<td>2</td>
<td>The one to the north of the one 161 in the right knee [no. 16]</td>
<td>Π 24</td>
<td>+20</td>
<td>&lt;3</td>
<td>Τaur</td>
</tr>
<tr>
<td>3</td>
<td>The one in advance of those in the Gorgon-head [noa. 12-15]</td>
<td>[3 stars, 2 of the fifth magnitude, 1 faint]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

12 Reading το βυ (with D,E,T,Ger) for τον βυ (those in) at H64,125.

13 The variant 16 is found in the later Arabie tradition according to S 19.

14 See XXIII 21. The magnitude there is given as 3. The star is also known as γ Aurigae, but today is included in the constellation Taurus.

15 ηυπέρηχον. Auriga ('the charioteer') is depicted as wearing a long tunic reaching to the feet, like the well-known bronze Delphic charioteer (see e.g. Richter, Handbook of Greek Art, Fig. 113 p. 485).
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>The southernmost of the 3 stars between the tail and the rhombus</td>
<td>Ξ 17</td>
<td>+30</td>
<td>6</td>
<td>η Del</td>
</tr>
<tr>
<td>9</td>
<td>The more advanced of the 2 to the north</td>
<td>Ξ 17</td>
<td>+31</td>
<td>6</td>
<td>ζ Del</td>
</tr>
<tr>
<td>10</td>
<td>The remaining, rearmost one</td>
<td>Ξ 19</td>
<td>+31</td>
<td>6</td>
<td>θ Del</td>
</tr>
<tr>
<td></td>
<td>[10 stars, 5 of the third magnitude, 2 of the fourth, 3 of the sixth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[XVIII] Constellation of Equuleus(^1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The more advanced of the 2 stars in the head</td>
<td>Ξ 26</td>
<td>+20</td>
<td>E</td>
<td>α Equ</td>
</tr>
<tr>
<td>2</td>
<td>The rearmost of them</td>
<td>Ξ 28</td>
<td>+20</td>
<td>E</td>
<td>β Equ</td>
</tr>
<tr>
<td>3</td>
<td>The more advanced of the two stars in the month</td>
<td>Ξ 26</td>
<td>+25</td>
<td>E</td>
<td>γ Equ</td>
</tr>
<tr>
<td>4</td>
<td>The rearmost of them</td>
<td>Ξ 27</td>
<td>+25</td>
<td>E</td>
<td>δ Equ</td>
</tr>
<tr>
<td></td>
<td>[4 stars, [all] faint]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[XIX] Constellation of Pegasus(^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The star on the navel, which is [applied in] common to the head of Andromeda</td>
<td>Ξ 17</td>
<td>+26</td>
<td>&lt;2</td>
<td>α And</td>
</tr>
<tr>
<td>2</td>
<td>The star on the ramp and the wing-tip</td>
<td>Ξ 12</td>
<td>+12</td>
<td>&lt;2</td>
<td>γ Peg</td>
</tr>
<tr>
<td>3</td>
<td>The star on the right shoulder and the place where the leg joins [it]</td>
<td>Ξ 2</td>
<td>+31</td>
<td>&lt;2</td>
<td>ζ Peg</td>
</tr>
<tr>
<td>4</td>
<td>The star on the place between the shoulders and the shoulder-part of the wing</td>
<td>Ξ 26</td>
<td>+19</td>
<td>&lt;2</td>
<td>θ Peg</td>
</tr>
<tr>
<td>5</td>
<td>The northernmost of the two stars in the body under the wing</td>
<td>Ξ 4</td>
<td>+25</td>
<td>4</td>
<td>ι Peg</td>
</tr>
<tr>
<td>6</td>
<td>The southernmost of them</td>
<td>Ξ 5</td>
<td>+25</td>
<td>4</td>
<td>υ Peg</td>
</tr>
<tr>
<td>7</td>
<td>The northernmost of the two stars in the right knee</td>
<td>Ξ 29</td>
<td>+35</td>
<td>3</td>
<td>η Peg</td>
</tr>
<tr>
<td>8</td>
<td>The southernmost of them</td>
<td>Ξ 28</td>
<td>+34</td>
<td>5</td>
<td>ο Peg</td>
</tr>
</tbody>
</table>

\(^1\) The variant 34 occurs in the Greek (C, '31' in D) and Arabic traditions (see S 286).

\(^2\) Literally 'horse', but the references to its wings make it clear that it is depicted as Pegasus. The identification was made as early as Aratus (216-24).

The star is also known as δ Pegasi, but in modern times is defined as being in Andromeda.

---

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>The more advanced of the two stars close together in the chest</td>
<td>Ξ 29</td>
<td>+29</td>
<td>4</td>
<td>λ Peg</td>
</tr>
<tr>
<td>10</td>
<td>The rearmost of them</td>
<td>Ξ 27</td>
<td>+29</td>
<td>4</td>
<td>μ Peg</td>
</tr>
<tr>
<td>11</td>
<td>The more advanced of the 2 stars close together in the neck</td>
<td>Ξ 18</td>
<td>+18</td>
<td>4</td>
<td>ζ Peg</td>
</tr>
<tr>
<td>12</td>
<td>The rearmost of them</td>
<td>Ξ 20</td>
<td>+15</td>
<td>5</td>
<td>η Peg</td>
</tr>
<tr>
<td>13</td>
<td>The southernmost of the two stars on the mane</td>
<td>Ξ 20</td>
<td>+16</td>
<td>5</td>
<td>θ Peg</td>
</tr>
<tr>
<td>14</td>
<td>The northernmost of them</td>
<td>Ξ 20</td>
<td>+15</td>
<td>5</td>
<td>ι Peg</td>
</tr>
<tr>
<td>15</td>
<td>The northernmost of the two stars close together on the head</td>
<td>Ξ 20</td>
<td>+16</td>
<td>5</td>
<td>υ Peg</td>
</tr>
<tr>
<td>16</td>
<td>The southernmost of them</td>
<td>Ξ 20</td>
<td>+16</td>
<td>4</td>
<td>η Peg</td>
</tr>
<tr>
<td>17</td>
<td>The star in the muzzle</td>
<td>Ξ 20</td>
<td>+14</td>
<td>&gt;4</td>
<td>η Peg</td>
</tr>
<tr>
<td>18</td>
<td>The star in the right hand</td>
<td>Ξ 20</td>
<td>+15</td>
<td>&gt;4</td>
<td>η Peg</td>
</tr>
<tr>
<td>19</td>
<td>The star on the left knee</td>
<td>Ξ 20</td>
<td>+16</td>
<td>&gt;4</td>
<td>η Peg</td>
</tr>
<tr>
<td>20</td>
<td>The star in the left hock</td>
<td>Ξ 20</td>
<td>+16</td>
<td>&gt;4</td>
<td>η Peg</td>
</tr>
</tbody>
</table>

[XX] Constellation of Andromeda

---

\(^1\) Most Greek ms (A'BC) and Is have 94. Heiberg adopts the reading of D,L.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>The northernmost of the 3 following this</td>
<td>182 14 13</td>
<td>+261 48 30</td>
<td>4</td>
<td>λ Ser</td>
</tr>
<tr>
<td>9</td>
<td>The middle one of the three</td>
<td>182 14 25</td>
<td>+251 48 30</td>
<td>3</td>
<td>α Ser</td>
</tr>
<tr>
<td>10</td>
<td>The southernmost of them</td>
<td>182 14 24</td>
<td>+241 48 30</td>
<td>3</td>
<td>μ Ser</td>
</tr>
<tr>
<td>11</td>
<td>The star after the next bend, which is in advance of the left hand of Ophiuchus</td>
<td>182 14 28</td>
<td>+161 48 30</td>
<td>4</td>
<td>ε Ser</td>
</tr>
<tr>
<td>12</td>
<td>The star to the rear of those in the hand of Ophiuchus</td>
<td>182 14 8</td>
<td>+131 48 30</td>
<td>5</td>
<td>Ω Ophi</td>
</tr>
<tr>
<td>13</td>
<td>The one after the back of the right thigh of Ophiuchus</td>
<td>182 14 23</td>
<td>+101 48 30</td>
<td>4</td>
<td>ν Ser</td>
</tr>
<tr>
<td>14</td>
<td>The southernmost of the 2 to the rear of the latter</td>
<td>182 14 27</td>
<td>+81 48 30</td>
<td>4</td>
<td>ζ Ser</td>
</tr>
<tr>
<td>15</td>
<td>The northernmost of them</td>
<td>182 14 1</td>
<td>+201 48 30</td>
<td>4</td>
<td>η Ser</td>
</tr>
<tr>
<td>16</td>
<td>The one after the right hand of Ophiuchus, on the bend in the tail</td>
<td>182 14 8</td>
<td>+211 48 30</td>
<td>4</td>
<td>η Ser</td>
</tr>
<tr>
<td>17</td>
<td>The one to the rear of this, likewise on the tail</td>
<td>182 14 11</td>
<td>+271 48 30</td>
<td>4</td>
<td>η Ser</td>
</tr>
<tr>
<td>18</td>
<td>The star on the tip of the tail</td>
<td>182 14 18</td>
<td>+271 48 30</td>
<td>4</td>
<td>η Ser</td>
</tr>
</tbody>
</table>

[XV] Constellation of Sagitta

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The lone star on the arrow-head</td>
<td>182 14 10</td>
<td>+391 48 30</td>
<td>4</td>
<td>γ Ser</td>
</tr>
<tr>
<td>2</td>
<td>The rearmost of the three stars in the shaft</td>
<td>182 14 6</td>
<td>+391 48 30</td>
<td>6</td>
<td>ζ Ser</td>
</tr>
<tr>
<td>3</td>
<td>The middle one</td>
<td>182 14 51</td>
<td>+391 48 30</td>
<td>5</td>
<td>δ Ser</td>
</tr>
<tr>
<td>4</td>
<td>The most advanced of the three</td>
<td>182 14 31</td>
<td>+391 48 30</td>
<td>5</td>
<td>α Ser</td>
</tr>
<tr>
<td>5</td>
<td>The star on the end of the notch</td>
<td>182 14 38</td>
<td>+381 48 30</td>
<td>5</td>
<td>β Ser</td>
</tr>
</tbody>
</table>

[XVI] Constellation of Aquila

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The star in the middle of the head</td>
<td>182 14 71</td>
<td>+262 48 30</td>
<td>4</td>
<td>τ Aql</td>
</tr>
<tr>
<td>2</td>
<td>The one in advance of this, on the neck</td>
<td>182 14 41</td>
<td>+271 48 30</td>
<td>3</td>
<td>β Aql</td>
</tr>
<tr>
<td>3</td>
<td>The bright star on the place between the shoulders, called Aquila</td>
<td>182 14 31</td>
<td>+291 48 30</td>
<td>2&gt;2</td>
<td>α Aql</td>
</tr>
</tbody>
</table>

[XVII] Constellation of Aquila

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The one close to this towards the north</td>
<td>182 14 41</td>
<td>+301 48 30</td>
<td>&lt;3</td>
<td>o Aql</td>
</tr>
<tr>
<td>5</td>
<td>The more advanced of the 2 in the left shoulder</td>
<td>182 14 31</td>
<td>+311 48 30</td>
<td>3</td>
<td>γ Aql</td>
</tr>
<tr>
<td>6</td>
<td>The rearmost of them</td>
<td>182 14 6</td>
<td>+311 48 30</td>
<td>5</td>
<td>ζ Aql</td>
</tr>
<tr>
<td>7</td>
<td>The more advanced of the two in the right shoulder</td>
<td>182 14 21</td>
<td>+281 48 30</td>
<td>5</td>
<td>δ Aql</td>
</tr>
<tr>
<td>8</td>
<td>The rearmost of them</td>
<td>182 14 14</td>
<td>+261 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
<tr>
<td>9</td>
<td>The star some distance under the tail of Aquila, touching the Milky Way</td>
<td>182 14 21</td>
<td>+361 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
</tbody>
</table>

[XVII] Constellation of Delphinus

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The most advanced of the 2 stars south of the head of Aquila</td>
<td>182 14 31</td>
<td>+211 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
<tr>
<td>11</td>
<td>The rearmost of them</td>
<td>182 14 81</td>
<td>+191 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
<tr>
<td>12</td>
<td>The star to the south and west of the right shoulder of Aquila</td>
<td>182 14 26</td>
<td>+251 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
<tr>
<td>13</td>
<td>The one to the south of this</td>
<td>182 14 28</td>
<td>+201 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
<tr>
<td>14</td>
<td>The one to the south again of the latter</td>
<td>182 14 29</td>
<td>+151 48 30</td>
<td>5</td>
<td>η Aql</td>
</tr>
<tr>
<td>15</td>
<td>The star most in advance of all</td>
<td>182 14 211</td>
<td>+181 48 30</td>
<td>3</td>
<td>η Aql</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The star in the rhomboid quadrilateral:</td>
<td>182 14 18</td>
<td>+321 48 30</td>
<td>&lt;3</td>
<td>β Del</td>
</tr>
<tr>
<td>5</td>
<td>The southernmost one on the advance side</td>
<td>182 14 20</td>
<td>+331 48 30</td>
<td>&lt;3</td>
<td>δ Del</td>
</tr>
<tr>
<td>6</td>
<td>The southernmost one on the rear side of the rhombus</td>
<td>182 14 23</td>
<td>+331 48 30</td>
<td>&lt;3</td>
<td>γ Del</td>
</tr>
</tbody>
</table>

131 I'arberg’s emendation (following Bode, who in fact conjectured '131'). All mss. have 161. See the discussion of P-K, pp. 99-100.

132 The variant 16 is found in the Greek ms. D and in the later Arabic tradition (see S 24).

133 This is the reading of D, adopted by Heiberg, where most Greek mss. have 371. The Arabic tradition varies between 381 and 383 (see S 25).

134 The variant 16 is found in the Greek ms. D and in the later Arabic tradition (see S 24).

135 Heiberg adopted K5 (20) as an emendulation of Bode, but it is in fact found in all Arabic mss. I have examined.
**[Number in constellation]** | **Description** | **Longitude in degrees** | **Latitude in degrees** | **Magnitude** | **[Modern designation]**
--- | --- | --- | --- | --- | ---
3 | The one close again to the latter | *P 241| | -9j | 4 | ζ Tau
4 | The southernmost of the 4 | Τ′ 241 | -9j | 4 | ζ Tau
5 | The one to the rear of those, on the right shoulder-blade | Τ′ 241 | -9j | 5 | 30(c) Tau
6 | The star in the chest | Τ′ 241 | -8j | 3 | λ Tau
7 | The star on the right knee | Τ′ 241 | -12j | 3 | μ Tau
8 | The star on the right hock | Τ′ 241 | -14j | 3 | ν Tau
9 | The star on the left knee | Τ′ 241 | -10j | 4 | 90(c) Tau
10 | The star on the left lower leg | Τ′ 241 | -13j | 4 | 88(d) Tau
11-15 | The stars in the face, called 'the Hyades': | Τ′ 241 | -| 4 | 4τ Tau
11 | the one on the nostrils | Τ′ 241 | -5j | 3 | γ Tau
12 | the one between this and the northern eye | Τ′ 241 | -4j | 3 | δ′ Tau
13 | the one between it (no. 11) and the southern eye | Τ′ 241 | -5j | 3 | δ′ Tau
14 | the bright star of the Hyades, the reddish one on the southern eye | Τ′ 241 | -5j | 1 | α Tau
15 | the remaining one, on the northern eye | Τ′ 241 | -3j | 3 | ε Tau
16 | The star on the place where the southern horn and the ear join [the head] | Τ′ 241 | -4j | 4 | 97(i) Tau
17 | The southernmost of the 2 stars on the southern horn | Τ′ 241 | -5j | 5 | 104(m) Tau
18 | The southernmost of these | Τ′ 241 | -3j | 5 | 106(i) Tau
19 | The star on the tip of the southern horn | Τ′ 241 | -7j | 3 | ζ Tau
20 | The star on the place where the northern horn joins [the head] | Τ′ 241 | -1j | 4 | τ Tau
21 | The star on the tip of the northern horn, which is the same as the one on the right foot of Auriga [XI no. 11] | Τ′ 241 | -5j | 3 | β Tau

---

**[Number in constellation]** | **Description** | **Longitude in degrees** | **Latitude in degrees** | **Magnitude** | **[Modern designation]**
--- | --- | --- | --- | --- | ---
22 | The northernmost of the 2 stars close together in the northern ear | Τ′ 241 | -0j | 5 | u Tau
23 | The southernmost of them | Τ′ 241 | -0j | 5 | k Tau
24 | The more advanced of the 2 small stars in the neck | Τ′ 241 | -0j | 5 | 37(A) Tau
25 | The remotest of them | Τ′ 241 | -0j | 6 | ω Tau
26-29 | The quadrilateral in the neck: | Τ′ 241 | -5j | 5 | 44(p) Tau
26 | the southernmost star on the advance side | Τ′ 241 | -5j | 5 | η Tau
27 | the northernmost star on the advance side | Τ′ 241 | -5j | 5 | η Tau
28 | the southern star on the rear side | Τ′ 241 | -5j | 5 | η Tau
29 | the northernmost on the rear side | Τ′ 241 | -5j | 5 | η Tau
30-33 | The Pleiades: | Τ′ 241 | -| 5 | *19 Tau
30 | the northern end of the advance side | Τ′ 241 | -4j | 5 | *23 Tau
31 | the southern end of the advance side | Τ′ 241 | -4j | 5 | 23 Tau
32 | the remotest and narrowest end of the Pleiades | Τ′ 241 | -4j | 5 | BSC 1188
33 | the small star outside of the Pleiades, towards the north | Τ′ 241 | -4j | 5 | 1188

---

173 P–K adopt 241, the reading of Ar, which is no doubt the origin of the corruption 211 in D.
174 The variant 01 is found in part of the Arabic tradition according to S 28. Manitius (p. 401) changes to 101 (1 γ for τ), with no ms. authority.
175 The variant 101 occurs in the later Arabic tradition (see S 29).
176 The variant 14 occurs in the earlier Arabic tradition according to S 30.
177 Reading 'γ' (with D, Ar, adopted by P–K) for 'γ' (I7j) at H90, 4.
178 The variant 4 is found in some Greek mss. (BC) and in the whole of the Arabic tradition according to S 32. 01 is undoubtedly correct, but the latitude might be north instead of south (see P–K on no. 399 p. 101, Manitius pp. 401-2).
179 In Auriga (XII, 11) the magnitude is given as > 3.

---
Description

1. The star in the apex of the triangle
2. The most advanced of the 3 on the base
3. The middle one of these
4. The rearmost of the three

[XXII] Constellation of Aries

1. The more advanced of the 2 stars on the horn
2. The rearmost of them
3. The northernmost of the 2 stars on the muzzle
4. The southernmost of them

[XXIII] Constellation of Taurus

1. The northernmost of the 4 stars in the cut-off
2. The one close by this

Also known as υ Piscium, but within the constellation Andromeda according to the modern boundaries.
### Constellation of Cancer

**[XXV]** Constellation of Cancer

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The middle of the nebulous mass in the chest, called Perseus²²³</td>
<td>101°</td>
<td>+6°19'38&quot;</td>
<td>neb.</td>
<td>CGal 2632 (Messier 44)</td>
</tr>
<tr>
<td>2-5</td>
<td>The quadrilateral containing the nebula [no. 1]; the northernmost of the two stars in advance; the southernmost of the two stars in advance; the northernmost of the rear 2 stars on the quadrilateral, which are called 'Apsib'²²⁹</td>
<td>71°</td>
<td>+1°40'</td>
<td>&lt;4</td>
<td>ζ Cnc</td>
</tr>
<tr>
<td>6</td>
<td>The star on the southern claw</td>
<td>8°</td>
<td>-1°40'</td>
<td>&lt;4</td>
<td>θ Cnc</td>
</tr>
<tr>
<td>7</td>
<td>The star on the northern claw</td>
<td>101°</td>
<td>+21°40'</td>
<td>&gt;4</td>
<td>γ Cnc</td>
</tr>
<tr>
<td>8</td>
<td>The star on the northern back leg</td>
<td>81°</td>
<td>+11°40'</td>
<td>&gt;2</td>
<td>μ Cnc</td>
</tr>
<tr>
<td>9</td>
<td>The star on the southern back leg</td>
<td>71°</td>
<td>+27°00'</td>
<td>&gt;4</td>
<td>β Cnc</td>
</tr>
</tbody>
</table>

### Stars around Cancer outside the constellation:

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>The star over the joint in the southern claw</td>
<td>197°23'</td>
<td>-2°20'</td>
<td>&lt;4</td>
<td>α Cnc</td>
</tr>
<tr>
<td>11</td>
<td>The star to the rear of the tip of the southern claw</td>
<td>219°</td>
<td>-5°40'</td>
<td>&lt;4</td>
<td>κ Cnc</td>
</tr>
<tr>
<td>12</td>
<td>The more advanced of the two stars over the nebula and to the rear of it</td>
<td>14°</td>
<td>+27°12'</td>
<td>5</td>
<td>ψ Cnc</td>
</tr>
<tr>
<td>13</td>
<td>The rearmost of these [two]</td>
<td>17°</td>
<td>+4°30'</td>
<td>5</td>
<td>χ Cnc</td>
</tr>
</tbody>
</table>

²²³ ἀσκάλων ('manger'). Manitius and P-K identify this star Cnc, which is indeed in the middle of the galactic cluster, but Ptolemy is clearly not referring to an individual star.

²²⁹ ἀπόσις ('asses').

²³⁰ On the large error in latitude see P-K no. 457 p. 102.

²³¹ The variant 19 is found in some Greek mss. (B) and in the earlier Arabic tradition. According to S 40, the Ishaq translation and Thabit's revision of it had 15. Extant Arabic mss. (except at-Ṭfāl's revision, which has 14) exhibit 19. If we accept the latter, the most probable identification is ζ Cnc (adopted by Manitius); P-K adopt 15(!) and identify the star as α1 + α2 Cnc.

²³² Following P-K and Manitius (who does it without comment), I have obviously transposed the latitudes of nos. 12 and 13, which then fit the actual positions of ν and ρ Cnc fairly well. There is no ms. authority for this.

### Constellation of Leo

**[XXVI]** Constellation of Leo

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The star on the tip of the nostrils</td>
<td>181°</td>
<td>+10°</td>
<td>4</td>
<td>λ Leo</td>
</tr>
<tr>
<td>2</td>
<td>The star in the gaping jaws</td>
<td>213°</td>
<td>+7°</td>
<td>4</td>
<td>Υ Leo</td>
</tr>
<tr>
<td>3</td>
<td>The northernmost of the two stars in the head</td>
<td>241°</td>
<td>+12°</td>
<td>3</td>
<td>μ Leo</td>
</tr>
<tr>
<td>4</td>
<td>The southernmost of these</td>
<td>241°</td>
<td>+9°</td>
<td>&gt;3</td>
<td>ε Leo</td>
</tr>
<tr>
<td>5</td>
<td>The northernmost of the 3 stars in the neck</td>
<td>241°23'</td>
<td>+11°40'</td>
<td>3</td>
<td>ζ Leo</td>
</tr>
<tr>
<td>6</td>
<td>The one close to this, the middle one of the three</td>
<td>22°</td>
<td>+8°</td>
<td>2</td>
<td>γ Leo</td>
</tr>
<tr>
<td>7</td>
<td>The southernmost of them</td>
<td>22°</td>
<td>+4°</td>
<td>3</td>
<td>η Leo</td>
</tr>
<tr>
<td>8</td>
<td>The star on the heart, called 'Regulus'</td>
<td>22°</td>
<td>+6°</td>
<td>1</td>
<td>α Leo</td>
</tr>
<tr>
<td>9</td>
<td>The one south of this, about on the chest</td>
<td>23°</td>
<td>+12°</td>
<td>4</td>
<td>31(A) Leo</td>
</tr>
<tr>
<td>10</td>
<td>The star a little in advance of the star on the heart [no. 8]</td>
<td>0°</td>
<td>0°</td>
<td>5</td>
<td>ν Leo</td>
</tr>
<tr>
<td>11</td>
<td>The star on the northern back leg</td>
<td>271°</td>
<td>0°</td>
<td>5</td>
<td>ψ Leo</td>
</tr>
<tr>
<td>12</td>
<td>The star on the right front claw-clutch²³⁴</td>
<td>224°</td>
<td>-3°45'</td>
<td>5°215</td>
<td>ζ Leo</td>
</tr>
<tr>
<td>13</td>
<td>The star on the left front claw-clutch</td>
<td>270°</td>
<td>-4°40'</td>
<td>4</td>
<td>ο Leo</td>
</tr>
<tr>
<td>14</td>
<td>The star on the left [front] knee</td>
<td>21°</td>
<td>-4°40'</td>
<td>4</td>
<td>π Leo</td>
</tr>
<tr>
<td>15</td>
<td>The star on the left arm pit</td>
<td>92°</td>
<td>-0°40'</td>
<td>4</td>
<td>ρ Leo</td>
</tr>
<tr>
<td>16</td>
<td>The most advanced of the three stars in the belly</td>
<td>7°</td>
<td>+4°</td>
<td>6</td>
<td>466(i) Leo</td>
</tr>
<tr>
<td>17</td>
<td>The northernmost of the other, rearmost 2</td>
<td>103°23'</td>
<td>+5°40'</td>
<td>6</td>
<td>52(b) Leo</td>
</tr>
<tr>
<td>18</td>
<td>The southernmost of these [two]</td>
<td>122°</td>
<td>+21°40'</td>
<td>6</td>
<td>53(d) Leo</td>
</tr>
<tr>
<td>19</td>
<td>The more advanced of the two stars on the rump</td>
<td>111°</td>
<td>+12°40'</td>
<td>&gt;6°17</td>
<td>60(b) Leo</td>
</tr>
<tr>
<td>20</td>
<td>The rearmost of them</td>
<td>141°</td>
<td>+13°</td>
<td>&lt;2</td>
<td>δ Leo</td>
</tr>
<tr>
<td>21</td>
<td>The northernmost of the 2 stars in the buttocks</td>
<td>141°</td>
<td>+11°40'</td>
<td>5</td>
<td>81 Leo²³⁵</td>
</tr>
<tr>
<td>22</td>
<td>The southernmost of them</td>
<td>161°</td>
<td>+9°40'</td>
<td>3</td>
<td>θ Leo</td>
</tr>
</tbody>
</table>

²³⁴ The variant 14 occurs in the early Arabic tradition according to S 41.

²³⁵ Πάτων, literally 'grasping hand'. The lion is represented with claws out and hooked, as in Thiele Fig. 26 p. 99.

²³⁶ All mss. except D give magnitude 6 here. Heilerg adopts 5 to reach agreement with the sub-total for the constellation. P-K adopt 6 here and 5 at no. 19 (from the Arabic), perhaps rightly.

²³⁷ The variant 13 occurs in the Arabic tradition (see S 42).

²³⁸ Cf. n. 215 on no. 12. All Greek mss. have 6, but the Arabic tradition is unanimous for 5. If correctly identified as 60 Leonis, this star has, by modern definition and measurement, magnitude 44.'
<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stars around Taurus outside the constellation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>The star under the right foot and 33 the shoulder-blade</td>
<td>Π 25</td>
<td>-17°</td>
<td>4</td>
<td>10 Tau</td>
</tr>
<tr>
<td>35</td>
<td>The most advanced of the 3 stars over the southern horn</td>
<td>Π 20°</td>
<td>-2°</td>
<td>5</td>
<td>ε Gem</td>
</tr>
<tr>
<td>36</td>
<td>The middle one of the three</td>
<td>Π 24°</td>
<td>-12°</td>
<td>5</td>
<td>δ Gem</td>
</tr>
<tr>
<td>37</td>
<td>The remnant of them</td>
<td>Π 26</td>
<td>-2°</td>
<td>5</td>
<td>γ Gem</td>
</tr>
<tr>
<td>38</td>
<td>The northermost of the 2 stars under the tip of the southern horn</td>
<td>Π 29</td>
<td>-6°</td>
<td>5</td>
<td>δ Tau</td>
</tr>
<tr>
<td>39</td>
<td>The southermost of them</td>
<td>Π 29</td>
<td>-7°</td>
<td>5</td>
<td>η Tau</td>
</tr>
<tr>
<td>40-44</td>
<td>The 5 stars under and to the rear of the northern horn:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>The most advanced</td>
<td>Π 27</td>
<td>0°</td>
<td>5</td>
<td>α Gem</td>
</tr>
<tr>
<td>41</td>
<td>The one to the rear of this</td>
<td>Π 29</td>
<td>1°</td>
<td>5</td>
<td>ε Gem</td>
</tr>
<tr>
<td>42</td>
<td>The one to the rear again of the latter</td>
<td>Π 21</td>
<td>1°</td>
<td>5</td>
<td>δ Gem</td>
</tr>
<tr>
<td>43</td>
<td>The northermost of the remaining, remnant 2</td>
<td>Π 21</td>
<td>1°</td>
<td>5</td>
<td>γ Gem</td>
</tr>
<tr>
<td>44</td>
<td>The southermost of these two</td>
<td>Π 31</td>
<td>1°</td>
<td>5</td>
<td>η Tau</td>
</tr>
<tr>
<td>[11 stars, 1 of the fourth magnitude, 10 of the fifth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H94</td>
<td>[XXIV] Constellation of Gemini</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

192) Talgren (see Kunitzsch, Der Almagest no. 295 p. 270) suggested emending κατ at H90,8 to κατα ("opposite the shoulder-blade"). This may be correct, but the rest of the Arabic tradition is based on κατ (see Kunitzsch, Ibn al-Salah p. 59 n.91).
193) The variant 16 is found in the earlier Arabic tradition according to S 37.
194) Reading κατ (with Α' and the latter Arabic tradition, see S 38) for κατ (21) at H91,10. Corrected by Manilius.
195) D, Ar have the variant Κη, adopted by P-K.
196) Most Greek mss. have 26, Heiberg's text is the reading of D, Ar. The identification of this star is very uncertain.
197) Most mss., both Greek and Arabic, have 3 γ (Γ) appears in G and as a variant in A', and is adopted by Heiberg and P-K.

<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>The star on the left knee of the advance twin</td>
<td>Π 13</td>
<td>1°</td>
<td>3</td>
<td>ε Gem</td>
</tr>
<tr>
<td>11</td>
<td>The star under the left knee of the rear twin</td>
<td>Π 13</td>
<td>2°</td>
<td>3</td>
<td>δ Gem</td>
</tr>
<tr>
<td>12</td>
<td>The star in the left groin of the rear twin</td>
<td>Π 21</td>
<td>0°</td>
<td>3</td>
<td>γ Gem</td>
</tr>
<tr>
<td>13</td>
<td>The star over the bend in the right knee[38] of the same [rear] twin</td>
<td>Π 21°</td>
<td>0°</td>
<td>3</td>
<td>λ Gem</td>
</tr>
<tr>
<td>14</td>
<td>The star on the forward foot[39] of the advance twin</td>
<td>Π 11</td>
<td>1°</td>
<td>4</td>
<td>η Gem</td>
</tr>
<tr>
<td>15</td>
<td>The one to the rear of this on the same foot</td>
<td>Π 11</td>
<td>1°</td>
<td>4</td>
<td>υ Gem</td>
</tr>
<tr>
<td>16</td>
<td>The star on the right foot of the advance twin</td>
<td>Π 12</td>
<td>1°</td>
<td>4</td>
<td>ε Gem</td>
</tr>
<tr>
<td>17</td>
<td>The star on the left foot of the rear twin</td>
<td>Π 14°</td>
<td>0°</td>
<td>4</td>
<td>η Gem</td>
</tr>
<tr>
<td>18</td>
<td>The star on the right foot of the rear twin</td>
<td>Π 21°</td>
<td>0°</td>
<td>4</td>
<td>ε Gem</td>
</tr>
<tr>
<td>[18 stars, 2 of the second magnitude, 5 of the third, 9 of the fourth, 2 of the fifth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>The star in advance of the forward foot of the advance twin</td>
<td>Π 4°</td>
<td>-0°</td>
<td>4</td>
<td>(14) Gem</td>
</tr>
<tr>
<td>20</td>
<td>The bright star in advance of the advance knee</td>
<td>Π 6°</td>
<td>5°</td>
<td>4</td>
<td>η Aur</td>
</tr>
<tr>
<td>21</td>
<td>The star in advance of the left knee of the rear twin</td>
<td>Π 15°</td>
<td>-2°</td>
<td>5</td>
<td>(6d) Gem</td>
</tr>
<tr>
<td>22</td>
<td>The northermost of the three stars in a straight line to the right of the arm of the rear twin</td>
<td>Π 28°</td>
<td>-11°</td>
<td>5</td>
<td>ζ Gem</td>
</tr>
<tr>
<td>23</td>
<td>The southermost of them, near the forearm of the [right] arm</td>
<td>Π 26°</td>
<td>-3°</td>
<td>5</td>
<td>(74) Gem</td>
</tr>
<tr>
<td>24</td>
<td>The bright star to the rear of the above-mentioned 3</td>
<td>Π 26°</td>
<td>-4°</td>
<td>5</td>
<td>ζ Gem</td>
</tr>
<tr>
<td>25</td>
<td>The one to the rear again of the latter</td>
<td>Π 26°</td>
<td>-6°</td>
<td>5</td>
<td>κ Gem</td>
</tr>
<tr>
<td>[7 stars, 4 of the fourth magnitude, 4 of the fifth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

198) D has ιπετα ("over"), and this was the reading behind the Arabic ("fawqa").
199) P-K emend to Ε (the reading of Gey; the rest of the Arabic tradition has Ε, on the grounds that the fraction is not used in the longitudes (the only other examples are XXVI 17 and 29 and XXIII 36). But in X 1 (p. 469) Ptolemy gives the longitude of this star as 18°, and that position is confirmed by his subsequent computations.
200) δ Γεμ, This would normally mean 'elbow', and is so translated by Manilius. But the position of the star on the figure shows that it must be on the leg, and therefore we must refer to the bud in the leg (as in animal figures, e.g. Aries, XXII 12).
201) The variant 211 is found in D, Ar.
202) This is the reading of D, Ar (5°, F). Most Greek mss. have θ ε (θ + ι), which is very strange, as ι is normally written as ι.
203) "forward foot": πρόσος, also used e.g. as the spur of a mountain. The twin is depicted with one foot (or leg) advanced before the other. πρόσος was used as a name for this particular star, see Hipparchus, Comm. in Astr. 3.4.12 (ed. Manilius 268,38).
204) Ε has 3, adopted by P-K.
205) D, Ar, adopted by P-K.
206) D has 3, adopted by Manilius. P-K adopt 51 (from Gey). All the Arabic mss. I have seen have 61.
[Number in constellation] | Description | Longitude in degrees | Latitude in degrees | Magnitude | [Modern designation]
--- | --- | --- | --- | --- | ---
20 | The star on the left knee | $\Delta 1^h$ | $-1^m$ | 5 | *86 Vir
21 | The star in the back of the right thigh | $\mu 28^h$ | $+8^m$ | 5 | *90(p) Vir
22 | The middle star of the 3 in the garment-bem round the feet | $\Delta 6^h$ | $+7^m$ | 4 | $\lambda$ Vir
23 | The southernmost of them | $\Delta 7^h$ | $+2^m$ | 4 | $\kappa$ Vir
24 | The northernmost of the three | $\Delta 8^h$ | $+11^m$ | 4 | $\varphi$ Vir
25 | The star on the left, southern foot | $\Delta 10^h$ | $+9^m$ | 4 | $\lambda$ Vir
26 | The star on the right, northern foot | $\Delta 12^h$ | $+5^m$ | 4 | $\mu$ Vir

[26 stars, 1 of the first magnitude, 6 of the third, 7 of the fourth, 10 of the fifth, 2 of the sixth]

Stars around Virgo outside the constellation:
27 | The most advanced of the three in a straight line under the left forearm | $\mu 14^h$ | $-3^m$ | 5 | $\chi$ Vir
28 | The middle one of these | $\mu 19$ | $-3^m$ | 5 | $\psi$ Vir
29 | The remotest of the 3 | $\mu 22^h$ | $-3^m$ | 5 | $\nu$ Vir
30 | The most advanced of the 3 stars almost on a straight line under Spica | $\mu 27^h$ | $-2^m$ | 6 | $\zeta$ Vir
31 | The middle one of these, which is a double star | $\mu 28^h$ | $-8^m$ | 5 | *61 + 63 Vir
32 | The remotest of the three | $\Delta 5^h$ | $-7^m$ | 6 | 89 Vir


VIII. Constellation XXVIII : Libra

1 [Tabular layout of the constellations in the southern hemisphere]

[Number in constellation] | Description | Longitude in degrees | Latitude in degrees | Magnitude | [Modern designation]
--- | --- | --- | --- | --- | ---
1-2 | [XXVIII] Constellation of Libra
1 | Stars on the tip of the southern claw: the bright one | $\Delta 18^h$ | $+0^m$ | 2 | $\alpha$ Lib
2 | the star to the north of this and fainter than it | $\Delta 17^h$ | $+2^m$ | 5 | $\mu$ Lib
3-4 | Stars on the tip of the northern claw: the bright one | $\Delta 22^h$ | $+8^m$ | 2 | $\beta$ Lib
4 | the faint star in advance of this | $\Delta 17^h$ | $+8^m$ | 5 | $\varphi$ Lib
5 | The star in the middle of the southern claw | $\Delta 24^h$ | $-1^m$ | 4 | $\xi$ Lib
6 | The one in advance of this on the same claw | $\Delta 21^h$ | $+4^m$ | 4 | $\gamma$ Lib
7 | The star in the middle of the northern claw | $m_3$ | $+3^m$ | <4 | $\delta$ Lib
8 | The one to the rear of this on the same claw | $[8$ stars, 2 of the second magnitude, 4 of the fourth, 2 of the fifth]
9 | Stars around Libra outside the constellation: the most advanced of the 3 stars north of the northern claw | $\Delta 26^h$ | $+9^m$ | 5 | 37 Lib
10 | The southernmost of the remotest 2 [of these] | $m_3$ | $+6^m$ | <4 | 48 Lib
11 | The remotest of them | $m_3$ | $+9^m$ | <4 | $\xi$ Sco
12 | The remotest of the 3 stars between the claws | $m_3$ | $+0^m$ | 6 | $\lambda$ Lib
13 | The remotest of the other 2 in advance [of the latter] | $m_0^h$ | $+0^m$ | 5 | *41 Lib

1 χραλιοι, literally 'claws' (of Scorpius). Both χραλις ('balance', hence Libra) and χραλια are found in the Greek texts, but Ptolemy always uses the latter except at IX 7 (H567,14), which is a quotation from an earlier observation. See Boll-Gundel cols. 963-5.
2 The variant 3 is found in the Greek ms. B and in part of the Arabic tradition (see S 53).
3 The identification of nos. 13 and 14 is highly uncertain. The stars I have designated are in approximately the same relative positions as Ptolemy indicates. But, if the identifications are correct, why does Ptolemy mention $\xi$ Lib? P-K identify as $\chi$ Lib and B Arg 14782 (which is BSC 5810, adopted by me), Manitius as 41 Lib and $\kappa$ Lib. Another problem is the magnitudes: $\kappa$ is 4.72, 41 is 5.38, and BSC 5810 only 5.94.
<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>The star in the hind thigh(^{219})</td>
<td>Ω 20(^{220})</td>
<td>+5 (^{221})</td>
<td>3</td>
<td>λ Leo</td>
</tr>
<tr>
<td>24</td>
<td>The star in the hind legs</td>
<td>Ω 21(^{220})</td>
<td>+1</td>
<td>4</td>
<td>α Leo</td>
</tr>
<tr>
<td>25</td>
<td>The one south of this, about in the lower legs</td>
<td>Ω 24</td>
<td>−0 (^{221})</td>
<td>4</td>
<td>τ Leo</td>
</tr>
<tr>
<td>26</td>
<td>The star on the hind claw-clutches</td>
<td>Ω 27(^{221})</td>
<td>+3 (^{221})</td>
<td>5</td>
<td>υ Leo</td>
</tr>
<tr>
<td>27</td>
<td>The star on the end of the tail</td>
<td>Ω 24(^{221})</td>
<td>+11</td>
<td>&lt;1</td>
<td>β Leo</td>
</tr>
</tbody>
</table>

\(^{219}\)The lion is represented with both hind legs together. Cf. nos. 24 to 26, and e.g. Thiele Fig. 26 p. 99.

\(^{220}\)The variant 24\(^{220}\) occurs in the Greek (\(\Lambda\)\(\omicron\)) and later Arabic traditions (see S 43).

\(^{221}\)Reading ζ\(^{219}\) (with D, adopted by Manilius) for γ\(^{219}\) (\(\omicron\)) at H101.6. The latter fraction would be unique in the whole catalogue. The Arabic tradition (see S 44) varies between Ω\(^{219}\) and P–K adopt the latter, which might be correct.

\(^{219}\)The variant Ω\(^{219}\) occurs in the later Arabic tradition (see S 45).

\(^{220}\)One can make out many of the stars of this cluster with the naked eye. But it is dubious whether one should identify the points named by Ptolemy with individual stars, as I have done following Manilius and P–K. For here Ptolemy uses the neuter (τὸ ἄπειρον), not the masculine (which would imply ἄρσης, 'star'). The group was named κλόκωμος (‘lock’) in honour of the lock of Berenice by Conon: see the poem of Callimachus, Αἰσχρ. fr. 110.

\(^{219}\)On the peculiar designation of the magnitude in most Greek ms., namely δημίος (‘tame’) with δήμαρχος (‘bright’) over it, see P–K p. 103.

\(^{220}\)The longitude of no. 1 should be greater than that of no. 2, but the only alternative mss. reading and longitude of no. 1, 29 (\(\Lambda\)\(\omicron\)) is even smaller. Hence P–K interchange the longitudes of the two stars. Manilius (p. 403) would prefer to correct the longitude of no. 2 to 26.

<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The southern most of the 2 stars in the top of the skull</td>
<td>ζ 26(^{225})</td>
<td>+4</td>
<td>5</td>
<td>ν Vir</td>
</tr>
<tr>
<td>2</td>
<td>The northern most of them</td>
<td>θ 27</td>
<td>+5</td>
<td>5</td>
<td>ζ Vir</td>
</tr>
</tbody>
</table>

\(^{219}\)Reading ζ\(^{219}\) at H103.7 for γ\(^{219}\) (\(\omicron\)), the reading of D. Ω\(^{219}\) is the reading of the Greek ms. BC (confirmed by CCAG I cod. 12, 142, 13) and the later Arabic tradition (see S 40). It is adopted by P–K. A and the rest of the Arabic tradition have 6.

\(^{220}\)Reading ζ\(^{220}\) (with all mss. except D) for a ζ\(^{220}\) (\(\omicron\)) at H103.8. Corrected by P–K.

\(^{221}\)Reading β\(^{219}\) for a ζ\(^{219}\) (\(\omicron\)) at H103.10. Corrected by P–K.

\(^{219}\)The variant 11 occurs in the early Arabic tradition according to S 47.

\(^{221}\)The variant 13, adopted by P–K, is the reading of Ar.

\(^{221}\)The variant 16 is found in the Greek (\(\Lambda\)\(\omicron\)) and later Arabic traditions (see S 48).

\(^{221}\)The group consists of two stars, one of which is a true quadrilateral. To remedy this P–K (no. 515 p. 194) suggest an implausible interchange in the coordinates of nos. 19 and 20.

\(^{219}\)The variant 6 is found in part of the Arabic tradition (see S 49).

\(^{221}\)The variant 01 is found in the Arabic tradition (see S 51).
<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H114</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>The middle one of these</td>
<td>$\ddagger$ 171</td>
<td>+11</td>
<td>4</td>
<td>α Sgr</td>
</tr>
<tr>
<td>11</td>
<td>The remotest of the three</td>
<td>$\ddagger$ 194</td>
<td>+12</td>
<td>4</td>
<td>δ Sgr</td>
</tr>
<tr>
<td>12</td>
<td>The southernmost of the 3 stars in the northern cloak-attachment(^{11})</td>
<td>$\ddagger$ 211</td>
<td>+12</td>
<td>5</td>
<td>43(d) Sgr</td>
</tr>
<tr>
<td>13</td>
<td>The middle one of these</td>
<td>$\ddagger$ 221</td>
<td>+11</td>
<td>4</td>
<td>δ Sgr</td>
</tr>
<tr>
<td>14</td>
<td>The northernmost of the three</td>
<td>$\ddagger$ 222</td>
<td>+11</td>
<td>4*</td>
<td>ζ Sgr</td>
</tr>
<tr>
<td>15</td>
<td>The faint star to the rear of these three</td>
<td>$\ddagger$ 234</td>
<td>+11</td>
<td>6</td>
<td>β Sgr</td>
</tr>
<tr>
<td>16</td>
<td>The northernmost of the 2 stars on the southern cloak-attachment</td>
<td>$\ddagger$ 241</td>
<td>+11</td>
<td>5</td>
<td>55(e) Sgr</td>
</tr>
<tr>
<td>17</td>
<td>The southernmost of them</td>
<td>$\ddagger$ 271</td>
<td>+2</td>
<td>6</td>
<td>*57 Sgr</td>
</tr>
<tr>
<td>18</td>
<td>The star on the right shoulder</td>
<td>$\ddagger$ 221*</td>
<td>-12</td>
<td>5</td>
<td>*2 Sgr</td>
</tr>
<tr>
<td>19</td>
<td>The star on the right elbow</td>
<td>$\ddagger$ 241</td>
<td>-12</td>
<td>4</td>
<td>*51(b) + 52(b(^{2})) Sgr</td>
</tr>
<tr>
<td>20-22</td>
<td>The three stars in the back:</td>
<td>* &amp; 261(^{18}) &amp; * 261(^{19})</td>
<td>-21</td>
<td>2</td>
<td>β Sgr</td>
</tr>
<tr>
<td>20</td>
<td>the one just above the place between the shoulders</td>
<td>$\ddagger$ 20</td>
<td>-21</td>
<td>5</td>
<td>ψ Sgr</td>
</tr>
<tr>
<td>21</td>
<td>the middle one, just above the shoulder-blade</td>
<td>$\ddagger$ 171</td>
<td>-4</td>
<td>3*</td>
<td>η Sgr</td>
</tr>
<tr>
<td>22</td>
<td>the other one, under the armpit</td>
<td>$\ddagger$ 161</td>
<td>-6</td>
<td>3</td>
<td>ζ Sgr</td>
</tr>
<tr>
<td>23</td>
<td>The star on the front left hock</td>
<td>$\ddagger$ 171</td>
<td>-23</td>
<td>2</td>
<td>β Sgr</td>
</tr>
<tr>
<td>24</td>
<td>The one on the knee of the same leg</td>
<td>$\ddagger$ 17</td>
<td>-18</td>
<td>4</td>
<td>η Sgr</td>
</tr>
<tr>
<td>25</td>
<td>The star on the front right hock</td>
<td>$\ddagger$ 61</td>
<td>-13</td>
<td>3</td>
<td>ζ Sgr</td>
</tr>
<tr>
<td>26</td>
<td>The star on the left thigh</td>
<td>$\ddagger$ 271</td>
<td>-131</td>
<td>3</td>
<td>*κ + * ε Sgr</td>
</tr>
</tbody>
</table>

\(^{11}\)ιππωτικ. This word is mistranslated in the dictionaries. It is a piece of cloth which was attached (hence the name) to a mounted soldier's cloak at the shoulder, and which, in theory, used to wrap round the arm as a guard (declined by Pollux IV 116, ed. Bethe I 235, αυτπεκτλίδς η ἔναετος), but in practice was largely decorative, being often of purple or embroidered (see Athenaeus V 194f and 1961, passages from Hellenistic authors), and streaming from the shoulder as the wearer galloped. This is how it (or they, one on each shoulder) appeared in the depictions of Sagittarius (e.g. Thiele Fig. 42 on p. 117), where they may be a Greek adaptation of the wings of a Babylonian original (see e.g. King, Babylonian Boundary Stones Pl. XXIX A; but it is only a plausible conjecture that this figure represents a constellation. See Secll, Hesperia XVII [17], with further literature). This attribute of Sagittarius is as early as Hipparchus (e.g. from, in Ast. 2.5.16, ed. Manitius 198,27). I do not know whether Hephaestion (ed. Pingree 1,3,10), in referring to "wings or cloak-attachment", preserves a Babylonian tradition or is misinterpreting a picture of Sagittarius.

\(^{18}\)S 56 records the variant I (?) in the earlier Arabic tradition.

\(^{19}\)S 51 in some Greek mss. 253 in AlAr.

\(^{20}\)Ar has 221, adopted by P-K.

\(^{21}\)For the variants 3 and 4 in the Arabic tradition see S 58.

---

<table>
<thead>
<tr>
<th>[Number in constellation]</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H116</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>The star on the right hind lower leg</td>
<td>* &amp; 261(^{18}) &amp; * 261(^{19})</td>
<td>3</td>
<td>t Sgr</td>
<td></td>
</tr>
<tr>
<td>28-31</td>
<td>The four stars [forming a quadrilateral] in the place where the tail joins [the body]:</td>
<td>* &amp; 271(^{18}) &amp; * 271(^{19})</td>
<td>-4</td>
<td>5</td>
<td>α Sgr</td>
</tr>
<tr>
<td>28</td>
<td>the advance star on the northern side</td>
<td>$\ddagger$ 261</td>
<td>-4</td>
<td>5</td>
<td>60(A) Sgr</td>
</tr>
<tr>
<td>29</td>
<td>the rear star on the northern side</td>
<td>$\ddagger$ 261</td>
<td>-4</td>
<td>5</td>
<td>59(b) Sgr</td>
</tr>
<tr>
<td>30</td>
<td>the advance star on the southern side</td>
<td>$\ddagger$ 261</td>
<td>-5</td>
<td>5</td>
<td>62(c) Sgr</td>
</tr>
<tr>
<td>31</td>
<td>the rear star on the southern side</td>
<td>$\ddagger$ 291</td>
<td>-6</td>
<td>5</td>
<td>62(c) Sgr</td>
</tr>
</tbody>
</table>

**XXXII** Constellation of Capricornus

1. The northernmost of the 3 stars in the rear horn | $\ddagger$ 71 | +71 | 3 | α + α Cap |
2. The middle one of these | $\ddagger$ 71 | +61 | 6 | v Cap |
3. The southernmost of the three | $\ddagger$ 71 | +5 | 3 | β Cap |
4. The star on the tip of the advance horn | * & 9\(^{20}\) | +8 | 6 | ζ + ξ Cap |
5. The southernmost of the 3 stars in the muzzle | $\ddagger$ 9 | +61 | 6 | η Cap |
6. The star on the advance star of the two | $\ddagger$ 81 | +11 | 6 | δ Cap |
7. The remotest of these | $\ddagger$ 81 | +11 | 6 | δ Cap |
8. The star in advance of the [above] 3, under the right eye | $\ddagger$ 81 | +11 | 6 | δ Cap |
9. The northernmost of the 2 stars in the neck | $\ddagger$ 111 | +31 | 6 | t Cap |
10. The southernmost of them | $\ddagger$ 111 | * 101\(^{22}\) | 5 | u Cap |
11. The star on the left, doubled-up knee\(^{21}\) | $\ddagger$ 111 | -81 | 4 | η Cap |
12. The star, under the right knee | $\ddagger$ 101 | -82 | 4 | η Cap |
13. The star on the left shoulder | $\ddagger$ 161 | -71 | 4 | 24(A) Cap |
14. The more advanced of the 2 stars close together under the belly | $\ddagger$ 201 | -61 | 4 | 3 ζ Cap |

\(^{18}\)Reading κζ β′ γ′ (with Ar, adopted by P-K) for κζ β′ γ′ (231) at H115.18.

\(^{19}\)Most Greek mss. have κζ (26). 201 is the reading of Ar. (except for 3’, which agrees with D in 41).

\(^{20}\)This is the reading of D,Ar. Most Greek mss. have 271.

\(^{21}\)This is the reading of D and most of the Arabic tradition (L,T,F). The other Greek mss. and some Arabic (Ger) have 9.

\(^{22}\)This is the reading of D,Ar. Other Greek mss. have κζ (3’ I 4’), but that is not the way 3 is normally written.

\(^{23}\)Compare Thiele Fig. 41 on p. 116, where, however, it is the right knee which is doubled up (cf. Introduction p. 15).
### VIII. Constellations XXX: Scorpius

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>The southernmost of three</td>
<td>m_2281</td>
<td>18°</td>
<td>3</td>
<td>GL6411</td>
</tr>
<tr>
<td>15</td>
<td>The most advanced of the 3 stars south of the southern claw</td>
<td>p 11</td>
<td>18°</td>
<td>3</td>
<td>GL6411</td>
</tr>
<tr>
<td>16</td>
<td>The southernmost of the other, rear 2</td>
<td>p 11</td>
<td>18°</td>
<td>3</td>
<td>GL6411</td>
</tr>
<tr>
<td>17</td>
<td>The southernmost of them</td>
<td>p 11</td>
<td>18°</td>
<td>3</td>
<td>GL6411</td>
</tr>
<tr>
<td>[9 stars, 1 of the third magnitude, 5 of the fourth, 2 of the fifth, 1 of the sixth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. This is the reading of all Greek manuscripts except D, which has 81 (so too Ar; adopted by P-K).
2. This variant 61 is found in D, Ar.
3. It is generally agreed that nos. 14 and 15 are to be identified with ζ² and ζ Sco, but it is not clear which is which. Furthermore what Ptolemy calls the southernmost star (in the notes a more northerly latitude (-18°) than what he calls the northern one (-18°)). I have therefore, dubiously, reversed the data of 14 and 15. Manitius reverses the latitudes only. P-K (no. 560 on p. 105) identify 14 as ζ² and 15 as ζ, emending -18 to -19. Everything is uncertain.

### VIII. Constellations XXXII: Sagittarius

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>The one after that, in the 5th joint</td>
<td>m 211</td>
<td>-18°</td>
<td>3</td>
<td>0 Sco</td>
</tr>
<tr>
<td>18</td>
<td>The next one again, in the 6th joint</td>
<td>p 11</td>
<td>-18°</td>
<td>3</td>
<td>1 Sco</td>
</tr>
<tr>
<td>19</td>
<td>The star in the 7th joint, the joint next to the sting</td>
<td>m 181</td>
<td>-18°</td>
<td>3</td>
<td>2 Sco</td>
</tr>
<tr>
<td>20</td>
<td>The rearmost of the 2 stars in the sting</td>
<td>m 181</td>
<td>-18°</td>
<td>3</td>
<td>3 Sco</td>
</tr>
<tr>
<td>21</td>
<td>The more advanced of these</td>
<td>m 271</td>
<td>-18°</td>
<td>3</td>
<td>4 Sco</td>
</tr>
<tr>
<td>[21 stars, 1 of the second magnitude, 13 of the third, 5 of the fourth, 2 of the fifth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. This is the reading in all Greek manuscripts except D, which has 81 (so too Ar; adopted by P-K).
2. This variant 61 is found in D, Ar.
3. It is generally agreed that nos. 14 and 15 are to be identified with ζ² and ζ Sco, but it is not clear which is which. Furthermore what Ptolemy calls the southernmost star (in the notes a more northerly latitude (-18°) than what he calls the northern one (-18°)). I have therefore, dubiously, reversed the data of 14 and 15. Manitius reverses the latitudes only. P-K (no. 560 on p. 105) identify 14 as ζ² and 15 as ζ, emending -18 to -19. Everything is uncertain.
This page contains tables with latitude and longitude data for stars in the constellation Aquarius. The tables also include notes and references to other versions and sources. The text is dense and technical, discussing the identification and translation of star names and positions.

The tables are as follows:

**Table H122**
- **Description**: Positions of stars in the constellation Aquarius.
- **Columns**: Longitude in degrees, Latitude in degrees, Magnitude, [Modern designation].

**Table H124**
- **Description**: Additional positions and notes for stars in Aquarius.
- **Columns**: Longitude in degrees, Latitude in degrees, Magnitude, [Modern designation].

**Table H126**
- **Description**: More detailed positions and notes for stars in Aquarius.
- **Columns**: Longitude in degrees, Latitude in degrees, Magnitude, [Modern designation].

The text contains references to other works and figures, such as Thiele Fig. 35 on p. 110, and notes on the identification of stars in the constellation. The technical nature of the content suggests it is aimed at readers with a specific knowledge of astronomy and classical texts.
The variant 23 occurs in the later Arabic tradition (see S 59).

2) The direction 'south' attached to the coordinate '0' perhaps indicates that the star is very slightly south of the ecliptic. Contrast no. 26.

3) 'Spine' (οξέον) here means a projection from the fish-tail. Maniuus (p. 404) emends υοξεον ('southern') to υοξεον (projecting on [projecting from] the back), comparing H12.8.1 εις τετραχος, δεκαδοκεως, and H16.6.2 εις τετραχος, δεκαδοκεως. Although the conjecture is superficially attractive, the location of this projection on the figure seems indeed to be south of the main tail: see Thieb Fig. 41 on p. 116.

4) For this meaning of ουσαίον I give the variant 21 found in all Greek mss. except P 24, Ar.

5) For this meaning of κεφαλα ως Μενικτους (XXXIV) nos. 21 and 22, and l.SJ s.v. oloupaiov, 2.

6) The variant 23 occurs in the later Arabic tradition according to S 60.

7) The Syriac translation had the variant 18 2 according to S 61.

---

<table>
<thead>
<tr>
<th>Number in</th>
<th>Description</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Magnitude</th>
<th>[Modern</th>
<th>designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constellation]</td>
<td>in degrees</td>
<td>in degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The star on the head of Aquarius</td>
<td>0°</td>
<td>+15°</td>
<td>5</td>
<td>23(d) Aqr</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The brightest of the 2 stars in the right shoulder</td>
<td>6°</td>
<td>+11°</td>
<td>3</td>
<td>α Aqr</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>The fainter one, under it</td>
<td>5°</td>
<td>+9°</td>
<td>5</td>
<td>μ Aqr</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The star in the left shoulder</td>
<td>6°</td>
<td>+12°</td>
<td>3</td>
<td>β Aqr</td>
<td></td>
</tr>
</tbody>
</table>

---

[XXXII] Constellation of Aquarius

1) Reading versio for βoλη ('to the north') at H120.10. Although all Greek mss. have βoλη, the sense requires the emendation, which is confirmed unanimously by the Arabic translations.

2) Αριστερα πεσε δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτινα δεπτι

3) The northernmost of them, on the end of the tail-fins? [128 stars, 3 of the third magnitude, 9 of the fourth, 9 of the fifth, 6 of the sixth]

---

<table>
<thead>
<tr>
<th>Number in</th>
<th>Description</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Magnitude</th>
<th>[Modern</th>
<th>designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constellation]</td>
<td>in degrees</td>
<td>in degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>The one under that, in the back, approximately under the armpit</td>
<td>7°</td>
<td>+6°</td>
<td>5</td>
<td>Ξ Aqr</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>The remotest of the three stars in the left arm, on the coat</td>
<td>7°</td>
<td>+4°</td>
<td>3</td>
<td>ν Aqr</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>The middle one of these</td>
<td>6°</td>
<td>+1°</td>
<td>4</td>
<td>μ Aqr</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>The most advanced of the three</td>
<td>6°</td>
<td>+1°</td>
<td>3</td>
<td>ε Aqr</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>The star in the right forearm</td>
<td>8°</td>
<td>+1°</td>
<td>3</td>
<td>γ Aqr</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>The northernmost of the 3 stars on the right hand</td>
<td>9°</td>
<td>+1°</td>
<td>3</td>
<td>π Aqr</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>The most advanced of the other 2 to the south</td>
<td>10°</td>
<td>+1°</td>
<td>3</td>
<td>ζ Aqr</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>The remotest of these</td>
<td>12°</td>
<td>+1°</td>
<td>4</td>
<td>θ Aqr</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>The more advanced of the 2 stars close together in the hollow of the right hand</td>
<td>7°</td>
<td>+1°</td>
<td>4</td>
<td>αιρ</td>
<td></td>
</tr>
</tbody>
</table>

---

[XXXIII] Constellation of Aquarius

1) Reading versio for βολη ('to the north') at H120.10. Although all Greek mss. have βολη, the sense requires the emendation, which is confirmed unanimously by the Arabic translations.

2) Ar should have a latitude somewhat south of 0 Aqr (no. 13). Hence Manius (p. 404) interchanges the latitudes of 13 and 14. Perhaps it would be preferable to adopt, for 14, the latitude 24, found in E, Gen.

3) Of βολη for the latitudinal direction (adopted by Manius). The reading 4 is found in all Arabic mss. I have examined.

4) A has 6, BC, gr. cf. the exactly similar formulation Pisces (XXXIII) no. 20, where πεξαγωγεύον cannot be interpreted as here (see n.51 there).
### Constellation of Orion

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The nebulous star in the head of Orion</td>
<td><strong>27</strong></td>
<td>-13′</td>
<td>nch.</td>
<td>*γ Ori, ο Ori</td>
</tr>
<tr>
<td>2</td>
<td>The bright, reddish star on the right shoulder</td>
<td>Ω 2</td>
<td>-17″</td>
<td>&lt;1</td>
<td>α Ori</td>
</tr>
</tbody>
</table>

### Stars in the pelvis

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>The star on the left shoulder</td>
<td>Ω 24</td>
<td>-17′</td>
<td>2</td>
<td>*γ Ori</td>
</tr>
<tr>
<td>4</td>
<td>The one under this to the rear</td>
<td>Ω 25</td>
<td>-18</td>
<td>&lt;4</td>
<td>32(A) Ori</td>
</tr>
<tr>
<td>5</td>
<td>The star on the right elbow</td>
<td>Ω 41</td>
<td>-14′</td>
<td>4</td>
<td>μ Ori</td>
</tr>
<tr>
<td>6</td>
<td>The star on the right forearm</td>
<td>Ω 61</td>
<td>-111″</td>
<td>6</td>
<td>74(k) Ori</td>
</tr>
<tr>
<td>7</td>
<td>The quadrilateral in the right hand</td>
<td>Ω 61</td>
<td>-10</td>
<td>4</td>
<td>*ε Ori</td>
</tr>
<tr>
<td>8</td>
<td>the rear, double star on the southern side</td>
<td>Ω 61</td>
<td>-9′</td>
<td>4</td>
<td>ν Ori</td>
</tr>
<tr>
<td>9</td>
<td>the advance star on the southern side</td>
<td>Ω 71</td>
<td>-8′</td>
<td>4</td>
<td>73(p) Ori</td>
</tr>
<tr>
<td>10</td>
<td>the rear one on the northern side</td>
<td>Ω 61</td>
<td>-6′</td>
<td>6</td>
<td>60(p) Ori</td>
</tr>
<tr>
<td>11</td>
<td>the advance one on the northern side</td>
<td>Ω 11</td>
<td>-3″</td>
<td>5</td>
<td>χ Ori</td>
</tr>
<tr>
<td>12</td>
<td>the more advanced of the 2 stars in the tail</td>
<td>Ω 47′</td>
<td>-41</td>
<td>5</td>
<td>χ Ori</td>
</tr>
<tr>
<td>13</td>
<td>The remnant of them</td>
<td>Ω 27′</td>
<td>-19′</td>
<td>6</td>
<td>38(n) Ori</td>
</tr>
<tr>
<td>14</td>
<td>The one in advance of this</td>
<td>Ω 26′</td>
<td>-20′</td>
<td>6</td>
<td>33(n) Ori</td>
</tr>
<tr>
<td>15</td>
<td>The one in advance again of this</td>
<td>Ω 24′</td>
<td>-20</td>
<td>5</td>
<td>ψ Ori</td>
</tr>
<tr>
<td>16</td>
<td>The last and most advanced of the 4</td>
<td>Ω 24′</td>
<td>-20</td>
<td></td>
<td>8 Ori</td>
</tr>
</tbody>
</table>

---

* ξ Orion is not a double star, but there are two small stars close together just below it, which may have led to this description.
* The variant 111 is attested in the earlier Arabic tradition (see § 68).
### constellation of Aquarius (XXXIV)

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>the southernmost of them</td>
<td>*K 23(^{10})</td>
<td>-5</td>
<td>6</td>
<td>ϊ Psc</td>
</tr>
<tr>
<td>16</td>
<td>the most advanced of the 3 stars after the head</td>
<td>K 26(^{10})</td>
<td>-8(^{11})</td>
<td>4</td>
<td>μ Psc</td>
</tr>
<tr>
<td>17</td>
<td>the middle one of these</td>
<td>K 28(^{10})</td>
<td>-4(^{11})</td>
<td>4</td>
<td>ν Psc</td>
</tr>
<tr>
<td>18</td>
<td>the remnant of the three</td>
<td>Π 0i</td>
<td>-7(^{12})</td>
<td>4</td>
<td>ξ Psc</td>
</tr>
<tr>
<td>19</td>
<td>the star on the knot joining the 2 fishing-lines</td>
<td>Ρ 2(^{11})</td>
<td>-8(^{11})</td>
<td>3</td>
<td>α Psc</td>
</tr>
<tr>
<td>20-23</td>
<td>Stars in the northern fishing-line:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>the first(^{13}) in the section beginning at the knot</td>
<td>Π 0i</td>
<td>-1(^{12})</td>
<td>4</td>
<td>ο Psc</td>
</tr>
<tr>
<td>21</td>
<td>the southernmost of the 3 stars following after that</td>
<td>Π 0i</td>
<td>-1(^{12})</td>
<td>5</td>
<td>π Psc</td>
</tr>
<tr>
<td>22</td>
<td>the middle one of these</td>
<td>*ξ 0i (^{12})</td>
<td>+3(^{12})</td>
<td>3</td>
<td>η Psc</td>
</tr>
<tr>
<td>23</td>
<td>the northernmost of the 3, which is also on the end of the tail</td>
<td>Π 0i</td>
<td>+9(^{12})</td>
<td>4</td>
<td>ρ Psc</td>
</tr>
<tr>
<td>24</td>
<td>the northernmost of 2 stars in the mouth of the rear fish</td>
<td>Π 2(^{12})</td>
<td>+21 (^{12})</td>
<td>5</td>
<td>θ Psc</td>
</tr>
<tr>
<td>25</td>
<td>the southernmost of them</td>
<td>Π 11(^{12})</td>
<td>+21(^{12})</td>
<td>5</td>
<td>ι Psc</td>
</tr>
<tr>
<td>26</td>
<td>the remnant of the 3 small stars in the head</td>
<td>K 28(^{12})</td>
<td>+20(^{12})</td>
<td>6</td>
<td>66(^{12}) Psc</td>
</tr>
<tr>
<td>27</td>
<td>the middle one of these</td>
<td>Β 27(^{12})</td>
<td>+19(^{12})</td>
<td>6</td>
<td>67(^{12}) Psc</td>
</tr>
<tr>
<td>28</td>
<td>the most advanced of the three</td>
<td>Ρ 27(^{12})</td>
<td>+14(^{12})</td>
<td>4</td>
<td>ω Psc</td>
</tr>
<tr>
<td>29</td>
<td>the most advanced of the 3 stars on the spine in the back, following</td>
<td>Ρ 27(^{12})</td>
<td>+14(^{12})</td>
<td>4</td>
<td>ω Psc</td>
</tr>
<tr>
<td>30</td>
<td>the middle one of the three</td>
<td>Ρ 26(^{12})</td>
<td>+13(^{12})</td>
<td>4</td>
<td>ω Psc</td>
</tr>
<tr>
<td>31</td>
<td>the remnant of the three</td>
<td>Ρ 27(^{12})</td>
<td>+12(^{12})</td>
<td>4</td>
<td>*ω Psc(^{12})</td>
</tr>
</tbody>
</table>

**Notes:**

- \(^{10}\) So D,L,T,F,Ger; 261\(^{12}\) A,B,C.
- \(^{12}\) So ADAr; 281 BC.
- \(^{12}\) αποσπονταγός, which is normally 'the most advanced', but that cannot be so here, since no. 20 is 'to the rear' of no. 21. Perhaps one should emend to αποσπονταγός, cf. Aquarius (XXXIX) no. 23, with n.34.
- \(^{12}\) The variant 'northern' is found in the Greek (BC) and Arabic traditions (see S 66).
- \(^{12}\) So ABC; if D, Ar, adopted by P-K.
- \(^{12}\) The identification of nos. 31 and 34 are very uncertain. See 125 (P-K) nos. 779, 780, p. 108.
- \(^{12}\) AD, 13 BC, Ar.
- \(^{12}\) The quadrilateral under the advance fish: the most advanced of the 2 northern stars the remnant of them the remnant one on the southern side of the tail. [XXXI Constellation of Cetus]
- \(^{12}\) The most advanced star on the southern side of the tail. [XXXI Constellation of Cetus]
- \(^{12}\) So Ar and most Greek ms.; 161 BC.

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### constellation of Pisces (XXXIV)

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>the northernmost of the 2 stars in the belly</td>
<td>Π 24(^{12})</td>
<td>+17 (^{12})</td>
<td>4</td>
<td>ι Psc</td>
</tr>
<tr>
<td>33</td>
<td>the southernmost of them</td>
<td>K 28(^{12})</td>
<td>+151 (^{12})</td>
<td>4</td>
<td>*ι Psc(^{12})</td>
</tr>
<tr>
<td>34</td>
<td>the star in the rear spine, near the tail</td>
<td>Π 0</td>
<td>+111 (^{12})</td>
<td>4</td>
<td>*ι Psc(^{12})</td>
</tr>
<tr>
<td>35-38</td>
<td>Stars round Pisces outside the constellation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>the quadrilateral under the advance fish</td>
<td>Κ 11</td>
<td>-21 (^{12})</td>
<td>4</td>
<td>27 Psc</td>
</tr>
<tr>
<td>36</td>
<td>the more advanced of the 2 northern stars</td>
<td>Κ 21</td>
<td>-21 (^{12})</td>
<td>4</td>
<td>29 Psc</td>
</tr>
<tr>
<td>37</td>
<td>the remnant of them</td>
<td>Κ 0i</td>
<td>-5 (^{12})</td>
<td>4</td>
<td>30 Psc</td>
</tr>
<tr>
<td>38</td>
<td>the more advanced star on the southern side</td>
<td>Κ 21</td>
<td>-5 (^{12})</td>
<td>4</td>
<td>33 Psc</td>
</tr>
<tr>
<td>39-44</td>
<td>Stars of the fourth magnitude</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>[Total for the zodiac: 340 stars, 5 of the first magnitude, 9 of the second, 64 of the third, 133 of the fourth, 105 of the fifth, 27 of the sixth, 3 nebulous, and Gama [Herseus]]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

- \(^{12}\) The most advanced star on the southern side of the tail. [XXXI Constellation of Cetus]
- \(^{12}\) The star on the tip of the nostrils | Π 17\(^{12}\) | -7\(^{12}\)         | 4         | λ Cat              |
- \(^{12}\) The star on the middle of the mouth | Π 17\(^{12}\) | -12\(^{12}\)       | 3         | α Cat              |
- \(^{12}\) The remnant, on the end of the jaw | Π 12\(^{12}\) | -14\(^{12}\)       | 4         | ς Cat              |
- \(^{12}\) The star on the eyes | Π 10\(^{12}\) | -14\(^{12}\)       | 4         | ς Cat\(^{12}\)     |
- \(^{12}\) The star on the crown of the head | Π 12\(^{12}\) | -14\(^{12}\)       | 4         | ς Cat\(^{12}\)     |
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>The southernmost of them</td>
<td>θ 5</td>
<td>-51°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>29</td>
<td>The rearmost of the next 2 stars after the bend</td>
<td>ϕ 281</td>
<td>-54°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>30</td>
<td>The more advanced of them</td>
<td>ϕ 252</td>
<td>-54°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>31</td>
<td>The rearmost of the 3 stars in the next interval</td>
<td>ϕ 172</td>
<td>-53°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>32</td>
<td>The middle one</td>
<td>ϕ 142</td>
<td>-53°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>33</td>
<td>The most advanced of the three</td>
<td>ϕ 112</td>
<td>-52°</td>
<td>4</td>
<td>α Eri</td>
</tr>
<tr>
<td>34</td>
<td>The last star of the river, the bright one</td>
<td>ϕ 02</td>
<td>-53°</td>
<td>13</td>
<td>α Eri</td>
</tr>
</tbody>
</table>

[XXXVII] Constellation of Lepus

1-4 The quadrilateral just over the ears:
1. the northern star on the advance side | θ 191 | -35° | 5 | α Lep |
2. the southern star on the advance side | θ 191 | -36° | 5 | α Lep |
3. the southern star on the rear side | θ 211 | -35° | 5 | α Lep |
4. the star on the side | θ 211 | -36° | 5 | α Lep |
5. the star on the cheek | θ 191 | -39° | 4 | μ Lep |
6. The star on the left front foot | θ 161 | -45° | 4 | τ Lep |
7. The star on the middle of the body | ϕ 251 | -41° | 3 | ζ Lep |
8. The star under the belly | ϕ 241 | -41° | 3 | η Lep |
9. The northernmost of the 2 stars in the hind legs | η 1 | -44° | 4 | δ Lep |
10. The southernmost of them | θ 29 | -45° | 4 | γ Lep |
11. The star on the rump | θ 0 | -38° | 4 | ζ Lep |
12. The star on the tip of the tail | θ 21 | -38° | 4 | η Lep |

[XXXVIII] Constellation of Canis Major

1. The star in the mouth, the brightest, which is called 'the Dog' and is reddish | η 171 | -39° | 1 | α CMa |
2. The star on the ears | η 191 | -35° | 4 | θ CMa |
3. The star on the head | η 211 | -36° | 5 | μ CMa |
4. The northernmost of the 2 stars in the neck | η 231 | -37° | 4 | γ CMa |
5. The southernmost of them | η 231 | -37° | 4 | γ CMa |
6. The star on the chest | η 231 | -37° | 4 | γ CMa |
7. The northernmost of the 2 stars on the right knee | η 161 | -41° | 5 | π CMa |
8. The southernmost of them | η 161 | -41° | 5 | π CMa |
9. The star on the end of the front leg | η 11 | -43° | 5 | Δ CMa |
10. The more advanced of the 2 stars in the left knee | η 141 | -46° | 5 | Δ CMa |
11. The rearmost of them | η 161 | -43° | 5 | Δ CMa |
12. The rearmost of the 2 stars in the left shoulder | η 241 | -46° | 5 | Δ CMa |
13. The more advanced of them | η 241 | -46° | 5 | Δ CMa |
14. The star in the place where the left thigh joins [the body] | η 261 | -48° | <3 | δ CMa |
15. The star below the belly, in the middle of the thighs | η 231 | -51° | 3 | ε CMa |
16. The star on the joint of the right leg | η 231 | -55° | 4 | η CMa |
17. The star on the end of the right leg | η 21 | -53° | 3 | ζ CMa |
18. The star on the tail | η 21 | -50° | <3 | η CMa |

[18 stars, 1 of the first magnitude, 5 of the third, 5 of the fourth, 7 of the fifth] Stars round Canis Major outside the constellation:
19. The star on the north of the top of Canis | η 191 | -25° | 4 | 22 Mon |

Footnotes:
14 On alternative identifications for nos. 31-3 see P-K. Their identifications correspond to BSC 1214 (Lacaille i), BSC 1195 (Lacaille g) and BSC 1143 (Lacaille h).
15 θ Eri is not of 1st magnitude, but a double star of 3rd and 4th magnitudes (combined magnitude 2.9). Hence P-K suggest emending θ to α (A to Δ).
13 This is contradicted by the subtotals, but see p. 365 n. 191.
12 This is the reading of all Greek ms., E, F and Ger. The variant 241, found in T.L., is adopted by P-K.
22 Ptolemy calls it simply 'the dog' ('κόναν), since to the constellation now known as 'Canis Minor' he gives the name 'Procyon'.
21 Ptolemy calls this star θο ("the dog"), not Σιρις ("Sirius"), although the latter name is as old as Hesiod (Works and Days 587). By 'brightest' he means 'brightest of all fixed stars'. Although Sirius is not a red star today, there is considerable evidence that it was in antiquity (cf. See, 'Change in the Color of Sirius').
20 This coordinate is greatly in error, but is found in all mss. Manitius adopts 21, on no authority, P-K 201, from as-Sulî. The error may be Ptolemy's.
19 This is the reading of all ms. P-K emend to 21.
18 The variant 651 is found in the Arabic tradition (see S 71).
17 So P-K. Manitius has 19 Monocerotis.
### H136

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>The middle one</td>
<td>8 271</td>
<td>-24</td>
<td>2</td>
<td>ε Ori</td>
</tr>
<tr>
<td>28</td>
<td>The rearmost of the three</td>
<td>8 261</td>
<td>-24</td>
<td>2</td>
<td>ζ Ori</td>
</tr>
<tr>
<td>29</td>
<td>The star near the handle of the dagger</td>
<td>8 261</td>
<td>-24</td>
<td>3</td>
<td>η Ori</td>
</tr>
<tr>
<td>30</td>
<td>The northernmost of the 3 stars joined together at the tip of the dagger</td>
<td>8 261</td>
<td>-24</td>
<td>4</td>
<td>δ Orion</td>
</tr>
<tr>
<td>31</td>
<td>The middle one</td>
<td>8 261</td>
<td>-24</td>
<td>*4 ξ <em>4 45 Ori</em></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>The southernmost of the three</td>
<td>8 261</td>
<td>-24</td>
<td>*4 ζ <em>4 Ω Ori</em></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>The rearmost of the 2 stars under the tip of the dagger</td>
<td>8 261</td>
<td>-24</td>
<td>3</td>
<td>ξ Ori</td>
</tr>
<tr>
<td>34</td>
<td>The more advanced of them</td>
<td>8 261</td>
<td>-24</td>
<td>4</td>
<td>ζ Ori</td>
</tr>
<tr>
<td>35</td>
<td>The bright star in the left foot, which is [applied in] common to the water [of Eridanus]</td>
<td>8 261</td>
<td>-31</td>
<td>3</td>
<td>θ Ori</td>
</tr>
<tr>
<td>36</td>
<td>The star to the north of it in the lower leg, over the ankle-join</td>
<td>8 21</td>
<td>-301</td>
<td>&gt;3</td>
<td>ι Ori</td>
</tr>
<tr>
<td>37</td>
<td>The star under the right heel, outside</td>
<td>8 21</td>
<td>-301</td>
<td>4</td>
<td>ι Ori</td>
</tr>
<tr>
<td>38</td>
<td>The star under the right knee</td>
<td>8 21</td>
<td>-33</td>
<td>3</td>
<td>Ρ Ori</td>
</tr>
</tbody>
</table>

[38 stars, 2 of the first magnitude, 4 of the second, 8 of the third, 15 of the fourth, 3 of the fifth, 5 of the sixth, [1] nebulous]

### H138

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>The rearmost of the next 2 in order again</td>
<td>8 131</td>
<td>-254</td>
<td>4</td>
<td>μ Eri</td>
</tr>
<tr>
<td>6</td>
<td>The more advanced of them</td>
<td>8 101</td>
<td>-254</td>
<td>4</td>
<td>ν Eri</td>
</tr>
<tr>
<td>7</td>
<td>The rearmost of the 3 stars after this</td>
<td>8 61</td>
<td>-26</td>
<td>5</td>
<td>ξ Eri</td>
</tr>
<tr>
<td>8</td>
<td>The middle one of these</td>
<td>8 51</td>
<td>-27</td>
<td>4</td>
<td>ζ Eri</td>
</tr>
<tr>
<td>9</td>
<td>The most advanced of the three</td>
<td>8 21</td>
<td>-27</td>
<td>4</td>
<td>η Eri</td>
</tr>
<tr>
<td>10</td>
<td>The rearmost of the four stars in the next interval</td>
<td>8 27</td>
<td>-32</td>
<td>3</td>
<td>η Eri</td>
</tr>
<tr>
<td>11</td>
<td>The one in advance of this</td>
<td>8 24</td>
<td>-31</td>
<td>4</td>
<td>η Eri</td>
</tr>
<tr>
<td>12</td>
<td>The one in advance of this</td>
<td>8 24</td>
<td>-30</td>
<td>3</td>
<td>η Eri</td>
</tr>
<tr>
<td>13</td>
<td>The most advanced of the 4</td>
<td>8 22</td>
<td>-28</td>
<td>3</td>
<td>η Eri</td>
</tr>
<tr>
<td>14</td>
<td>The rearmost of the 4 stars in the next interval again</td>
<td>8 12</td>
<td>-25</td>
<td>3</td>
<td>η Eri</td>
</tr>
<tr>
<td>15</td>
<td>The one in advance of this</td>
<td>8 14</td>
<td>-23</td>
<td>4</td>
<td>η Eri</td>
</tr>
<tr>
<td>16</td>
<td>The one in advance of this</td>
<td>8 12</td>
<td>-23</td>
<td>3</td>
<td>η Eri</td>
</tr>
<tr>
<td>17</td>
<td>The most advanced of the 4</td>
<td>8 101</td>
<td>-23</td>
<td>4</td>
<td>η Eri</td>
</tr>
<tr>
<td>18</td>
<td>The first star in the bend of the river, which [star] touches the chest of Cygnus</td>
<td>8 52</td>
<td>-31</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>19</td>
<td>The one to the rear of this</td>
<td>8 52</td>
<td>-31</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>20</td>
<td>The most advanced of the next [group of] three</td>
<td>8 52</td>
<td>-30</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>21</td>
<td>The middle one of these</td>
<td>8 52</td>
<td>-30</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>22</td>
<td>The rearmost of the three</td>
<td>8 172</td>
<td>-39</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>23-26</td>
<td>The next four stars, nearly forming a trapezium:</td>
<td>8 21</td>
<td>-41</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>23</td>
<td>the northern one on the advance side</td>
<td>8 21</td>
<td>-41</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>24</td>
<td>the southernmost on the advance side</td>
<td>8 21</td>
<td>-41</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>25</td>
<td>the more advanced one on the rear side</td>
<td>8 21</td>
<td>-41</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>26</td>
<td>the last of the 4, the rear one on that side</td>
<td>8 21</td>
<td>-41</td>
<td>4</td>
<td>τ Eri</td>
</tr>
<tr>
<td>27</td>
<td>The northernmost of the 2 stars close together at some distance to the east</td>
<td>8 41</td>
<td>-50</td>
<td>4</td>
<td>υ Eri</td>
</tr>
</tbody>
</table>

### H140

This is the reading of A. ηΣ(16), the reading of the other Greek mss., cannot be right, since that would not be "more advanced".

The identification of nos. 15 to 17 are of the utmost uncertainty. I give these dubious proposed by P-K (see their discussion pp. 108-9). Manilius gives 15 = p₁, 16 = p₂ (these are certainly wrong, but one might reverse them), 17 = η. One might also consider, for 17, BSG: Τιάν. The "bend", ἐνεργοῦον, i.e. a change of direction (see Bayer Tab. 36), in contrast to ἐσκερτοῦον 'curve', in no. 2.

Go D,Ar (50-30 T; P), 531 the other Greek mss.

Considerable confusion arises in the identifications of nos. 27-33 from differences in the modern nomenclature of these stars (see P-K on nos. 718-804, p. 110). Thus Manilius' identifications appear to be completely different from those of P-K, but in fact are really partly so. To avoid this confusion I give the BSG (and where applicable) the F1 (amended) nos. of my identifications, which are those of P-K, though named differently. 27 = BSG: 1453 + Π50; 28 = BSG: 1464 + Π52; 29 = BSG: 1393 = Π45, 30 = BSG: 1347 + Π41; 31 = BSG: 1190; 32 = BSG: 1143; 33 = BSG: 1190.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>the more advanced of the 2 faint stars under the bright one</td>
<td>ο 18 l</td>
<td>-60</td>
<td>5</td>
<td>α Pup (BSC 3080)</td>
</tr>
<tr>
<td>19</td>
<td>the rearmost of them</td>
<td>ο 21</td>
<td>-59 l</td>
<td>5</td>
<td>β Pup (BSC 3162)</td>
</tr>
<tr>
<td>20</td>
<td>the more advanced of the 2 stars over the above-mentioned bright one</td>
<td>ο 23 l</td>
<td>-56 i</td>
<td>5</td>
<td>γ Pup (BSC 3225)</td>
</tr>
<tr>
<td>21</td>
<td>the rearmost of them</td>
<td>ο 24 l</td>
<td>-57 j</td>
<td>5</td>
<td>δ Pup (BSC 3253)</td>
</tr>
<tr>
<td>22</td>
<td>The northernmost of the 3 stars on the little shields, about on the mast-holder</td>
<td>ο 5 l</td>
<td>-51 i</td>
<td>&gt;4</td>
<td>BSC 3439</td>
</tr>
<tr>
<td>23</td>
<td>The middle one</td>
<td>ο 6 l</td>
<td>-55 l</td>
<td>&gt;4</td>
<td>d Vel (BSC 3477)</td>
</tr>
<tr>
<td>24</td>
<td>The southernmost of the three</td>
<td>ο 4</td>
<td>-57 l</td>
<td>&gt;4</td>
<td>e Vel (BSC 3426)</td>
</tr>
<tr>
<td>25</td>
<td>The northernmost of the 2 stars close together under these</td>
<td>ο 9 l</td>
<td>-60 l</td>
<td>&gt;4</td>
<td>*a Vel (BSC 3487)</td>
</tr>
<tr>
<td>26</td>
<td>The southernmost of them</td>
<td>ο 9</td>
<td>-61 i</td>
<td>&gt;4</td>
<td>*b Vel (BSC 3445)</td>
</tr>
<tr>
<td>27</td>
<td>The southernmost of the 2 stars in the middle of the mast</td>
<td>ο 6 l</td>
<td>-51 l</td>
<td>&gt;4</td>
<td>BSC 3498</td>
</tr>
<tr>
<td>28</td>
<td>The northernmost of them</td>
<td>ο 29 l</td>
<td>-49 l</td>
<td>3</td>
<td>β Pyx</td>
</tr>
<tr>
<td>29</td>
<td>The more advanced of the 2 stars by the tip of the mast</td>
<td>ο 28</td>
<td>-43 l</td>
<td>4</td>
<td>γ Pyx</td>
</tr>
<tr>
<td>30</td>
<td>The rearmost of them</td>
<td>ο 29</td>
<td>-45 l</td>
<td>4</td>
<td>δ Pyx</td>
</tr>
<tr>
<td>31</td>
<td>The star below the 3rd and rearmost little shield</td>
<td>ο 14 l</td>
<td>-54 l</td>
<td>2</td>
<td>λ Vel</td>
</tr>
<tr>
<td>32</td>
<td>The star on the cut-off111 of the deck</td>
<td>ο 17 l</td>
<td>-51 l</td>
<td>&lt;2</td>
<td>ψ Vel</td>
</tr>
<tr>
<td>33</td>
<td>The star between the steering-oars,112 in the keel</td>
<td>ο 11 l</td>
<td>-63 l</td>
<td>4</td>
<td>*b Pup</td>
</tr>
<tr>
<td>34</td>
<td>The faint star to the rear of this</td>
<td>ο 19</td>
<td>-64 l</td>
<td>6</td>
<td>*p Pup</td>
</tr>
<tr>
<td>35</td>
<td>The bright star to the rear of this, under the deck</td>
<td>ο 0</td>
<td>-63 l</td>
<td>2</td>
<td>γ Vel</td>
</tr>
</tbody>
</table>

110 This is the reading of א,גא (most Arabic mss. have 23:0). The other Greek mss. have 26, adopted by P-K.
111 The identifications of nos. 25 and 26 are those of P-K, but it is possible that they are instead g Vel (BSC 3520) and h Vel respectively.
112 The constellation is represented as only the stern-half of the ship. Cl. Thick Fig. 48 on p. 123 and Pl. II, and p. 361 n.174.
113 Two steering-oars are clearly visible in the illustration Thick Fig. 67 on p. 157, less clearly in Fig. 48 on p. 123.

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>The bright star to the south of this, on the lower [part of the] keel</td>
<td>ο 8 l</td>
<td>-60 l</td>
<td>2</td>
<td>*γ Car</td>
</tr>
<tr>
<td>37</td>
<td>The most advanced of the 3 stars to the rear of this</td>
<td>ο 15 l</td>
<td>-65 l</td>
<td>3</td>
<td>*α Vel113</td>
</tr>
<tr>
<td>38</td>
<td>The middle one</td>
<td>ο 21 l</td>
<td>-65 l</td>
<td>3</td>
<td>*ε Vel</td>
</tr>
<tr>
<td>39</td>
<td>The rearmost of the three</td>
<td>ο 26 l</td>
<td>-67 l</td>
<td>2</td>
<td>*κ Car</td>
</tr>
<tr>
<td>40</td>
<td>The more advanced of the 2 stars to the rear of these, near the cut-off</td>
<td>μ 1</td>
<td>-62 l</td>
<td>3</td>
<td>κ Vel</td>
</tr>
<tr>
<td>41</td>
<td>The rearmost of them</td>
<td>μ 8</td>
<td>-62 l</td>
<td>3</td>
<td>N Vel (BSC 3383)</td>
</tr>
<tr>
<td>42</td>
<td>The more advanced of the 2 stars in the northern advance steering-oar</td>
<td>Ί 4</td>
<td>-65 l</td>
<td>&gt;4</td>
<td>η Col</td>
</tr>
<tr>
<td>43</td>
<td>The rearmost of them</td>
<td>Ί 20 l</td>
<td>-65 l</td>
<td>&gt;3</td>
<td>ν Pup</td>
</tr>
<tr>
<td>44</td>
<td>The more advanced of the 2 stars in the other steering-oar, called Canopus</td>
<td>Ί 17 l</td>
<td>-75 l</td>
<td>1</td>
<td>ι Car</td>
</tr>
<tr>
<td>45</td>
<td>The other, rearmost star</td>
<td>Ί 29 l</td>
<td>-71 l</td>
<td>&gt;3</td>
<td>t Pup</td>
</tr>
</tbody>
</table>

[XI.1] Constellation of Hydra114

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>The 5 stars in the head; the southernmost of the 2 advance ones, which is on the nostrils</td>
<td>ά 14</td>
<td>-15 l</td>
<td>4</td>
<td>α Hya</td>
</tr>
<tr>
<td>1</td>
<td>the southernmost of these [2], which is above the eye</td>
<td>ά 13 l</td>
<td>-13 l</td>
<td>4</td>
<td>δ Hya</td>
</tr>
<tr>
<td>2</td>
<td>the northernmost of these 2 to the rear of these, which is about on the skull</td>
<td>ά 15 l</td>
<td>-11 l</td>
<td>4</td>
<td>ε Hya</td>
</tr>
<tr>
<td>3</td>
<td>the southernmost of them, on the gaping jaws</td>
<td>ά 15 l</td>
<td>-11 l</td>
<td>4</td>
<td>η Hya</td>
</tr>
<tr>
<td>4</td>
<td>the rearmost of all, about on the cheek</td>
<td>ά 17 l</td>
<td>-12 l</td>
<td>4</td>
<td>ζ Hya</td>
</tr>
</tbody>
</table>

114 The identifications I give for nos. 37-9 are those of P-K. But the actual magnitude of 1 Carinae is much too small, and the positions are in poor agreement. Manitius gives δ, κ, η Vel, which produces better agreement for the magnitudes but even worse for the positions.
115 The water-snake. Ptolemy, like Hipparchus (e.g. Comh. in Ast. 1.11.9, ed. Manitius 116.5) calls it ἑδρος (masculine); but it is feminine (ἐδρα) in Aratus. 444. Somewhat confusingly, there is a different modern constellation called Hydrus (far south of this).
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H146</td>
<td>The southernmost of the 4 stars almost on a straight line under the hind legs</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>0 Col</td>
</tr>
<tr>
<td>21</td>
<td>The one north of this</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>k Col</td>
</tr>
<tr>
<td>22</td>
<td>The one north again of this</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>k Col</td>
</tr>
<tr>
<td>23</td>
<td>The last and northernmost of the 4</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>l CMa</td>
</tr>
<tr>
<td>24</td>
<td>The most advanced of the 3 stars almost on a straight line to the west of</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>µ Col</td>
</tr>
<tr>
<td>25</td>
<td>The middle one</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>µ Col</td>
</tr>
<tr>
<td>26</td>
<td>The rearmost of the three</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>2</td>
<td>β Col</td>
</tr>
<tr>
<td>27</td>
<td>The rearmost of the 2 bright stars under these</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>2</td>
<td>α Col / γ Col</td>
</tr>
<tr>
<td>28</td>
<td>The more advanced of them</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>c Col</td>
</tr>
<tr>
<td>29</td>
<td>The last star, to the south of the above</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>c Col</td>
</tr>
<tr>
<td>[XXXIX] Constellation of Camis Minor</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>c Col</td>
</tr>
<tr>
<td>1</td>
<td>The star in the neck</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>β CMi</td>
</tr>
<tr>
<td>2</td>
<td>The bright star just over the headquarters, called Procyon</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>2</td>
<td>α CMi</td>
</tr>
<tr>
<td>[XI] Constellation of Argus</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>4</td>
<td>c Col</td>
</tr>
<tr>
<td>1</td>
<td>The more advanced of the 2 stars in the stern-ornament</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>5</td>
<td>11(c) Pup</td>
</tr>
<tr>
<td>2</td>
<td>The rearmost of them</td>
<td><img src="image.png" alt="Image" /></td>
<td><img src="image.png" alt="Image" /></td>
<td>3</td>
<td>p Pup</td>
</tr>
</tbody>
</table>

105-108 Reading µ (vith all ms., Greek and Arabic, except D) for µζ (46l), the reading of D, at H147.18. Corrected by P-K.
109 Reading µζ (with AD, Ar, adopted by P-K) at H149.4 for µζ (49l) of the other Greek ms.
110 This is the reading of D, Ar, the other Greek ms. have 49l.
111 The top of the post on the stern, which was often given this shape. See LSJ s.v. II for other references.
112 This seems the only likely candidate in the right region. P-K assign to the star they identify (Piazzi VII 277) the magnitude 6.5. Perhaps Peters confounded the two stars (very close together) SBC 3113 (mag. 4.78) and SBC 3099 (mag. 6.30, which is too faint to be considered).
113 This might be any of the 5th-magnitude stars (all close together) SBC. 2019, 2023, 2934, or some combination of them. All are in the modern constellation Canis Major.
114 The variant 16 occurs in the Greek (AD) and later Arabic traditions (see S 73).
115 This may include more of the numerous small stars close together (P-K give d' studied by)
### H158

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>The star in the breast</td>
</tr>
<tr>
<td>4</td>
<td>The star in the advance, right wing</td>
</tr>
<tr>
<td>5</td>
<td>The more advanced of the 2 stars in the rear wing</td>
</tr>
<tr>
<td>6</td>
<td>The rearmost of them</td>
</tr>
<tr>
<td>7</td>
<td>The star on the end of the leg, which is applied in common to Hydra</td>
</tr>
<tr>
<td></td>
<td>[7 stars, 5 of the third magnitude, 1 of the fourth, 1 of the fifth]</td>
</tr>
</tbody>
</table>

**[XLIV] Constellation of Centaurus**

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The southernmost of the 4 stars in the head</td>
</tr>
<tr>
<td>2</td>
<td>The northernmost of them</td>
</tr>
<tr>
<td>3</td>
<td>The more advanced of the other, middle 2</td>
</tr>
<tr>
<td>4</td>
<td>The rearmost of these, the last of the 4</td>
</tr>
<tr>
<td>5</td>
<td>The star on the left, advance shoulder</td>
</tr>
<tr>
<td>6</td>
<td>The star on the right shoulder</td>
</tr>
<tr>
<td>7</td>
<td>The star on the left shoulder-shoulder</td>
</tr>
</tbody>
</table>

**8-11 The star in the breast**

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>The northernmost of the advance 2</td>
</tr>
<tr>
<td>9</td>
<td>The southernmost of these</td>
</tr>
<tr>
<td>10</td>
<td>The last one, south of the latter</td>
</tr>
<tr>
<td>11</td>
<td>The most advanced of the 3 stars in the right side</td>
</tr>
<tr>
<td>12</td>
<td>The middle one</td>
</tr>
<tr>
<td>13</td>
<td>The rearmost of the three</td>
</tr>
</tbody>
</table>

### H160

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>The star on the right upper arm</td>
</tr>
<tr>
<td>16</td>
<td>The star on the right forearm</td>
</tr>
<tr>
<td>17</td>
<td>The star in the right hand</td>
</tr>
<tr>
<td>18</td>
<td>The bright star in the place where the human body joins [the horse's]</td>
</tr>
<tr>
<td>19</td>
<td>The rearmost of the 2 faint stars to the north of this</td>
</tr>
<tr>
<td>20</td>
<td>The more advanced of them</td>
</tr>
<tr>
<td>21</td>
<td>The star on the place where the back joins [the horse's body]</td>
</tr>
<tr>
<td>22</td>
<td>The star in advance of this, on the horse's back</td>
</tr>
<tr>
<td>23</td>
<td>The rearmost of the stars on the rump</td>
</tr>
<tr>
<td>24</td>
<td>The middle one</td>
</tr>
<tr>
<td>25</td>
<td>The most advanced of the three</td>
</tr>
<tr>
<td>26</td>
<td>The more advanced of the 2 stars close together on the right thigh</td>
</tr>
<tr>
<td>27</td>
<td>The rearmost of them</td>
</tr>
<tr>
<td>28</td>
<td>The star in the chest, under the horse's armpit</td>
</tr>
<tr>
<td>29</td>
<td>The more advanced of the 2 stars under the belly</td>
</tr>
<tr>
<td>30</td>
<td>The rearmost of them</td>
</tr>
<tr>
<td>31</td>
<td>The star on the knee-bend of the right [hind] leg</td>
</tr>
<tr>
<td>32</td>
<td>The star in the hock of the same leg</td>
</tr>
<tr>
<td>33</td>
<td>The star under the knee-bend of the left [hind] leg</td>
</tr>
<tr>
<td>34</td>
<td>The star on the frog of the hoof[^{130}] on the same leg</td>
</tr>
<tr>
<td>35</td>
<td>The star on the end of the right front leg</td>
</tr>
</tbody>
</table>

---

\[^{129}\] The thyrsus was a branch carried by followers of Dionysus, tipped with vine-leaves, pine-cone, or other Dionysiac emblems. See A. J. Reinach s.v. in Daremberg-Saglio V, 287-96, with illustrations. The attribution to a centaur is rare, but attested (ibid. 293 n.20).

\[^{130}\] Manitius and P-K identify this as c' Cen, but c' and c' are so close together that one cannot decide between them: it is better to assume that Ptolemy refers to both.

---

**[Number in constellation]**  **Description**  **Longitude in degrees**  **Latitude in degrees**  **Magnitude**  **[Modern designation]**

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>The star on the right upper arm</td>
</tr>
<tr>
<td>16</td>
<td>The star on the right forearm</td>
</tr>
<tr>
<td>17</td>
<td>The star in the right hand</td>
</tr>
<tr>
<td>18</td>
<td>The bright star in the place where the human body joins [the horse's]</td>
</tr>
<tr>
<td>19</td>
<td>The rearmost of the 2 faint stars to the north of this</td>
</tr>
<tr>
<td>20</td>
<td>The more advanced of them</td>
</tr>
<tr>
<td>21</td>
<td>The star on the place where the back joins [the horse's body]</td>
</tr>
<tr>
<td>22</td>
<td>The star in advance of this, on the horse's back</td>
</tr>
<tr>
<td>23</td>
<td>The rearmost of the stars on the rump</td>
</tr>
<tr>
<td>24</td>
<td>The middle one</td>
</tr>
<tr>
<td>25</td>
<td>The most advanced of the three</td>
</tr>
<tr>
<td>26</td>
<td>The more advanced of the 2 stars close together on the right thigh</td>
</tr>
<tr>
<td>27</td>
<td>The rearmost of them</td>
</tr>
<tr>
<td>28</td>
<td>The star in the chest, under the horse's armpit</td>
</tr>
<tr>
<td>29</td>
<td>The more advanced of the 2 stars under the belly</td>
</tr>
<tr>
<td>30</td>
<td>The rearmost of them</td>
</tr>
<tr>
<td>31</td>
<td>The star on the knee-bend of the right [hind] leg</td>
</tr>
<tr>
<td>32</td>
<td>The star in the hock of the same leg</td>
</tr>
<tr>
<td>33</td>
<td>The star under the knee-bend of the left [hind] leg</td>
</tr>
<tr>
<td>34</td>
<td>The star on the frog of the hoof[^{130}] on the same leg</td>
</tr>
<tr>
<td>35</td>
<td>The star on the end of the right front leg</td>
</tr>
</tbody>
</table>

---

\[^{131}\] Reading ι' γ' at H161,8 (with Ar, adopted by P-K). The Greek mss. have the reading γ' (33), but, with these identifications of nos. 19 and 20, the Arabic tradition is almost certainly the correct one. Manitius identifies 19 as u and 20 as u', but u is definitely 'to the rear' of u'.

\[^{132}\] As P-K note, ι is not a single star, but a globular cluster (no. 5339).

\[^{133}\] Reading ι' γ' (40), which is abundantly attested in the Arabic tradition (see S 81) at H161,12 for γ (43) of the Greek tradition. P-K also adopt 40.

\[^{134}\] For the identifications of nos. 28-37 see P-K. nos. 962-71 on p. 112. The identifications they suggest are probably correct, in spite of the large errors in the coordinates, which are perhaps due to the difficulty of observing stars with extreme southern declinations.

\[^{135}\] Πατόπτυχος. The Oxford English Dictionary defines 'frog' (s.v. 2) as 'an elastic, horny substance growing in the middle of the sole of a horse’s hoof'.

\[^{136}\] This is the reading of D,Ar and an alternative reading in A. Other Greek mss. have 441. -411 is more correct, but all other stars in this group are assigned too great a southern latitude, so -441 may have been Ptolemy's measurement. It is adopted by P-K.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>The more advanced of the 2 stars in the place where the neck joins [the head]</td>
<td>20</td>
<td>-14° 17'</td>
<td>5</td>
<td>ι Hya</td>
</tr>
<tr>
<td>7</td>
<td>The rearmost of them</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>The middle star of the following three in the bend of the neck</td>
<td>26</td>
<td>-15°</td>
<td>4</td>
<td>ιι Hya</td>
</tr>
<tr>
<td>9</td>
<td>The rearmost of the 3</td>
<td>0</td>
<td>-14°</td>
<td>4</td>
<td>ιιι Hya</td>
</tr>
<tr>
<td>10</td>
<td>The southernmost of them</td>
<td>28</td>
<td>-17°</td>
<td>4</td>
<td>ιιιι Hya</td>
</tr>
<tr>
<td>11</td>
<td>The faint, northernmost star of the 2 close together to the south</td>
<td>29</td>
<td>-19°</td>
<td>6</td>
<td>BSC 3750^109</td>
</tr>
<tr>
<td>12</td>
<td>The bright one of these two close stars</td>
<td>0</td>
<td>-20° 26'</td>
<td>2</td>
<td>ι Hya</td>
</tr>
<tr>
<td>13</td>
<td>The most advanced of the 3 stars after the bend [in the neck]</td>
<td>6</td>
<td>-26°</td>
<td>4</td>
<td>ιι Hya</td>
</tr>
<tr>
<td>14</td>
<td>The middle one</td>
<td>8</td>
<td>-26°</td>
<td>4</td>
<td>ιιι Hya</td>
</tr>
<tr>
<td>15</td>
<td>The rearmost of the three</td>
<td>11</td>
<td>-23° 14'</td>
<td>4</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>16</td>
<td>The most advanced of the next 3 stars almost on a straight line</td>
<td>18</td>
<td>-24°</td>
<td>3</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>17</td>
<td>The middle one</td>
<td>20</td>
<td>-23°</td>
<td>4</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>18</td>
<td>The rearmost of the three</td>
<td>23</td>
<td>-22° 13'</td>
<td>3</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>19</td>
<td>The northernmost of the 2 stars after [i.e. to the rear of] the base of Grater^124</td>
<td>26</td>
<td>-23°</td>
<td>4</td>
<td>β Gr</td>
</tr>
<tr>
<td>20</td>
<td>The southernmost of them</td>
<td>21</td>
<td>-30°</td>
<td>4</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>21</td>
<td>The most advanced of the 3 stars after these, as it were in a triangle</td>
<td>12</td>
<td>-31°</td>
<td>4</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>22</td>
<td>The middle and southernmost one</td>
<td>14</td>
<td>-31°</td>
<td>4</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>23</td>
<td>The rearmost of the three</td>
<td>16</td>
<td>-31°</td>
<td>3</td>
<td>ιιιιι Hya</td>
</tr>
<tr>
<td>24</td>
<td>The star after Corvus, in the section by the tail</td>
<td>0</td>
<td>-13°</td>
<td>4</td>
<td>γ Hya</td>
</tr>
</tbody>
</table>

^117 The variant 14 is occurs in the later Arabic tradition (see S 74).
^118 The variant 19 is attested for the later Arabic tradition by S 75.
^119 BSC 3750 is P-K’s W 43. Another possible identification is 28 Hya. 29 Hya, adopted by Manittius, is impossible, since it is south of ι Hya (no. 12).
^11A P-K’s emendation, κυ (23) for κ λ, is very plausible.
^112 The Greek mss are unanimous for 261 (so too 1*). Heilberg adopts 231 from an emendation by Bode, which is however found in the Arabic tradition (L,T,E,F).
^113 The variant 281 was found in the margin of Isaghi’s autograph according to S 78.
^114 The figures of Crater (the mixing-bowl) and Corvus (the raven, cf. no. 24) were depicted as sitting on the back of Hydra: see Thiele Fig. 54 on p. 129 and Pl. V (lower).

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>The star on the tip of the tail [25 stars, 1 of the second magnitude, 3 of the third, 19 of the fourth, 1 of the fifth, 1 of the sixth]</td>
<td>13</td>
<td>-17° 125'</td>
<td>&gt;4</td>
<td>π Hya</td>
</tr>
<tr>
<td>26</td>
<td>The star in the southern part of the head</td>
<td>12</td>
<td>-23°</td>
<td>3</td>
<td>BSC 3314^119</td>
</tr>
<tr>
<td>27</td>
<td>The star some distance to the rear of those in the neck [nos. 6-15] [2 stars of the third magnitude]</td>
<td>11</td>
<td>+16°</td>
<td>3</td>
<td>α Sc^117</td>
</tr>
</tbody>
</table>

[XLI] Constellation of Crater

1 The star in the base of the bowl, which is [applied in] common to Hydra 23° -23° 4 α Ct
2 The southernmost of the 2 stars in the middle of the bowl 23° -19° 4 γ Ct
3 The northernmost of them 0 -18° 4 δ Ct
4 The star on the southern rim of the mouth 7 -18° 4 ζ Ct
5 The star on the northern rim 29° -13° 4 τ Ct
6 The star on the southern handle 9° -16° <4 η Ct
7 The star on the northern handle 11° -11° 4 δ Ct

[XLI. III] Constellation of Corvus

1 The star in the beak, which is [applied in] common to Hydra 15° -21° 3 α Cv
2 The star in the neck, by the head 14° -19° 3 η Cv

Since there is no doubt about the identification, the latitude is so wrong that one should consider emendation. Manittius (p. 405) suggests 13° 40', no no authority.

This identification is the same as that of Manittius and P-K, who use the obsolete nomenclature 30 Monocerotis (the star is now included in the constellation Hydra).

The identification is highly uncertain. My suggestion has coordinates not impossibly different from Ptolemy’s, but its magnitude is less than 5. P-K suggest 24 Sex, but this involves emending the latitude to 10° (adopting the variant found in D,Ar οτ (16) for οτ γ), and the magnitude is still bad. Their alternative, α Sex, is not much better. Should one emend the magnitude to 6 (ο for γ)?

For the description of nos. 1 and 7 cf. p. 392 n. 124 and Thiele Fig. 54 on p. 129, which depicts the raven standing on and pecking the water-snake.
<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The one to the rear again of this</td>
<td>$-20$</td>
<td>$4$</td>
<td></td>
<td>$\zeta$ G A</td>
</tr>
<tr>
<td>5</td>
<td>The one after this, before the knee of Sagittarius</td>
<td>$-18.1$</td>
<td>$5$</td>
<td></td>
<td>$\delta$ G A</td>
</tr>
<tr>
<td>6</td>
<td>The one after this, which is north of the bright star in the knee of</td>
<td>$-17.1$</td>
<td>$4$</td>
<td></td>
<td>$\beta$ G A</td>
</tr>
<tr>
<td>7</td>
<td>The star to the north of this</td>
<td>$-16.2^m$</td>
<td>$4$</td>
<td></td>
<td>$\alpha$ G A</td>
</tr>
<tr>
<td>8</td>
<td>The one to the north again of this</td>
<td>$-15.4$</td>
<td>$4$</td>
<td></td>
<td>$\gamma$ G A</td>
</tr>
<tr>
<td>9</td>
<td>The rearmost of the 2 stars after this, in advance, in the northern rim</td>
<td>$-15.1$</td>
<td>$6$</td>
<td></td>
<td>$\zeta$ G A</td>
</tr>
<tr>
<td>10</td>
<td>The more advanced of these 2 faint stars</td>
<td>$-14.3$</td>
<td>$5$</td>
<td></td>
<td>$\delta$ G A</td>
</tr>
<tr>
<td>11</td>
<td>The star quite some distance in advance of this</td>
<td>$-14.1$</td>
<td>$5$</td>
<td></td>
<td>$\gamma$ G A</td>
</tr>
<tr>
<td>12</td>
<td>The one in advance again of this</td>
<td>$-14.1$</td>
<td>$5$</td>
<td></td>
<td>$\beta$ G A</td>
</tr>
<tr>
<td>13</td>
<td>The last one, which is south of the aforementioned star</td>
<td>$-13.5$</td>
<td>$5$</td>
<td></td>
<td>$\alpha$ G A</td>
</tr>
<tr>
<td>14</td>
<td>[13 stars, 5 of the fourth magnitude, 6 of the fifth, 2 of the sixth]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[XLVIII] Constellation of Piscis Australis

1. The star in the mouth, which is the same as the beginning of the water

2. The most advanced of the 3 stars on the southern rim of the head

3. The middle one

4. The rearmost of the three

5. The star by the gills

6. The star on the southernmost spine on the back

7. The rearmost of the 2 stars in the belly

8. The more advanced of them

9. The rearmost of the 3 stars on the northern spine

10. The middle one

11. The most advanced of the three

[12 stars, 9 of the fourth magnitude, 2 of the fifth]

Stars round Piscis Australis outside the constellation:

12. The star on the tip of the tail

13. The most advanced of the 3 bright stars in advance of Piscis [Australis]

14. The middle one

15. The rearmost of the three

16. The faint star in advance of this

17. The southernmost of the remaining 2 stars to the north

18. The northernmost of them

[6 stars, 3 of the third magnitude, 2 of the fourth, 1 of the fifth]

[Total for the southern region 316 stars, 7 of the first magnitude, 18 of the second, 63 of the third, 164 of the fourth, 54 of the fifth, 9 of the sixth, 1 nebulous]

[Total for all stars 1022, 15 of the first magnitude, 45 of the second, 208 of the third, 674 of the fourth, 217 of the fifth, 49 of the sixth, 9 faint, 5 nebulous, plus Gama [Berenices]]

138 The variant 20 is found in the earliest Arabic tradition according to S 85.
139 This is the star which P-K call 'v Coronae Australis'; I do not know what their authority for this appellation is.
140 This is P-K's Lac. 7748. Manitius suggests $\pi$ G A, which is certainly possible.
141 In Aquarius (XXXII 42) this is called 'the end of the water'.

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>Modern designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>The star on the tip of the tail</td>
<td>$-20.4$</td>
<td>$4$</td>
<td></td>
<td>$\gamma$ Gru</td>
</tr>
<tr>
<td>13</td>
<td>The most advanced of the 3 bright stars in advance of Piscis [Australis]</td>
<td>$-22.4$</td>
<td>$&lt;3$</td>
<td></td>
<td>$\eta$ Mic</td>
</tr>
<tr>
<td>14</td>
<td>The middle one</td>
<td>$-22.4$</td>
<td>$&lt;3$</td>
<td></td>
<td>$\kappa$ Mic</td>
</tr>
<tr>
<td>15</td>
<td>The rearmost of the three</td>
<td>$-21.1$</td>
<td>$&lt;3$</td>
<td></td>
<td>$\delta$ Gru</td>
</tr>
<tr>
<td>16</td>
<td>The faint star in advance of this</td>
<td>$-20.4$</td>
<td>$5$</td>
<td></td>
<td>$\eta$ Mic</td>
</tr>
<tr>
<td>17</td>
<td>The southernmost of the remaining 2 stars to the north</td>
<td>$-17.4$</td>
<td>$4$</td>
<td></td>
<td>$\gamma$ Mic</td>
</tr>
<tr>
<td>18</td>
<td>The northernmost of them</td>
<td>$-14.4$</td>
<td>$4$</td>
<td></td>
<td>$\alpha$ Mic</td>
</tr>
</tbody>
</table>

[Total for all stars 1022, 15 of the first magnitude, 45 of the second, 208 of the third, 674 of the fourth, 217 of the fifth, 49 of the sixth, 9 faint, 5 nebulous, plus Gama [Berenices]]
### [XI.V] Constellation of Lupus

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>The star on the knee of the left [front] leg</td>
<td>+24°</td>
<td>+4°</td>
<td>2</td>
<td>β Cer</td>
</tr>
<tr>
<td>37</td>
<td>The star outside, under the right hind leg</td>
<td>*+21°17'</td>
<td>-4°</td>
<td>4</td>
<td>µ Gru</td>
</tr>
</tbody>
</table>

**Note:**

113) D has 114 as P-K remark (no. 971 on p. 112), this would be more consistent than 114 with the errors of the other stars in this group.

114) Ptolemy does not identify this as a wolf or any particular animal, but calls it the 'head' (μυατοσκός). It is depicted as being held by its hind legs in the right hand of Centaurus; see Table Fig. 53 on p. 128, and cf. no. 1 here.

115) The mss. are unanimous in 22 (including the Arabic, despite the statement of P-K, no. 982 on pp. 112-13, that they have 20 instead). Peters emends to 26 instead of 20. Much of the Arabic tradition, and (Ger, have 34 (see S 87).

116) Reading α Τέλεσκοπιον, actually an incense-burner. It is depicted upside-down (i.e. base towards the north).

117) The ms. are unanimous for 22 (including the Arabic, despite the statement of P-K, no. 982 on pp. 112-13, that they have 201). Peters emends to 26 without authority. The identification of this star is dubious: see P-K's discussion, cf. Manitius' identifications, here and elsewhere in Lupus, are mostly unacceptable.

118) For the identifications of nos. 16 and 17 P-K prefer χ and ξ Lupi, but mine (which are also proposed by Manitius) seem more in accord with the relative positions.

### [XI.V] Constellation of Aurigae

<table>
<thead>
<tr>
<th>Number in constellation</th>
<th>Description</th>
<th>Longitude in degrees</th>
<th>Latitude in degrees</th>
<th>Magnitude</th>
<th>[Modern designation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>The rearmost of them</td>
<td>26°</td>
<td>-11°</td>
<td>4</td>
<td>α Ara</td>
</tr>
<tr>
<td>18</td>
<td>The southernmost of the 2 stars in the front leg</td>
<td>*+2°27'</td>
<td>-11°26'</td>
<td>&gt;4</td>
<td>10 Lup</td>
</tr>
<tr>
<td>19</td>
<td>The northernmost of them</td>
<td>26°</td>
<td>-10°</td>
<td>&gt;4</td>
<td>20 Lup</td>
</tr>
</tbody>
</table>

**Note:**

119) L'E. Ger have 21, adopted by P-K.

120) T.E. Ger have 11, adopted by P-K.

121) Βαματσοκόν, actually an incense-burner. It is depicted upside-down (i.e. base towards the mouth).

122) BC have 31. Much of the Arabic tradition, and Ger, have 01, but 3 is also found (see S 84).

123) BCx is found as an alternative reading in A, and in Is. It is adopted by P-K. "The little altar" (βαματσοκόν) is evidently the same as the 'brazier' (Τέλεσκοπιον) in no. 4: see p. 400 n. 160.

124) Reading λ y (with Ar) at H166, for α y (11), the unanimous reading of the Greek ms. Heidrun (ad loc.) realized that this correction should be made, and Manitius made it.

125) This is the reading of A, 31 R1(3), 34 Ar, adopted by P-K.

126) Reading αιτιβοmuş (implied by Ar) at H166, for αιτιβοmuş 'that one of these', which has no reference.
2. On the situation of the circle of the Milky Way

Such, then, is the way in which we may set out the order of the fixed stars. To this we shall join, as the logical order demands, our discussion of the disposition of the circle of the Milky Way, to the best of our ability, with our observations of each of its sections: we shall try to describe the form which the individual parts appear to take.

Now the Milky Way is not strictly speaking a circle, but rather a belt of a sort of milky colour overall (whence it got its name); moreover this belt is neither uniform nor regular, but varies in width, colour, density and situation, and in one section is bifurcated. [All] that is very apparent even to the casual eye, but the details, which can only be determined by a more careful examination, we find to be as follows.

The bifurcated part of the belt has one of its ‘forks’, so to speak, near Ara, and the other in Cygnus. But, whereas the advance [part of the] belt is in no way attached to the other part, since it forms gaps both at the fork by Ara and at the fork by Cygnus, the rearmost part is joined to the remainder of the Milky Way and forms [with it] a single belt, through which the great circle drawn approximately along the middle of it would pass. It is this belt which we shall describe first, beginning with its southernmost section.

This [section] goes through the legs of Centaurus, and is rather less dense and less bright [than the rest]. The star on the knee-bend of the right hind leg [XLIV 31] is a little farther south than the line [bounding] the milk to the north, and so are the star on the left front knee [XLIV 36] and the star under the right hind hock [XLIV 32]. But the star in the left hind lower leg [XLIV 33] lies in the middle of the milk, and the stars on the hock of the same leg [XLIV 34] and on the right front hock [XLIV 35] are to the north of its southern rim, by about 2° (where the great circle is 360°). It is slightly denser in the region near the hind legs.

Next in order, the northern rim of the milk is about 1° from the star on the rump of Lupus [XLV 10], and the southern rim encloses the star on the burning-apparatus of Ara [XLVI 7], but just grazes the northernmost of the two stars close together in the brazier [XLVI 16] and the southernmost of the two stars in the base [XLVI 2], while the star in the northern part of the brazier and the one in the middle of the brazier [XLVI 4.3] lie right in the milk. These sections are rather less dense.

Next, the northern part of the milk encloses the three joints before the sting of Scorpius [XXIX 17, 18, 19] and the nebulous mass to the rear of the sting [XXIX 22], while the southern rim touches the star in the right front hock of Sagittarius [XXX 25], and encloses the star on his left hand [XXX 2]. The star on the southern portion of the bow [XXX 3] is outside the milk, but the star
on the point of the arrow [XXX 1] lies in the middle of it, while the stars in the northern part of the bow [XXX 4, 5] also lie in it, each of them being a little more than 1° removed from one of the rims, the southern star from the southern rim, the northern star from the opposite rim. The area [of the Milky Way] near the three joints [of Scorpion] is somewhat denser, while the area round the point [of the arrow of Sagittarius] is very dense indeed and appears smoky.

The following section is a little less dense. It extends along [the constellation] Aquila, maintaining about the same width throughout. The star on the tip of the tail of the snake [Serpens. XIV 18] held by Ophiuchus lies in the open, 162 a little more than one degree away from the advance rim of the milk, while the two most advanced of the bright stars below it lie right in the milk: the southern one [XVI 15] is 1° from the rear rim, and the northern one [XVI 12], 2° [from it]. The rearmost of the [two] stars in the right shoulder of Aquila [XVI 8] touches the same rim, while the more advanced one [XVI 7] is cut off inside it, as is also the more advanced, bright star of those in the left wing [XVI 5]. 163 Furthermore, the bright star on the place between the shoulders [XVI 3] and the two stars which lie on a straight line with it 164 fall a little short of touching the same rim. Next, Sagitta is enclosed entirely within the milk. The star on the arrowhead [XV 1] lies one degree from the eastern rim, while the star on the notch [XV 5] lies two degrees from the western rim. The section round Aquila is slightly denser, and the remainder slightly less dense.

Next the milk extends towards Cygnus. Its north-western rim is defined in a reentrant angle 165 by the star in the southern shoulder of Cygnus [IX 11], 166 the star under it in the same [southern] wing [IX 10], and the two stars on the southern leg [IX 13, 14]. Its south-eastern rim is defined by the star in the tip of the southern wing-leathers [IX 12], and encloses the two stars under the same wing outside the constellation [IX 18, 19], which are about 2° from it [the rim]. The section around the wing is slightly denser. The next section is continuous with that belt, but is much denser and seems to have a different starting-point. 167 For it points towards the end parts of the other belt, 168 but leaves a gap between it [and itself]: on its southern side it joins the belt which we are currently describing, which is very rarefied at the junction; but after the point where it forms a gap with the other belt it gets denser.

162 Literally 'in the open air', i.e. outside the Milky Way.
163 In the catalogue these stars are described as being 'in the left shoulder'.
164 This does not correspond to any description in the catalogue. Manitius identifies the two stars as XVI 2 and 4 β and o Aql). These are indeed approximately on a straight line with XVI 3 (α Aql), but they hardly fit the rest of the description, since β Aql lies well outside the Milky Way as viewed by Ptolemy. More appropriate would be φ Aql (XVI 6) and u Aql. However, the latter star seems not to be mentioned in the catalogue.
165 έπικομίατο. Explained by what follows: this is where the other (western) branch of the Milky Way joins: since, according to Ptolemy, the part north of this is aligned with the end of that branch, it forms a reentrant angle with the present, eastern branch. This is best seen on a star globe.
166 In the catalogue this is called 'the star in the middle of the left wing'.
167 Translating Heiberg's emendation, ὀρῷμενα (supported by Is: 'ibtada' a') for the ὀρῷμενα of the Greek mss. and L. The latter could perhaps be translated as 'and is seen, as it were, from a different starting-point', but this is very harsh.
168 I.e. the other branch of the Milky Way which is mentioned above (p. 400) and described below (p. 403).
beginning from the bright star in the rump of Cygnus [IX 5] and the nebulous mass in the northern knee [IX 17]. Then it makes a slight bend as far as the star on the southern knee [IX 14], and continues, gradually diminishing in density, up to the tiara of Cepheus. The northern side is delimited by the southernmost of the three stars in the tiara [IV 9] and the star to the rear of those three [IV 13], at which it also forms two outrunners, one verging to the north and east, the other to the south and east.

Next the milk encloses the whole of Cassiopeia except for the star in the end of the leg [X 7]. The southern rim is defined by the star in the head of Cassiopeia [X 1], and the northern rim by the star in the foot of the throne [X 11] and the star in the lower leg of Cassiopeia [X 6]. The other stars of Cassiopeia and all those round about this constellation lie in the milk. The areas near the rims are of thinner consistency, but those at the middle of Cassiopeia display a dense patch running the length of the Milky Way.

Next, the right-hand parts of Perseus are enclosed in the milk. Furthermore, its northern edge, which is very rarefied, is defined by the lone star outside the right knee of Perseus [XI 28], and its southern edge, which is very dense, by the bright star on his right side [XI 7] and by the two rearmost stars of the three to the south of that bright star [XI 9, 10]. Enclosed in it also are the nebulous mass on the hilt [XI 1], the star in the head [XI 5], the star in the right shoulder [XI 3] and the star on the right elbow [XI 2]. The quadrilateral in the right knee [XI 16, 17, 18, 19] and also the star on the same [right] calf [XI 20] lie in the midst of the milk, while the star in the right heel [XI 21] is also inside it, a little distance from the southern border.

Next the belt goes through Auriga, displaying a slightly thinner consistency. The star on the left shoulder, called Capella [XII 3], and the two stars on the right forearm [XII 5, 6] fall just short of touching the north-eastern rim of the milk, while the small star over the left foot in the lower hem of the garment [XII 14] defines the south-western edge. The star over the right foot [XII 12] lies half a degree within the same edge, and the two stars close together on the left forearm, called Haedi [XII 8, 9], lie in the middle of the belt.

Next the milk goes through the legs of Gemini, displaying a certain amount of density in elongated form just over the stars at the ends of the legs. Now the advance edge of the milk is defined by the rearmost of the 3 stars on a straight line under the right foot of Auriga [XXIV 19], by the rearmost star of the two in the staff of Orion [XXXV 12] and by the northernmost [two] of the four stars on his [Orion's] hand [XXXV 9, 10]; the brilliant star under the right hand of Auriga [XXIV 20] and the star in the rear foot of the rear twin [XXIV 18] are approximately 1° inside the rear edge, while the stars in the other feet [XXIV 14, 15, 16, 17] lie in the midst of the milk.

Thence the belt passes by Canis Minor [Procyon] and Canis Major: it leaves the whole of Canis Minor outside the milk no small distance to the east, and
leaves Canis Major too outside to the west, almost in its entirety; for the star on its ears\(^\text{172}\) [XXXVIII 2] is caught by a sort of cloud which projects [from the Milky Way] and which then almost touches the three stars in the neck of Canis Major next to that [star] towards the rear [XXXVIII 3, 4, 5], while the lone star over the head of Canis Major, outside it and at some distance [XXXVIII 19], is about 2\(^\circ\) inside the eastern rim. The consistency in this whole region\(^\text{173}\) is somewhat thinner.

After that the milk passes through Argo. The western rim of the belt is defined by the northernmost and most advanced of the stars in the little shield in the poop [XL 5]. The star in the middle of the little shield [XL 6], the two stars close together under it [XL 8, 9], the bright star at the beginning of the deck near the steering-oar [XL 17] and the midmost of the three stars in the keel [XL 38] are just short of touching the same [western] edge. The northernmost of the three stars in the mast-holder [XL 22] defines the eastern rim, while the bright star in the stern-ornament [XL 2] is 1\(^\circ\) within the same [eastern] edge, and the bright star under the rearmost little shield in the deck [XL 31] is the same amount, 1\(^\circ\), outside the same [eastern] edge. The southermost of the two brilliant stars in the middle of the mast [XL 27] touches the same edge, and the two bright stars at the point where the keel is cut off\(^\text{174}\) [XL 35, 36] are about 2\(^\circ\) inside the advance rim. At that point the milk joins the belt through the legs of Centaurus.\(^\text{175}\) The consistency in this area too, throughout Argo, is somewhat rarefied, but the sections of it around the little shield, the mast-holder and the point where the keel is cut off are more dense.

The belt we mentioned previously\(^\text{176}\) forms a gap, as we said, between [itself and] the one we have [just] described, at Ara. Beginning at that point, it encloses the three joints of Scorpius' [tail] nearest the body [XXIX 12, 13, 14], but leaves the rearmost star of the three in the body [XXIX 9] 1\(^\circ\) outside its western rim. The star in the fourth joint [XXIX 16] lies in the open space between the two belts, about the same distance from each, a little more than 1\(^\circ\).

After that the advance belt turns aside to the east, in the shape of a segment of a circle, defining the advance edge of the milk by the star on the right knee of Ophiuchus [XIII 12], and the rear edge by the star on the same [right] shin [XIII 13], while the most advanced of the stars at the end of the same [right] leg [XIII 14] touches that same [rear] edge. Subsequently the western rim is defined by the star under the right elbow of Ophiuchus [XIII 9], and the eastern rim by the more advanced of the two stars in the same [right] hand [XIII 10].

\(^{172}\) Reading ἐν τῶν ὄπων (with Ar: D\(^\prime\) has ἐν τῶν νῶτων) for ἐν τῷ νῷτῳ ("on the back") at H76,18. The correction was made by Kunitzsch, Der Almagest no. 533 on p. 322. It is confirmed by the whole context, and especially by the position of the star, θ CMa. Manitius identifies the star here with XXXVIII 12, which is said in the catalogue to be "in the left shoulder", but this star (ο\(^\prime\) CMa) lies well outside the Milky Way as viewed by Ptolemy.

\(^{173}\) Reading τὸ χῦμα δῖξον τοῦτῳ ἤρεμα ἀραμέτερον (with Di) at H76,24, to get a normal word order, for τὸ χῦμα τοῦτῳ ἤρεμα δῖξον ἀραμέτερον.

\(^{174}\) Reading ἐν τῇ ἀποτομῇ (with D, Ar) at H77,13-4 for ἐν τῇ ἀποτομῇ ("in the same cut-off of the keel"), which is senseless.

\(^{175}\) I.e. the point where Ptolemy began the description, p. 400.

\(^{176}\) I.e. the western 'fork' mentioned on p. 400. But it is tempting to follow Is, who has 'advance' (i.e. προπρογονένη) here and 'mentioned previously' below at H78,7 (i.e. προπρογονένη, 'advance', of the Greek mss.)
From that point on there is a considerable gap of open space, in which lie the two stars on the tail of Serpens [XIV 16, 17] next to the star in the tip [of the tail, XIV 18]. The whole of the section of this belt which we have [just] finished describing consists of an extremely fine and almost aery substance, except for the area enclosing the three joints [of Scorpius], which is somewhat more concentrated.

After the gap the milk again makes a fresh beginning at the four stars to the rear of the right shoulder of Ophiuchus [XIII 25, 26, 27, 28]. The eastern rim of this belt is defined (being just grazed) by the lone brilliant star under\(^ {177}\) the tail of Aquila [XVI 9], while the opposite rim is defined by the star which is some distance to the north of the four just mentioned [XIII 29]. From there on this belt, besides being rarefied, is also contracted into a narrow space in the area which is in advance of the star in the beak of Cygnus [IX 1], so as to produce the appearance of a gap. However, the remainder of it, from the star in the beak up to the star in the breast of Cygnus [IX 4], is wider and considerably denser. The star in the neck of Cygnus [IX 3] lies in the middle of the dense section. A rarefied section branches off to the north from the star\(^ {178}\) in the breast as far as the star in the shoulder of the right wing [IX 6] and the two stars close together in the right foot [IX 15, 16]. From this point, as we said, occurs a clear gap to the other belt, [a gap] stretching from the above-mentioned stars in Cygnus up to the bright star in the rump [IX 5].

3. \(\text{[On the construction of a solini globe]}\)\(^ {179}\)

Such, then, is the disposition of the phenomena associated with the Milky Way. But we also wish to provide a representation [of the fixed stars] by means of a solid globe in accordance with the hypotheses which we have demonstrated concerning the sphere of the fixed stars, according to which, as we saw, this sphere too, like those of the planets, is carried around by the primary [daily] motion from east to west about the poles of the equator, but also has a proper motion in the opposite direction about the poles of the sun’s, ecliptic circle. To this end we shall carry out the construction of the solid globe and the delineation of the constellations in the following fashion.

We make the colour of the globe in question somewhat deep, so as to resemble, not the daytime, but rather the nighttime sky, in which the stars actually appear. We take two points on it precisely diametrically opposite, and with these as poles draw a great circle: this will at all times be in the plane of the ecliptic. At right angles to the latter and through its poles we draw another [great] circle, and starting from one of the intersections of this with the first

\(^{177}\) Reading \(\text{ππο} (\text{with } D, \text{Ar}) \text{ for } \text{ππα (by') at H179.4. Compare the description of XVI 9 (p.}\)

\(^{178}\) Reading \(\text{ππο } \text{το } \text{δ } \text{ν } \text{ε ν } \text{ τ } \text{ο } \text{ν } \text{τ } \text{α } \text{τ } \text{η } \text{τ } \text{ε } \text{τ }\) (\text{with } \text{Ar}) \text{ at H179.14-15 for } \text{κακ } \text{τ } \text{ο } \text{ν } \text{ε ν } \text{ τ } \text{ο } \text{ν } \text{τ } \text{α } \text{τ } \text{η } \text{τ } \text{ε } \text{τ } . \text{Corrected by } \text{M} \text{ani} \text{tiu} \text{s } (\text{ππο already suggested by Heiberg \text{ad loc.)}}

\(^{179}\) On this ‘precession-globe’ see \(H. \text{Alm.} \text{II } 890-92, \) with Figs. 79-80 on p. 1399 (for an error in Neugebauer’s account see p. 405 n. 181). On the history of the star-globe in antiquity see Schlachter. \(D e r \text{ Globus.}\)
circle we divide the ecliptic into the [conventional] 360 degrees, and write by it
the numbers at intervals of however many degrees seems convenient. Then we
make, from a tough and unwarped\textsuperscript{180} material, two rings with rectangular
cross-section, accurately turned on the lathe in all dimensions: one should be
smaller [than the other], and fit closely to the globe on the whole of its inner
surface, while the other should be a little larger than this. In the middle of the
convex face of each ring we draw a line accurately bisecting its width. Using
these lines as guides, we cut out\textsuperscript{181} one of the latitudinal sections\textsuperscript{182} defined by
the line over half of the circumference, and divide [each of] the semi-circular
recessed sections [thus created] into 180 degrees. When this is done, we take the
smaller of the rings as the one which will always represent the circle through
both poles, that of the equator and that of the ecliptic, and also through the
solstitial points (this circle runs] along the plane surface of the above-
mentioned recessed section), and, boring holes through the middle of it at the
diametrically opposite points at the ends of the recessed section, we attach it, by
means of pins [through those holes], to the poles of the ecliptic which we took on
the globe, in such a way that the ring can revolve freely over the whole spherical
surface.

Since it is not reasonable to mark the solstitial and equinoctial points on the
actual zodiac of the globe (for the stars depicted [on the globe] do not retain a
constant distance with respect to these points), we need to take some fixed
starting-point in the delineated fixed stars. So we mark the brightest of them,
namely the star in the mouth of Canis Major [Sirius], on the circle drawn at
right angles to the ecliptic at the division forming the beginning of the
graduation, at the distance in latitude from the ecliptic towards its south pole
recorded [in the star catalogue]. Then, for each of the other fixed stars in the
catalogue in order, we mark the position by rotating the ring with the
graduated recessed face about the poles of the ecliptic: we turn the face of its
recessed section to that point on the [globe's] ecliptic which is the same distance
from the beginning of the numbered graduation (at Sirius) as the star in
question is from Sirius in the catalogue;\textsuperscript{183} then we go to that point on the

\textsuperscript{180}\textsuperscript{180}ευτόνου καὶ τεταμένης. The meaning of both adjectives is disputable. The context requires
that the material /certainly wood, although ἔλεγη does not mean wood here, pace Maniti"
be strong in
the sense that it can be cut into thin strips and bored through. Cf. Heron, Belopoeica 94, ed. Marsden
p.30.12, where the side-pieces of a catapult must be made εἰς ἔλεγην ἐξ ξύλου. ἔλεγην occurs
frequently in that work, and is usually applied to sinews or elements requiring elastic strength, e.g.
110. ibid.p.38.2: cf. Heron, Pneumatica, ed. Schmidt p. 200, where it is used of pieces of horn). But it
seems improbable that Ptolemy means 'flexible' wood here and the meaning 'rigidly strong' is
certain in one passage of Heron's Mechanic preserved in Pappus, Synagoge VIII. 1132, 6-14.
tεταμένης means literally 'stretched'. I know of no real parallel, but take it to be a synonym of
ἀστραβάτης, 'unwarped', found frequently in Theophrastus, Historia Plantarum, e.g. 5.2.1.

\textsuperscript{181}\textsuperscript{181}I.e., cut out along the central line so that half the width of the ring is removed for half the
circumference of the ring. The purpose of this is that the graduated face may be flush with the
surface of the globe, and coincide with a great circle. The result is depicted in H.A.I.1 Fig. 80A p.
1399, lower part. Neugebauer is wrong (p. 891) in saying that the text implies the making of a
central slit in the rings: he has been misled by Maniti"s translation.

\textsuperscript{182}\textsuperscript{182}Reading πλευράν (with D) for πλευράς at H181.5. Corrected by Maniti"us.

\textsuperscript{183}\textsuperscript{183}Since Sirius has in the catalogue (XXXVIII 1) the longitude Π 17°, this means that one
subtracts 77°40' from the catalogue longitudes. Wherever my translation has 'Sirius', Ptolemy has
κύων ('the Dog'). Cf. p. 387 n.88.
graduated face which we have [thus] positioned which is, again, the same distance from the ecliptic as the star is in the catalogue, either towards the north or towards the south pole of the ecliptic as the particular case may be, and at that point we mark the position of the star; then we apply to it a spot of yellow colouring (or, for some stars, the colour they are noted [in the catalogue] as having), of a size appropriate to the magnitude of each star.

As for the configurations of the shapes of the individual constellations, we make them as simple as possible, connecting the stars within the same figure only by lines, which moreover should not be very different in colour from the general background of the globe. The purpose of this is, [on the one hand], not to lose the advantages of this kind of pictorial description, and [on the other] not to destroy the resemblance of the image to the original by applying a variety of colours, but rather to make it easy for us to remember and compare when we actually come to examine [the starry heaven], since we will be accustomed to the unadorned appearance of the stars in their representation on the globe too.

We also, then, mark the location of the Milky Way on [the globe], in accordance with its positions, arrangements, densities and gaps as described above. Then we attach the larger of the rings, which will always represent a meridian, to the smaller ring which fits around the globe, on poles coinciding with those of the equator. These points [the poles of the equator] are, in the case of the larger, meridian [ring], attached, again, at the diametrically opposite ends of the recessed and graduated face (which will represent the [section of the meridian] above the earth); but in the case of the smaller ring, [which passes] through both poles, they will be fixed at the ends of the diametrically opposite arcs which stretch the 23°51' of the obliquity from each of the poles of the ecliptic. We leave small solid pieces in the recessed parts of the rings, to receive the bore-holes for the attachments [of the pins representing the poles].

Now the recessed face of the smaller of the rings must, clearly, always coincide with the meridian through the solstitial points. So on any occasion [when we want to use the globe], we set it to that point of the ecliptic graduation whose distance from the starting-point defined by Sirius is equal to the distance of Sirius from the summer solstice at the time in question (e.g. at the beginning of the reign of Antoninus, 12° in advance). Then we fix the meridian ring in position perpendicular to the horizon defined by the stand [of the globe], in such a way that it is bisected by the visible surface of the latter, but can be moved round in its own plane: this is in order that we may, for any particular application, raise the north pole from the horizon by the appropriate arc for the latitude in question, using the graduation of the meridian [to place the ring correctly].

We shall suffer no disadvantage from our inability to mark the equator and the solstitial points on the globe itself. For since the face of the meridian is graduated, the point between the poles of the equator which is 90° of the quadrant distant from both will be equivalent to points on the equator, while the points 23°51' distant from that point will be equivalent to points on the two solstitial circles, the one to the north to those on the summer solstitial circle, and

\[^{184}\text{This has not been described. For a schematic representation, with a suggestion for how the motion in the plane of the meridian may be achieved, see } H.A.M.1 \text{ p.1399 Fig. 80C.}\]
the one to the south to those on the winter solstitial circle. Thus, when any required star is rotated with the primary, east-to-west rotation to the graduated face of the meridian, we can again, by means of that same graduation, determine its distance from the equator or the solstitial circles, as measured on the great circle through the poles of the equator.

4. {On the configurations particular to the fixed stars}

Now that we have demonstrated the distinctive features of the pictorial representation of the fixed stars, it remains to discuss their configurations. The configurations involving the fixed stars, then, are, apart from those fixed configurations with respect to each other (e.g. such and such stars lie on a straight line, form a triangle, and the like), as follows:

[1] those considered with respect to the planets, sun and moon, or the parts of the zodiac alone;
[2] those considered with respect to the earth alone;
[3] those considered with respect both to the earth and at the same time to the planets, sun and moon, or the parts of the zodiac.

[1] Those configurations of the fixed stars with the planets and the parts of the zodiac alone which are accepted are

[a] for all stars in general, when fixed star and planet come to be on the same circle through the poles of the ecliptic, or on circles which are different, but at intervals [of a regular polygon] with three, four or six angles, i.e., which enclose an angle which is either a right angle or a third of a right angle greater or less than a right angle;

[b] for some stars in particular, those for which one of the planets can pass directly below it (these are the stars fixed in that narrow band of the zodiac containing the latitudinal motions of the planets) – for these, [configurations] with the five planets concern their apparent contacts or their occultations, and with the sun and moon, their last visibilities, conjunctions and first visibilities. We give the name ‘last visibility’ to the situation when a star falls within the rays of [one of] the luminaries and begins to become invisible; ‘conjunction’, when it is covered by the centre of [one of] them; and ‘first visibility’ when it escapes their rays and begins to be visible.

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185 These are the relationships trine, quartile and sextile, commonly applied in astrology: see Bouc-Leclercq, e.g. 165-79.
186 πρίσμα, literally ‘a sawn-out section’. This is probably the term that Ptolemy used for the ‘drums’ containing the planetary models in Bk. II of his Planetary Hypotheses (preserved only in Arabic translation); see e.g. Op. Min. p. 113. The word has nothing to do with the geometrical ‘prism’ here.
187 κολλάσεις. This is a technical term in astrology. It includes certain kinds of close approach, besides actual occultations. For details see Bouc-Leclercq 245, quoting Porphyrius. See also Vettius Valens, index p. 380, s.v. At Almagest IX 2 (H213,3), it appears to mean actual contact.
188 Reading ἀητός (with D) for ἀητός at H186,13.
189 Καταγείσσ. This is technically ‘rising’ (ἐπιτολή). For the planets Ptolemy uses the more appropriate word φάσις. For an explanation of the full panoply of terms associated in traditional Greek astronomy with the risings and settings of stars see below pp. 409-10, and cf. Autolycus peri ἐπιτολός I introduction (ed. Mogenet 214).
The configurations of the fixed stars with the earth alone are four in number. The term applied by some people to all in common is 'cardines'. Their individual titles are 'ascendant', 'culmination above the earth', 'descendant' and 'culmination below the earth'. Now in the region where the equator is in the zenith all the fixed stars rise and set and once in every revolution reach culmination above the earth, and once culmination below the earth; for in that situation the poles of the equator lie on the horizon, and do not make any of the parallel circles either always visible or always invisible. And in the regions where one of the poles is in the zenith, none of the fixed stars either rises or sets. For in that situation the equator assumes the position of the horizon, and one of the hemispheres into which it divides [the heavens] rotates always above the earth, while the other rotates always below the earth. Hence each star repeats the same type of culmination twice in one revolution, some reaching culmination above the earth twice, the others culmination below the earth twice. But at the other, intermediate, terrestrial latitudes, some of the [parallel] circles are always visible, and some always invisible; so the stars cut off between these and the poles neither rise nor set, and perform two culmination in each revolution; those stars in the region which is always visible [culminate twice] above the earth, and those in the region which is always invisible [culminate twice] below the earth. The remaining stars, which lie on parallels greater [than the always visible and invisible parallels], both rise and set, and culminate once above the earth and once below the earth in each revolution. For these stars the time [of travel] from any one of the cardines back to the same one is the same at every place: it comprises one revolution, to the senses. The time from one cardine to the one diametrically opposite is the same at every place when one considers meridian [passage], since it comprises half a revolution. When one considers horizon [passage] it is again constant where the equator is in the zenith: each of the two intervals [from rising to setting and from setting to rising] comprises half a revolution, since in that case all the parallel circles are bisected, not only by the meridian, but also by the horizon. However, at all other terrestrial latitudes, if one takes separately the time spent above the earth and the time spent below the earth [by a star], neither is the same for all stars [at a given latitude]; nor is the time spent above the earth for any particular star equal to the time it spends below the earth, except for those stars which happen to lie precisely on the equator; for the latter is the only circle which is bisected by the horizon at sphaera obliqua too, whereas all the other [parallels] are divided [by the horizon] into arcs which are neither similar nor equal. Furthermore, in accordance with this, the time from rising or setting to one or other of the culminations is equal to the time from the same culmination to setting or rising, since the meridian bisects those segments of the parallels which are above and below the earth; but the times from rising or setting to the two [opposite] culminations are unequal at sphaera obliqua, but equal at sphaera recta, since only

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190 κεφαλή. The primary importance of these points is in astrology: see Bouche-Philippe 257-9.
191 The two types of culmination are usually known in modern times as 'upper' and 'lower' culmination (see Introduction p. 19). I retain the literal terminology here for obvious reasons.
192 The qualification 'to the senses' is inserted because of precession (the effect of which is negligible over one daily revolution).
in the latter situation are the whole segments [of the parallel circles] above the earth equal to the segments below the earth. Hence, for *sphaera recta*, [heavenly bodies] which culminate simultaneously always rise and set simultaneously too (in so far as their motion about the poles of the ecliptic is imperceptible); but, for *sphaera obliqua*, [heavenly bodies] which culminate simultaneously neither rise nor set simultaneously, but the more southerly ones always rise later and set sooner than the more northerly.

[3] The accepted configurations of the fixed stars considered with respect to the earth and at the same time to the planets or the parts of the zodiac are:

[a] in general, their risings, culminations or settings which are simultaneous with those of one of the planets or with some part of the zodiac;

[b] in particular, their configurations with respect to the sun, which are of 9 types.

The first type of configuration is that called ‘dawn easterly’, when the star is on the eastern horizon together with the sun. One variety of this is called ‘dawn invisible later rising’, when the star, which is just at last visibility, rises immediately after the sun; another is called ‘dawn true simultaneous rising’, when the star arrives at the eastern horizon at precisely the same time as the sun; the third is called ‘dawn visible earlier rising’, when the star, which is just at first visibility, rises before the sun.

The second type of configuration is that called ‘dawn culmination’, when the sun is on the eastern horizon while the star is at the meridian, either above or below the earth. Of this too there are varieties: one is called ‘dawn invisible later culmination’, when the star culminates immediately after sunrise; a second is called ‘dawn true simultaneous culmination’, when the star culminates at the same time as the sun rises; and the third is called ‘dawn earlier culmination’, when the star culminates immediately before sunrise. When the latter is a culmination above the earth it is visible.

The third type of configuration is that called ‘dawn westerly’, when the sun is on the eastern horizon and the star on the western. This too has varieties: one is called ‘dawn invisible later setting’, when the star sets immediately after sunrise; a second is called ‘dawn true simultaneous setting’, when the star sets at exactly the same time as the sun rises; and the third is called ‘dawn visible earlier setting’, when the sun rises immediately after the star has set.

The fourth type of configuration is that called ‘meridian easterly’, when the sun is on the meridian and the star is on the eastern horizon. This too has varieties: one during the day and invisible, when the sun is culminating above the earth as the star is rising; the other during the night and visible, when the sun is culminating below the earth as the star is rising.

The fifth type of configuration is that called ‘meridian culmination’, when sun and star both reach the meridian at the same time. This too has varieties:

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*193 If $a$ is the time from rising to upper culmination, $b$ from upper culmination to setting, $c$ from setting to lower culmination, and $d$ from lower culmination to rising, then $a = b$ and $c = d$ but (at *sphaera obliqua*) $a \neq c$ and $b \neq d$.

*194 This implies that Ptolemy is thinking of planets as well as fixed stars.

*195 Reading διατείλατος (with D) for διατέλλαντος at H190.18. Corrected by Manitius.

*196 Reading καταδώναντος (with D) for καταδύοντος at H190.22.
two are during the day and invisible, when the sun is culminating above the earth and the star is either culminating above the earth together with the sun, or else culminating below the earth opposite it; and two are during the night, when the sun is culminating below the earth; of these one is invisible, when the star too culminates below the earth together with the sun, and the other is visible, when the star culminates above the earth opposite it.

The sixth type of configuration is that called 'meridian westerly', when the sun is on the meridian and the star is on the western horizon. This too has varieties: one during the day and invisible, when the sun is culminating above the earth as the star is setting; the other during the night and visible, when the sun is culminating below the earth as the star is setting.

The seventh type of configuration is that called 'evening easterly', when the sun is on the western horizon and the star on the eastern. This again has varieties: one is called 'evening visible later rising', when the star rises immediately after the sun has set; another is called 'evening true simultaneous rising', when the star rises at the same time as the sun sets; the third is called 'evening invisible earlier rising', when the sun sets immediately after the star has risen.

The eighth type of configuration is that called 'evening culmination', when the sun is on the western horizon and the star is on the meridian either above or below the earth. This too has varieties: one is called 'evening later culmination', when the star culminates immediately after sunset (when the culmination is above the earth, this is visible);\(^*\) another is called 'evening true simultaneous culmination', when the star culminates at the same time as the sun sets; the third is called 'evening invisible earlier culmination', when the sun sets immediately after the star has culminated.

The ninth type of configuration is that called 'evening westerly', when the star is on the western horizon together with the sun. This too has varieties: one is called 'evening visible later setting', when the star, just at last visibility, sets immediately after the sun; another is called 'evening true simultaneous setting', when the star sets at exactly the same time as the sun; and the third is called 'evening invisible earlier setting', when the star, which is just at first visibility, sets \([just]\) before the sun.

5. \{On simultaneous risings, culminations and settings of the fixed stars\}\(^{198}\)

Given the above definitions, the times of the true simultaneous risings, culminations and settings, which are taken with respect to the sun's centre, can be found by us immediately from the position of [the stars in question] in the delineation of the stars \([on the solid globe]\), by purely geometrical methods.

For the points on the ecliptic with which each fixed star simultaneously

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\(^{197}\) Adapting the reading of D,Ar, which omits ϕαινόμενον at H192,19 and adds κατ᾿ τὸ ὑπὲρ γῆν τοῦτον ϕαινόμενον γίνεται alter μεσοπαρανήσιαν at H192,20. The text printed by Heiberg falsely implies that both upper and lower culminations are visible.

\(^{198}\) See H.A.M.A 32-4, 39.
culminates, rises or sets can be derived geometrically by means of the theorems [already] established.\textsuperscript{199}

First, to demonstrate the simultaneous culminations, let [Fig. 8.1]\textsuperscript{200} the circle through both poles, that of the equator and that of the ecliptic, be ABGD. Let AEG be a semi-circle of the equator about pole Z, and BED a semi-circle of the ecliptic about pole H. Draw through the poles of the ecliptic the great circle segment H\(\Theta\)KL, and take on it point \(\Theta\) as the required fixed star (for it is with respect to such circles [i.e. great circles through the poles of the ecliptic] that we have observed and recorded the positions of the fixed stars). Also, draw through the poles of the equator and the star at \(\Theta\) the great circle segment Z\(\Theta\)MN.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig8.1.png}
\caption{Fig. 8.1}
\end{figure}

Now it is obvious that the star at \(\Theta\) culminates simultaneously with points M and N of the ecliptic and equator [respectively]. But these points, and arc \(\Theta\)N, are given, as will be clear from the following considerations. From what we proved at the beginning of our treatise [I 13], since the [two] great circle arcs HL and NZ have been drawn to meet the two great circle arcs AH and AN, Crd arc 2HA:Crd arc 2AZ = (Crd arc 2HL:Crd arc 2\(\Theta\)).(Crd arc 2N\(\Theta\):Crd arc 2ZN). [M.T. I]

But, immediately by hypothesis, each of the arcs AZ, ZN and HK are given as quadrants; from the catalogue, arc K\(\Theta\) is given from the star's latitude and arc KB from its longitude; and arc ZH and arc KL are given from the demonstrated obliquity of the ecliptic.\textsuperscript{201} Hence it is clear that, of the arcs in question, arc HA [ = arc AZ + arc ZH], arc AZ, arc HL [= arc HK + arc KL], arc L\(\Theta\)

\textsuperscript{199} In I 13, I 16 and II 7-8.

\textsuperscript{200} Heiberg's version of Fig. 8.1, derived from ms. A, is defective, since it contains a redundant point \(\Xi\). I follow the correct version in D.Ar.

\textsuperscript{201} arc ZH = \(\epsilon\), arc KL = \(\delta\) of point K.
[ = arc KL + arc KΘ] and also arc NZ are given. Hence the remaining arc, NΘ, will also be given.

Again, since

\[ \text{Crd arc } 2ZH : \text{Crd arc } 2HA = \]

\[ \text{H195} \]

\[ (\text{Crd arc } 2ZΘ : \text{Crd arc } 2ΘN) \cdot (\text{Crd arc } 2NL : \text{Crd arc } 2LA), \] [M.T. II]

and, by the above, of the arcs in question, arc ZH, arc HA, arc ZΘ [= arc ZN - arc NΘ] and arc ΘN are given, and arc LA is given from [the given] arc KB, by means of [the arcs of] the equator which rise together with [those of] the ecliptic at sphaera recta, the remaining arc, NL, will also be given. Similarly [by means of the rising-times at sphaera recta] arc MB of the ecliptic will be given from arc NA, the sum [of arc NL + arc LA].

Moreover the points on the equator and ecliptic which rise or set simultaneously with a fixed star can readily be found by means of the simultaneous culminations, in the following manner.

Let [Fig. 8.2] ABDG be a meridian, AEG a semi-circle of the equator about pole Z, and BED a semi-circle of the horizon. Let the star rise at point H of the horizon, and draw the great circle quadrant ZΘ through points Z, H.

\[ \text{Fig. 8.2} \]

Then again, since [two] great circle arcs ZΘ and EB have been drawn to meet two great circle arcs AZ and AE.

\[ \text{Crd arc } 2ZB : \text{Crd arc } 2BA = \]

\[ \text{H197} \]

\[ (\text{Crd arc } 2ZH : \text{Crd arc } 2HΘ) \cdot (\text{Crd arc } 2ΘE : \text{Crd arc } 2AE), \] [M.T. II]

But, of the arcs in question, arc ZA, arc ZΘ and arc EA each comprise a quadrant, arc ZB [and hence arc BA = arc ZA - arc ZB] is given from the elevation of the pole, and point Θ of the equator and arc ΘH [and hence arc HZ = arc ZΘ - arc ΘH] from the simultaneous culmination. Therefore the remaining [arc], ΘE, will be given.

For the simultaneous settings, too, it can easily be seen that if we cut off an arc, ΘK, in advance of Θ equal to arc ΘE, the star will set together with point K
of the equator. For in that situation the setting takes place on an arc [of the horizon measured from the meridian] equal to arc BH, and cuts off an angle in advance of the meridian equal to that enclosed to the rear [of it] by arc AZ and arc ZΘ in the present situation.

Furthermore, from the arcs of the equator and ecliptic which rise and set together which we have computed for each clima [II 8], there will immediately be given that point on the ecliptic which rises together with point E of the equator and the star, and that point which sets together with point K and the star. It is clear that at the moment when the sun is exactly in those points of the ecliptic, there will come to pass the risings, culminations and settings of the fixed star [in question] taken with respect to the sun’s centre which are called ‘true simultaneous cardinal positions’.

6. [On first and last visibilities of the fixed stars] 203

However, in the case of the first and last visibilities [of the fixed stars], we find that the geometrical method expounded [above], using only their position [in latitude and longitude], is no longer adequate. For it is not possible204 to find the size of the arc by which the sun must be removed below the horizon in order for a given star to have its first or last visibility by methods similar to the geometrical procedures by which, e.g., one demonstrates the point on the ecliptic with which that star rises. For that arc [the arcus visionis] cannot be the same for all stars nor the same for a given star at all places [on earth], but varies according to the magnitude of the star, its distance in latitude from the sun, and the change in the inclinations of the ecliptic [with respect to the horizon].

For if we imagine [Fig. 8.3] a meridian circle ABGD, a semi-circle of the ecliptic AEZG, and a semi-circle of the horizon BED about pole H. it is clear that, given a star rising simultaneously with point E of the ecliptic, 205 if a star of greater magnitude has its first visibility when the sun is at a distance of, e.g., arc EZ below the earth, a star of lesser magnitude, even one at an equal distance in latitude from the sun, will have its first visibility when the sun is at a greater distance than arc EZ, and [thus] the effect of its rays is weaker. Again, for stars of equal magnitude, if a star which is closer in latitude to point E has its first visibility at a distance [of the sun from the horizon] of arc EZ, a star which is farther than that [from point E in latitude] will have its first visibility at a lesser [solar] distance. For, given the same distance of the sun below the horizon, the rays in the vicinity of the ecliptic and of the sun itself are denser206 than those

202 Συγκεντρώσεις, cf. p. 408 n.190, on κέντρα.
203 See H.1.1 II 927–8.
204 Reading δυσώτων εἶναι with the mss. at H198.18. Heiberg deletes εἶναι, since one expects an indicative verb. But for the infinitive after words like εἶπεν ἐν oro ὀρικα see Kühner-Gerth II 551, quoting Xenophon, Mem. 1.2.13, ἐθαύμαζε... ἐπεὶ καὶ τοὺς μέγιστον φρονοῦντας οὐ ταύτα δοξάζειν ἀλλήλοις.
205 Ptolemy says ‘of those stars which rise simultaneously with point E’. However, he does not mean to compare a number of stars rising simultaneously with some fixed point of the ecliptic; for that would not allow the third situation envisaged, in which two different stars with the same latitude cross the horizon together with point E, and the angle at E is different in the two cases.
206 Literally ‘more numerous’.
farther away. [Finally], in the case of the stars of equal magnitude which rise at
equal distances in latitude [from the sun], the more the ecliptic is inclined to the
horizon, [thus] making angle DEZ smaller, the greater the [solar] distance EZ
at which the star will have its first visibility.

For if, as in the following figure [Fig. 8.4], we also draw in the semi-circle
HΘZK through the poles of the horizon and the sun at Z,\(^207\) which will,
obviously, be perpendicular to the horizon, the [vertical] distance of the sun
below the earth will always remain equal to Ze for the same star, since, for an
equal interval so taken, the [effect of] the rays above the earth will be similar;
but if arc ΘZ is kept constant, arc EZ will, as we said, become less as the ecliptic
is raised more towards a perpendicular position, and greater as it is more
inclined\(^208\) [to the horizon].

Therefore we need observations for each individual fixed star in order to
determine the [required] distance of the sun below the earth as measured along
the ecliptic. And if even the distance vertical to the horizon (for instance, in the
present figure [8.4], ZΘ) does not remain the same for the same stars at all
locations on earth, because the rays of similar density do not have the same
obscuring effect\(^209\) in the thicker air of the more northerly terrestrial latitudes,
we will need observations, not merely at one terrestrial latitude, but at each of
the others alike. However, if the arc corresponding to ZΘ remains constant
everywhere on earth for the same stars (as seems likely, since the fixed stars too
must be affected by the variation in the atmosphere in the same way as the rays
are), the distances observed at a single terrestrial latitude will suffice us to
determine those at the other latitudes: [we can do this] by geometrical methods,

\(^207\) Taking the reading of D at H200, 6, τοῦ κατὰ τὸ Z (for τὸ κατὰ τὸ Z), and at H200,7, HΘZK
(also in Ar) for ΘZK. Corrected by Manitius.

\(^208\) Reading ἐγκλινομένου (with D) for κεκλιμένου at H200,13.

\(^209\) καταλάμπειν, 'shine on so as to obscure'. See p. 470 n.8.
whether the variation in the inclination of the ecliptic is due to the terrestrial location or to the demonstrated motion of the sphere of the fixed stars towards the rear with respect to it [the ecliptic].

[To show this], in the figure described [Fig. 8.4], let the distance EZ be given from an observation at any one terrestrial latitude whatever. Then since, again, the [two great circle arcs] BΘ and ZA have been drawn to meet the two great circle arcs HB and HZ.

\[ \text{Crd arc } 2AB : \text{Crd arc } 2BH = \]
\[ (\text{Crd arc } 2AE : \text{Crd arc } 2EZ) \cdot (\text{Crd arc } 2Z\Theta : \text{Crd arc } 2\Theta H). \]  
[M.T. II]

But, of the arcs in question, arc BH and arc \( \Theta H \) are immediately [given, being] each a quadrant; and since point \( E \), with which the star rises, is given by hypothesis, \( A \), the culminating point, is also given, by means of the section on rising-times [II 9, p. 104]; thus arc AE too is given by this means, and arc EZ by the observation; and arc AH too [and hence arc \( AB = \text{arc } BH - \text{arc } AH \)] is given, being derived from the distance of point \( A \) from the equator (which is given from the Table of Inclination [I 15]) and from the distance of the equator from the zenith along the same meridian (which equals the elevation of the pole). Therefore the remaining [arc], \( Z\Theta \), will be given.

Once this [arc \( Z\Theta \)] has been found, and provided that it remains the same for all locations, we can use it to derive the amounts of arc EZ at [all] other terrestrial latitudes from the same considerations. For again [in Fig. 8.4]

\[ \text{Crd arc } 2HB : \text{Crd arc } 2AB = \]
\[ (\text{Crd arc } 2H\Theta : \text{Crd arc } 2Z\Theta) \cdot (\text{Crd arc } 2ZE : \text{Crd arc } 2EA). \]  
[M.T. II]

And, of the arcs in question, arc \( Z\Theta \) is now given by hypothesis; and since \( E \), the point which rises together with the star at the terrestrial latitude in question, is given by the procedure demonstrated above [VIII 5 p. 412], and similarly arcs
EA and BA are given, the remaining arc, which is arc EZ of the ecliptic, is also given.

We shall take the same method of operation for granted for the last visibilities, which occur near the setting-point. Practically the only difference will be that in the same figure [Fig. 8.4] the ecliptic will be drawn on the other side [of BED], in accordance with the way it is inclined when the horizon [arc] BD is taken as the western part [see Fig. N].

We think that the above suffices as an indication of the methods in this type of theoretical investigation, enough [at least] so that it cannot be said that we have neglected this topic. However, seeing that the computation of this kind of prediction is of great complexity, not only because of the great number of different terrestrial latitudes and inclinations of the ecliptic involved, but also because of the sheer multitude of the fixed stars: seeing, too, that, in respect of the actual observations of the phases it is laborious and uncertain, since [differences between] the observers themselves and the atmosphere in the regions of observation can produce variation in and doubt about the time of the first suspected occurrence, as has become clear, to me at least, from my own experience and from the disagreements in this kind of observations; seeing, furthermore, that because of the motion [through the ecliptic] of the sphere of

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210 As before, (p. 415), from E, the horoscope, we find A, the culminating point, by the procedure II 9 (p. 104). Thus we have arc EA = arc BH - arc AH, where arc BH = 90° and arc AH = φ - δ (A).

211 Reading κατ' αὐτὰς τῶν φάσεων τηρήσεις, with D, at H203.14, i.e. taking it as following ἐνεκεν and understanding τοῦ before ἑγγωδές τε εἶναι. Heiberg prints κατ' αὐτὰς τὰς τῶν (τῶν) ἀστέρων φάσεων τηρήσεις, presumably understanding τὰ πᾶρα before it, but this is very harsh. By phases (φάσεις) Ptolemy means here both first and last visibilities.
the fixed stars, even for the individual terrestrial latitudes the simultaneous risings, culminations and settings cannot remain forever identical with the present ones, which would take such a vast amount of numerical and geometrical computations to calculate, we have decided to dispense with such a time-consuming operation. For the time being we content ourselves with the approximate [phases] which can be derived either from earlier records or from actual manipulation of the [star-]globe for any particular star. Moreover, we notice that the prognostications concerning the states of the atmosphere derived from first or last visibilities (if indeed one assigns these as the cause of changes in the weather), and not rather the positions of the sun in the ecliptic, are almost always approximations, and do not exhibit a perfect regularity and invariability: it seems that this causal factor has only general application, and derives its strength, not so much from the actual times of the first or last visibility, as from the configurations with respect to the sun, taken as intervals in round numbers, and, in part, the inclinations of the moon at those configurations.

212 Reading ἀπό, with D, for ἀπ’ αὐτῶν at H204.3.
213 In his later work, Phases, of which only Bk. II is preserved, Ptolemy lists many of these.
214 προσειστείς. From the Tetrabiblos (II 13, ed. Boll-Boer 100,7–9) it appears that Ptolemy means the direction ('wind') towards which the moon 'points' in its motion in [argument of] latitude. But see also ibid. II 14,5 (ed. Boll-Boer 102,2–3) where it seems to be the direction towards which the sickle or gibbous moon points.
Book IX

1. [On the order of the spheres of sun, moon and the 5 planets]

Such, then, more or less, is the sum total of the chief topics one may mention as having to do with the fixed stars, in so far as the phenomena [observed] up to now provide the means of progress in our understanding. There remains, to [complete] our treatise, the treatment of the five planets. To avoid repetition we shall, as far as possible, explain the theory of the latter by means of an exposition common [to all five], treating each of the methods [for all planets] together.

First, then, [to discuss] the order of their spheres, which are all situated [with their poles] nearly coinciding with the poles of the inclined, ecliptic circle: we see that almost all the foremost astronomers agree that all the spheres are closer to the earth than that of the fixed stars, and farther from the earth than that of the moon, and that those of the three [outer planets] are farther from the earth than those of the other [two] and the sun, Saturn's being greatest, Jupiter's the next in order towards the earth, and Mars' below that. But concerning the spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's],¹ for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.²

And since there is no other way, either, to make progress in our knowledge of this matter, since none of the stars³ has a noticeable parallax (which is the only phenomenon from which the distances can be derived), the order assumed by the older [astronomers] appears the more plausible. For, by putting the sun in the middle, it is more in accordance with the nature [of the bodies] in thus

¹ There is a good deal of evidence for the identity of some of those who held the second opinion, including Plato, Eratosthenes and Archimedes. For details on this and other ancient arrangements see HAM.14 II 690-3.

² I.e. no transits of Venus or Mercury had been observed. Neugebauer has shown (HAMA 227-30) that transits are in fact predictable from Ptolemy's own theory. Ptolemy later seems to have realized this, for in the Planetary Hypotheses (ed. Goldstein 2,28,10-12) he says: 'if a body of such small size (as a planet) were to occult a body of such large size and with so much light (as the sun), it would necessarily be imperceptible, because of the smallness of the occulting body and the state of the parts of the sun's body which remain uncovered.' (Goldstein's translation here, p.6, is inaccurate).

³ This includes both fixed stars and planets.
separating those which reach all possible distances from the sun and those which do not do so, but always move in its vicinity; provided only that it does not remove the latter close enough to the earth that there can result a parallax of any size.\footnote{In his \textit{Planetary Hypotheses} (see Goldstein's edition) Ptolemy proposes a system in which the spheres of the planets are contiguous; thus the greatest distance from the earth attained by a planet is equal to the least distance attained by the one next in order outwards. This appears to provide support for the order he adopts here, since it results in a solar distance very nearly the same as that obtained by a different method in \textit{Almagest} V. 15. Since this system also brings Mercury, at its least distance, to the moon's greatest distance (64 earth-radii), Mercury ought to exhibit a considerable parallax, contrary to what is enunciated here.}

2. \textit{(On our purpose in the hypotheses of the planets)}

So much, then, for the arrangements of the spheres. Now it is our purpose to demonstrate for the five planets, just as we did for the sun and moon, that all their apparent anomalies can be represented by uniform circular motions, since these are proper to the nature of divine beings, while disorder and non-uniformity are alien to such beings. Then it is right that we should think success in such a purpose a great thing, and truly the proper end of the mathematical part of theoretical philosophy.\footnote{Cf. I 1. p. 35.} But, on many grounds, we must think that it is difficult, and that there is good reason why no-one before us has yet succeeded in it.\footnote{We cannot doubt that not only planetary theories but planetary tables had been constructed before Ptolemy: the proof is supplied by Indian astronomy, which is based on Greek theories which are largely, if not entirely, pre-Ptolemaic, and indeed by Ptolemy's own reference to the 'Aeon-tables' below (p. 422). What he means is that all previous efforts were, by his criteria, unsatisfactory.} For, [firstly], in investigations of the periodic motions of a planet, the possible [inaccuracy] resulting from comparison of [two] observations (at each of which the observer may have committed a small observational error) will, when accumulated over a continuous period, produce a noticeable difference [from the true state] sooner when the interval [between the observations] over which the examination is made is shorter, and less soon when it is longer. But we have records of planetary observations only from a time which is recent in comparison with such a vast enterprise: this makes prediction for a time many times greater [than the interval for which observations are available] insecure. [Secondly], in investigation of the anomalies, considerable confusion stems from the fact that it is apparent that each planet exhibits two anomalies, which are moreover unequal both in their amount and in the periods of their return: one [return] is observed to be related to the sun, the other to the position in the ecliptic; but both anomalies are continuously combined, whence it is difficult to distinguish the characteristics of each individually. [It is] also [confusing] that most of the ancient [planetary] observations have been recorded in a way which is difficult to evaluate, and crude. For [1] the more continuous series of observations concern stations and phases [i.e. first and last visibilities].\footnote{Ptolemy is certainly thinking of the Babylonian planetary observations, which are characteristically of this type. They have become available to us through the 'diaries' (see Sachs[2]), but to Ptolemy were probably known only through Hipparchus' compilation (see p. 421).} But detection of both of these particular...
phenomena is fraught with uncertainty: stations cannot be fixed at an exact moment, since the local motion of the planet for several days both before and after the actual station is too small to be observable; in the case of the phases, not only do the places [in which the planets are located] immediately become invisible together with the bodies which are undergoing their first or last visibility, but the times too can be in error, both because of atmospherical differences and because of differences in the [sharpness of] vision of the observers. [2] In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an angle of any size (hence one may expect considerable error in determining the position in latitude and longitude, due to the varying inclination of the ecliptic [to the horizon frame of reference]); but also because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven; hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

Hence it was, I think, that Hipparchus, being a great lover of truth, for all the above reasons, and especially because he did not yet have in his possession such a groundwork of resources in the form of accurate observations from earlier times as he himself has provided to us, although he investigated the theories of the sun and moon, and, to the best of his ability, demonstrated with every means at his command that they are represented by uniform circular motions, did not even make a beginning in establishing theories for the five planets, not at least in his writings which have come down to us. All that he did was to make a compilation of the planetary observations arranged in a more useful way, and to show by means of these that the phenomena were not in agreement with the hypotheses of the astronomers of that time. For, we may presume, he thought that one must not only show that each planet has a twofold anomaly, or that each planet has retrograde arcs which are not constant, and are of such and such sizes (whereas the other astronomers had constructed their geometrical proofs on the basis of a single unvarying anomaly and retrograde arc); nor [that it was sufficient to show] that these anomalies can in fact be represented either

\[\text{IX 2. Why Hipparchus did not establish theory of planets}\]

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8 This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance e.g. to the observations of Mercury's greatest elongations in IX 7. Ptolemy discusses it (as a theoretical problem) in some detail in Optics V 23-30 (ed. Lejeune 237-42).

9 This seems to imply that Hipparchus recorded planetary observations of his own, which Ptolemy used to establish his theories. This may be true, but it is strange that Ptolemy cites not a single such observation by Hipparchus. Could Ptolemy mean merely that Hipparchus had not 'yet' assembled the compilation of earlier planetary observations which he mentions just below?

10 The circulation of books in antiquity was so fortuitous that, even for one, like Ptolemy, who had access to the great resources of the libraries at Alexandria, this was a necessary caveat.

11 I have little doubt that all the other planetary observations cited in the Almagest are derived from this compilation (cf. p. 452 n.66), and that part of Hipparchus' 'rearrangement' was to give their dates in the Egyptian calendar. For a similar service he rendered for the listing of lunar eclipses see H.A.M.A 320-21.
by means of eccentric circles or by circles concentric with the ecliptic, and carrying epicycles, or even by combining both, the ecliptic anomaly being of such and such a size, and the synodic anomaly of such and such (for these representations have been employed by almost all those who tried to exhibit the uniform circular motion by means of the so-called 'Aeon-tables', but their attempts were faulty and at the same time lacked proofs: some of them did not achieve their object at all, the others only to a limited extent); but, [we may presume], he reckoned that one who has reached such a pitch of accuracy and love of truth throughout the mathematical sciences will not be content to stop at the above point, like the others who did not care [about the imperfections]; rather, that anyone who was to convince himself and his future audience must demonstrate the size and the period of each of the two anomalies by means of well-attested phenomena which everyone agrees on, must then combine both anomalies, and discover the position and order of the circles by which they are brought about, and the type of their motion; and finally must make practically all the phenomena fit the particular character of the arrangement of circles in his hypothesis. And this, I suspect, appeared difficult even to him.

The point of the above remarks was not to boast [of our own achievement]. Rather, if we are at any point compelled by the nature of our subject to use a procedure not in strict accordance with theory (for instance, when we carry out proofs using without further qualification the circles described in the planetary spheres by the movement [of the body, i.e.] assuming that these circles lie in the plane of the ecliptic. to simplify the course of the proof); or [if we are compelled] to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application, or to assume a type of motion or inclination of the circles which is not the same and unchanged for all planets; we may [be allowed to] accede [to this compulsion], since we know that this kind of inexact procedure will not affect the end desired, provided that it is not going to result in any noticeable error; and we know too that assumptions made without proof, provided only that they are found to be in agreement with the phenomena, could not have been found without some careful methodological procedure, even if it is difficult

12 διὰ τῆς καλομένης αἰονίου κανονοποιίας. In my opinion, Ptolemy is referring to a type of work in which the mean motions of the planets were represented by integer numbers of revolutions in some huge period, in which they all return to the beginning of the zodiac, and the planetary equations were calculated by a combination of epicycles or of eccentric and epicycle which was not reducible to a geometrically consistent kinematic model, i.e. to a class of Greek works which were the ancestors of the Indian siddhâantas. In this I am in agreement with van der Waerden, 'Ewige Tafeln', except that I believe that the αἰόνιος implied by the title of these tables does not mean 'eternity' (cf. the conventional translation, 'Eternal Tables', which is philologically possible, but not necessary), but refers to the immense common period in which the planets return (cf. the Greek inscription of Keskineto, HAM: 698-705, and the Indian Mahâyuga). The other two references to these tables in antiquity (P. Lond. 130, see Neugebauer-van Hoesen, Greek Horoscopes p. 21, 112-13, and Vettius Valens VI 1, ed. Kroll 243,8) are consistent with, but do not require, this interpretation.

13 Literally 'as if the circles were bare [circles'].

14 Ptolemy in fact carries out all the proofs involving the longitudinal motions of the planets (in Bks. IX-XII) as if the motions lay in the plane of the ecliptic.

15 The paradigm case of this is the introduction of the equant.

16 E.g. the special model for the longitudinal motions of Mercury, or the special inclinations attributed to the inner planets for their latitudinal motions.
to explain how one came to conceive them (for, in general, the cause of first principles is, by nature, either non-existent or hard to describe); we know, finally, that some variety in the type of hypotheses associated with the circles [of the planets] cannot plausibly be considered strange or contrary to reason (especially since the phenomena exhibited by the actual planets are not alike [for all]); for, when uniform circular motion is preserved for all without exception, the individual phenomena are demonstrated in accordance with a principle which is more basic and more generally applicable than that of similarity of the hypotheses [for all planets].

The observations which we use for the various demonstrations are those which are most likely to be reliable, namely [1] those in which there is observed actual contact or very close approach to a star or the moon, and especially [2] those made by means of the astrolabe instruments. [In these] the observer's line of vision is directed, as it were, by means of the sighting-holes on opposite sides of the rings, thus observing equal distances as equal arcs in all directions, and can accurately determine the position of the planet in question in latitude and longitude with respect to the ecliptic, by moving the ecliptic ring on the astrolabe, and the diametrically opposite sighting-holes on the rings through the poles of the ecliptic, into alignment with the object observed.

3. [On the periodic returns of the five planets] 18

Now that we have completed the above discussion, we will first set out, for each of the 5 planets, the smallest period in which it makes an approximate return in both anomalies, as computed by Hipparchus.19 These [periods] have been corrected by us, on the basis of the comparison of their positions which became possible after we had demonstrated their anomalies, as we shall explain at that point.20 However, we anticipate and put them here, so as to have the individual mean motions in longitude and anomaly set out in a convenient form for the calculations of the anomalies. But it would in fact make no noticeable difference in those calculations21 even if one used more roughly computed mean positions.

17 It is not clear why the plural ('rings') is used (contrast the singular at V 1, H354,13). Although the sights are attached only to ring 1 in Fig. F (p. 218), Ptolemy is presumably referring to both ring 1 and ring 2, since ring 2 has first to be moved to the correct sighting position on the ecliptic ring (no. 3).
19 If Ptolemy means, as we may presume, that the periods 'computed by Hipparchus' are the relationships in integers, '57 returns in anomaly correspond to 59 years and 2 revolutions in longitude', etc., then he seems ignorant of the fact that these are well-known (to us) Babylonian period relationships (for details see H.A.M.I 151).
20 This is a reference to the chapters on the 'corrections of the mean motions', IX 10, X 4, X9, XI 3 and XI7. The 'comparison' refers to the use in these chapters of two positions, separated by a long time-interval, to derive the mean motions. On the problem of the actual derivation of the corrections given here, and of the mean motions, see Appendix C.
21 Ptolemy means that where he uses the mean motions in determining the eccentricity (e.g. X 2, p. 484) over the short periods involved (a few years) one could use quite crude parameters (e.g. the mean motions given by the uncorrected Babylonian periods) without seriously affecting the final result. He is right (see p. 484 n.33). The corrected mean motions are given here merely for convenience. Cf. the procedure for the lunar mean motion table, p. 179.
As a general definition, we mean by 'motion in longitude' the motion of the centre of the epicycle around the eccentre, and by 'anomaly' the motion of the body around the epicycle.

We find, then, that

[1] for Saturn, 57 returns in anomaly correspond to 59 solar years (as defined by us, i.e. returns to the same solstice or equinox), plus about 14 days, and to 2 revolutions [in longitude] plus 143° (for in the case of the 3 planets which are always overtaken by the sun the number of revolutions of the sun during the period of return is always, for each of them, the sum of the number of revolutions in longitude and the number of returns in anomaly of the planet);

[2] for Jupiter, 65 returns in anomaly correspond to 71 solar years (defined as above) less about 14 days, and to 6 revolutions of the planet from a solstice back to the same solstice, less 48°;

[3] for Mars, 37 returns in anomaly correspond to 79 solar years (as defined by us) plus about 313 days, and to 42 revolutions of the planet from a solstice back to the same solstice, plus 314°;

[4] for Venus, 5 returns in anomaly correspond to 8 solar years (as defined by us) less about 18 days, and to a number of [longitudinal] revolutions of the planet equal to that of the sun, 8, less 21°;

[5] for Mercury, 145 returns in anomaly correspond to 46 of the same kind of years plus about 15 days, and to a number of [longitudinal] revolutions which is, again, equal to that of the sun, 46, plus 1°.

But if, for each planet, we reduce the period of return to days, in accordance with the length of the year as demonstrated by us, and the number of returns in anomaly to degrees according to the system in which a circle contains 360°, we will get:

for Saturn, 21551;18 and 20520° of anomaly
for Jupiter, 25927;37 and 23400° of anomaly
for Mars, 28857;43 and 13320° of anomaly
for Venus, 2919;40 and 1800° of anomaly
for Mercury, 16802;24 and 52200° of anomaly.

So we divide the degrees of anomaly proper to each by the appropriate number of days, and get the following for the approximate mean daily motions in anomaly:

Saturn 0:57.7,43,41,43,40°
Jupiter 0:54,9,2,46,26,0°

22 Περικαταλαμβανομένων. Cf. Περικατάληψις ΗI 24,13. This feature distinguishes the three outer planets from the two inner ones, since the latter (usually) overtake the sun.
23 Expressed by Ptolemy as $3 + \frac{1}{4} + \frac{1}{2}$. 
24 Expressed by Ptolemy as $2 + \frac{1}{2} + \frac{1}{3}$.
25 Reading $\frac{6}{2}$, with D. Ar., for $\frac{5}{4}$ (27400) at H216.1. Corrected by Manitius.
26 Reading $\frac{17}{2}$ for $\frac{15}{2}$ (53) at H216.2. Multiplying the Ptolemaic length of the year, 365;14,48°, by 79 and adding 3;13 produces 28857;42,12, of which 28857;43 is the rounding. The ms. tradition is solid for 53, but nothing in the previous or subsequent calculations favours it.
27 Precise calculation (cf. n.26) gives 16802;22,48. Possibly we should change 1\frac{1}{2} days (above) to 1\frac{1}{3} days (reading $\kappa$ for $\lambda$' at H215.11).
28 For the problem of precisely how Ptolemy arrives at the parameters he gives for the planetary mean motions, which is not as simple as it appears here, see Appendix C.
Mars 0;27,41,40,19,20,58°
Venus 0;36,59,25,53,11,28°
Mercury 3;6,24,6,59,35,50°.

For each of these we take $\frac{1}{12}$th to get the following mean hourly motions in anomaly:

Saturn 0;2,22,49,19,14,19,10°
Jupiter 0;2,15,22,36,56,5°
Mars 0;1,9,14,10,48,22,25°
Venus 0;1,32,28,34,42,58,40°
Mercury 0;7,46,0,17,28,59,35°.

Then we multiply the daily motion of each by 30 to get the following mean monthly motions in anomaly:

Saturn 28;33,51,50,51,50,0°
Jupiter 27;4,31,23,13,0,0°
Mars 13;50,50,9,40,29,0°
Venus 18,29,42,56,35,44,0°
Mercury 183;3,29,47,55,0°.

Similarly, we multiply the daily motions by 365, the number of days in one Egyptian year, to get the following mean yearly motions in anomaly:

Saturn 347;32,0,48,50,38,20°
Jupiter 329,25,1,52,28,10,0°
Mars 168,3,28,30,17,42,32,50°
Venus 225,1,32,28,34,39,15°29
Mercury 53,56,42,32,32,59,10° (increment [overcomplete circles]).

In the same way, we multiply each of the annual motions by 18 (just as we did in the construction of tables for the luminaries), to get the following increments in mean anomaly for the period of 18 Egyptian years:

Saturn 135;36,14,39,11,30,0°
Jupiter 169;30,33,44,27,0,0°
Mars 152;33,5,18,45,51,0°
Venus 90;27,44,34,23,46,30°
Mercury 251;0,45,45,53,45,0°.

We can also find the mean motions in longitude corresponding to the above without reducing the number of [longitudinal] revolutions to degrees and dividing them by [the number of days in] the period set out above for each planet. For Venus and in Mercury, it is obvious that we can do this by taking the same mean motions as we set out previously for the sun; for the other three planets, by taking the difference between the [mean motion in] anomaly and the corresponding solar [mean] motion for each individual entry.30 By this method we get the following mean motions in longitude:

39 This corresponds to a mean daily motion of 0;36,59,25,53,11,27°, i.e. one less in the last place than that given above. Thus the mean motion table of Venus is based on different parameters in different parts: on 28 in the last place for hours, days and months, and on 27 in the last place for years and 18-year periods. On the possible significance of this see Appendix C p. 671 n.11.
30 Venus and Mercury have the same mean motion in longitude as the sun. For the other planets, for any length of time, the sum of anomaly and mean motion equals the sun’s mean motion, because of the relationship stated at p. 424.
### IX 3. Mean motions of the planets

#### Daily:
- **Saturn**: 0;2,0,33,31,28,51°
- **Jupiter**: 0;4,59,14,26,46,31°
- **Mars**: 0;31,26,36,53,51,33°

#### Hourly:

<table>
<thead>
<tr>
<th></th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
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<tbody>
<tr>
<td>H219</td>
<td>0;0,5,1</td>
<td>0;12,28</td>
<td>0;1,18,36</td>
</tr>
<tr>
<td></td>
<td>23,48,42,7,30°</td>
<td>6,56,17,30°</td>
<td>32,14,38,52,30°</td>
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#### Monthly:

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<th></th>
<th>Saturn</th>
<th>Jupiter</th>
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<tbody>
<tr>
<td></td>
<td>1;0,16,45,44,25,30°</td>
<td>2;29,37,13,23,15,30°</td>
<td>15;43,18,26,55,46,30°</td>
</tr>
</tbody>
</table>

#### Yearly:

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<tr>
<th></th>
<th>Saturn</th>
<th>Jupiter</th>
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<tr>
<td></td>
<td>12;13,23,56,30,30,15°</td>
<td>30;20,22,52,58,35,32</td>
<td>191;16,54,27,38,35,45°</td>
</tr>
</tbody>
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#### For 18 years:

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<tr>
<th></th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
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<tr>
<td></td>
<td>220;1,10,57,9,4,30°</td>
<td>186;6,51,51,34,30°</td>
<td>203;4,20,17,34,43,30°</td>
</tr>
</tbody>
</table>

So once again, for the user's convenience, we shall set out, for each of the planets in order, tables of the above mean motions derived by successive summation [of the motions for the appropriate time-interval]. Like the other [mean motion tables], these will be in 45 lines and 3 sections: the first section will contain the entries (obtained by successive summation) for the 18-year periods; the second will contain those for the years and hours, and the third those for the months and days.

The tables are as follows.

<table>
<thead>
<tr>
<th></th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
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<tbody>
<tr>
<td>H220-49</td>
<td>4. {Tables of the mean motions in longitude and anomaly of the five planets}^{33}</td>
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<tr>
<td></td>
<td>[See pp. 427-41.]</td>
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<tr>
<td>H250</td>
<td>5. {Preliminary notions [necessary] for the hypotheses of the 5 planets}^{34}</td>
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</tbody>
</table>

Now that these [mean motions] have been tabulated, our next task is to discuss the anomalies which occur in connection with the longitudinal positions of the five planets. The way we have approached it, to give the general outlines, is as follows.

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^{31} Reading $\lambda_\eta \nu \beta$ $\lambda$ (38,52,30) for $\lambda_\theta$ (39) at H219,2, with D,Ar. Although the figure is rounded to 39 in the table, there is no reason why it should be (for Mars alone) here.

^{32} Reading $\nu \nu \nu \lambda \xi$ for $\nu \nu \lambda \xi$ (52,38,35) at H219,7, with D,Ar. Corrected by Manitius.

^{33} Corrections to Heiberg:
- H235,24 (Mars, longitude, 3°, last place) read $\nu \gamma$ for $\zeta$ (6). Misprint.
- H238,3 (Venus, epoch in longitude) read $\nu \mu \nu$ for $\nu \mu \nu$ (45°), with $D^2$.
  - Corrected by Manitius, but this is not (pace Manitius) a misprint in Heiberg.

^{34} On chs. 5 and 6 see HAM.1 149-50.
### Saturn's Mean Motion Tables

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<tbody>
<tr>
<td>18</td>
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<td>135°</td>
<td>14°</td>
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<tr>
<td>36</td>
<td>200°</td>
<td>271°</td>
<td>12°</td>
</tr>
<tr>
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**Saturn's mean motion tables**

**IX 4.**
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**Notes:**
- The table provides Venus' mean motion tables, detailing the longitude and anomaly at specific years.
- The data includes columns for single years, Venus longitude, and Venus anomaly, with corresponding values for each year.
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### IX 4. Mercury's mean motion tables

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</tbody>
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Note: The table above contains the Mercury's mean motion tables for the specified months and days, listing the longitude and anomaly values in degrees.
There are, as we said, two types of motion which are simplest and at the same time sufficient for our purpose, [namely] that produced by circles eccentric to [the centre of] the ecliptic, and that produced by circles concentric with the ecliptic but carrying epicycles around. There are likewise two apparent anomalies for each planet: [1] that anomaly which varies according to its position in the ecliptic, and [2] that which varies according to its position relative to the sun.

For [2] we find, from a series of different [sun-planet] configurations observed round about the same part of the ecliptic, that in the case of the five planets the time from greatest speed to mean is always greater than the time from mean speed to least. Now this feature cannot be a consequence of the eccentric hypothesis, in which exactly the opposite occurs, since the greatest speed takes place at the perigee in the eccentric hypothesis, while the arc from the perigee to the point of mean speed is less than the arc from the latter to the apogee in both [eccentric and epicyclic] hypotheses. But it can occur as a consequence of the epicyclic hypothesis, however only when the greatest speed occurs, not at the perigee, as in the case of the moon, but at the apogee; that is to say, when the planet, starting from the apogee, moves, not as the moon does, in advance [with respect to the motion] of the universe, but instead towards the rear. Hence we use the epicyclic hypothesis to represent this kind of anomaly.

But for [1], the anomaly which varies according to the position in the ecliptic, we find from [observations of] the arcs of the ecliptic between [successive] phases or [sun-planet] configurations of the same kind that the opposite is true: the time from least speed to mean is always greater than the time from mean speed to greatest. This feature can indeed be a consequence of either of the two hypotheses (in the way we described in our discussion of the equivalence of the hypotheses at the beginning of our treatise on the sun [III 3]). But it is more appropriate to the eccentric hypothesis, and that is the hypothesis which we actually use to represent this kind of anomaly, since, moreover, the other anomaly was found to be peculiar, so to speak, to the epicyclic hypothesis.

Now from prolonged application and comparison of observations of individual [planetary] positions with the results computed from the combination of [the above] hypotheses, we find that it will not work simply to assume [as one has hitherto] that the plane in which we draw the eccentric

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35 III 3 p. 141.
36 This eliminates the effect of the ecliptic anomaly. Examples would be observations of Mars at opposition, station and (by interpolation) conjunction all near the same point in the ecliptic.
37 Exsisting κατ’ before έκ των πέντε πλανώμενων at H250.17. (κατ’ was apparently omitted in the text translated by al-Hajjaj). One would have to translate Heiberg’s text ‘in the case of the five planets too’ (as well as the sun and moon). But the situation is precisely the opposite for the sun and moon (see e.g. III 4 p. 153). Perhaps the whole phrase κατ . . . πλανώμενων is an ancient interpolation.
38 See Ptolemy's discussion of this point at III 3 p. 144-5. However, as Neugebauer points out (H.A.M.A 149-50) it is perfectly possible for an eccentric model to represent the planetary motions, provided the apsidal line is allowed to move, and precisely this kind of eccentric model is described at XII 1, though even there Ptolemy restricts its applicability to the outer planets.
39 This eliminates the effect of the synodic anomaly. Examples would be observations of oppositions of Mars in different parts of the ecliptic (as in X 7).
40 Cf. III 4 p. 153, where Ptolemy prefers it on the ground that it is 'simpler'.
41 Literally 'that the assumption that . . . cannot progress so simply'.

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circles is stationary, and that the straight line through both centres (the centre of
the [planet's] eccentre and the centre of the ecliptic), which defines apogee and
perigee, remains at a constant distance from the solstitial and equinoctial
points; nor [to assume] that the eccentre on which the epicycle centre is carried
is identical with the eccentre with respect to the centre of which the epicycle
makes its uniform revolution towards the rear, cutting off equal angles in equal
times at [that centre]. Rather, we find that the apogee of the eccentric performs a
slow motion towards the rear with respect to the solstices, which is uniform
about the centre of the ecliptic, and comes to about the same for each planet as
the amount determined for the sphere of the fixed stars, i.e. 1° in 100 years (at
least, as far as can be estimated on the basis of available evidence). We find, too,
that the epicycle centre is carried on an eccentre which, though equal in size to
the eccentre which produces the anomaly, is not described about the same
centre as the latter. For all planets except Mercury the centre [of the actual
deferent] is the point bisecting the line joining the centre of the eccentre
producing the anomaly to the centre of the ecliptic. For Mercury alone, [the
centre of the deferent] is a point whose distance from the centre of the circle
about which it rotates is equal to the distance of the latter point towards the
apogee from the centre of the eccentre producing the anomaly, which in turn is
the same distance towards the apogee from the point representing the observer;
for also, in the case of this planet alone, we find that, just as for the moon, the
eccentre is rotated by [the movement of] the above-mentioned centre in the
opposite sense to the epicycle, [i.e.] in the advance direction, one rotation per
year. [This must be so] because the planet itself appears twice in the perigee in
the course of one revolution, just as the moon appears twice in the perigee in one
[synodic] month.

6. [On the type of and difference between the hypotheses]

One may more easily grasp the type of the hypotheses which we infer on the
basis of the preceding [phenomena] from the following description.

First for that of the [four planets] other [than Mercury], imagine [Fig. 9.1]
the eccentre ABG about centre D, with ADG as the diameter through D and the
centre of the ecliptic: on this let E be taken as the centre of the ecliptic, i.e. the
viewpoint of the observer, making A the apogee and G the perigee. Let DE be
bisected at Z, and with centre Z and radius DA draw a circle H0K, which
must, clearly, be equal to ABG. Then on centre O draw the epicycle LM, and
join LOMD.

First, then, although we assume that the plane of the eccentric circles is
inclined to the plane of the ecliptic, and also that the plane of the epicycle is
inclined to the plane of the eccentres, to account for the latitudinal motion of
the planets, in accordance with what we shall demonstrate concerning that
topic, nevertheless, for the motions in longitude, for the sake of convenience, let
us imagine that all [those planes] lie in a single [plane], that of the ecliptic, since
there will be no noticeable longitudinal difference, not at least when the
inclinations are as small as those which will be brought to light for each of the
planets. Next, we say that the whole plane [of the eccentre] moves uniformly about centre E towards the rear [i.e. in the order] of the signs, shifting the position of apogee and perigee 1° in 100 years, and that diameter LΘM of the epicycle rotates uniformly about centre D, again towards the rear [i.e. in the order] of the signs, with a speed corresponding to the planet's return in longitude, and that it carries with it points L and M of the epicycle, and centre Θ of the epicycle (which always moves on the eccentre HΘK), and also carries with it the planet; the planet, for its part, moves with uniform motion on the epicycle LM and performs its return always with respect to that diameter [of the epicycle] which points towards centre D, with a speed corresponding to the mean period of the synodic anomaly, and [a sense of rotation] such that its motion at the apogee L takes place towards the rear.

We can visualise the peculiar features of the hypothesis for Mercury as follows. Let [Fig. 9.2] the eccentre producing the anomaly be ABC about centre D, and let the diameter through D and centre E of the ecliptic be ADEG, [passing] through the apogee at A. On AG take DZ towards the apogee A, equal to DE. Then everything else remains the same, namely the whole plane, [revolving] about centre E, shifts the apogee towards the rear by the same amount as for the other planets, the epicycle is revolved uniformly about centre D towards the rear, as [here] by the line DB, and furthermore the planet moves on the epicycle in the same way as the others. But in this case the centre of the other eccentre, which is, again, equal in size to the first eccentre, and on which the epicycle centre is always located, is carried around point Z in the opposite sense to the motion of the epicycle, namely in advance [i.e. in the reverse order] of the signs, but uniformly and with the same speed as the epicycle, as [here] by the line ZΘ. Thus in one year each of the lines DB and ZΘ performs one
return with respect to a [given] point of the ecliptic, but, with respect to each other, obviously, two returns. And [the centre of that eccentric] will always be at a constant distance from point Z, and that distance too will be equal to both ED and DZ (as [here] ZH). Thus the small circle described by its motion in advance, with centre Z and radius ZH, always has on its boundary the point D (the centre of the first, fixed eccentric) too; and the moving eccentric, at any given moment, can be described with centre H and radius HΘ equal to DA (as here ΘK), the epicycle always having its centre on it (as here at point K).

We shall get an even clearer grasp of these hypotheses from the demonstrations we shall make [in determining] the parameters for each planet individually. In those demonstrations will also frequently become clear, [at least] in outline, the motives which somehow led us to adopt these hypotheses.

However, one must make the preliminary point that the longitudinal periods do not bring the planet back to the same position both with respect to a point on the ecliptic and [simultaneously] with respect to the apogee or perigee of the eccentric; this is due to the shift in position which we assign to the latter. Hence the mean motions in longitude which we tabulated above represent, not the returns [of the planets] defined with respect to the apogees of the eccentres, but the returns defined with respect to the solstitial and equinoctial points, agreeing with the length of the year as we have determined it.

Now we must prove first that from these hypotheses too it follows that, for equal distances of the planet in mean longitudinal motion on opposite sides of apogee or perigee, the equation of ecliptic anomaly on one side [of apogee or

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42 In other words, the mean motions tabulated by Ptolemy are tropical, not sidereal mean motions, and since the apogees are, by his definition, sidereally fixed, a return in longitude (to the same point in the ecliptic) must differ slightly from a return to the apogee.
IX 6. Symmetries in planetary model

perigee] is equal to that on the other side, and that the greatest elongation on the epicycle from the mean position [on one side is equal to that] in the same direction [on the other side].

Let [Fig. 9.3] the eccentric circle on which the epicycle centre moves be $ABGD$ on centre $E$, with diameter $AEG$, on which $Z$ is taken as the centre of the ecliptic, and $H$ as the centre of the eccentric producing the anomaly, i.e. the point about which we say the uniform motion of the epicycle takes place. Draw $BH\Theta$ and $DHK$ at equal distances from apogee $A$ (so that $\angle AHB = \angle AHD$), draw on points $B$ and $D$ epicycles of equal size, and join $BZ$ and $DZ$. From $Z$, the observer, draw $ZL$ and $ZM$ as tangents to the [two] epicycles in the same direction [i.e. both towards the perigee].

![Diagram of Fig. 9.3](image)

I say [1] that the angles of the equation of ecliptic anomaly

$\angle ZBH = \angle HDZ$

[2] similarly, that the greatest elongations on the epicycle

$\angle BZL = \angle DZM$.

(For, [if these statements are true], the amounts of the greatest elongations from the mean [position] resulting from the combination [of the hypotheses] will also be equal [on opposite sides of the apsides]).

[Proof:] Drop perpendiculars $BL$ and $DM$ from $B$ and $D$ on to $ZL$ and $ZM$, and perpendiculars $EN$ and $EX$ from $E$ on to $DK$ and $B\Theta$.

By 'in the same direction' is meant 'both towards apogee' or 'both towards perigee'. This is explained by Fig. 9.3. Ptolemy is carrying out the proof of symmetry analogous to that performed for the models of the sun and moon (III 3 pp. 151-3).

$\angle BZL$ etc. are the true maximum elongations (as seen from the earth). In what follows Ptolemy is going to compare the mean maximum elongations, and it is essential to his proof that these too be symmetrical about the line of the apsides. Since the latter differ from the angles $BZL$ etc. by an angle equal to the equation of centre, or $\angle ZBH$ etc., the symmetry is guaranteed by the equations [1] and [2].
Then, since $\angle XHE = \angle NHE$\textsuperscript{45} and the angles at N and X are right and EH is common to the equiangular triangles [NHE, XHE],
\[ \text{NH} = \text{XH} \]
and perpendicular $\text{EN} = \text{perpendicular EX}$. Therefore lines $\Theta B$ and $\text{DK}$ are equidistant from centre $E$. Therefore they are equal to one another,\textsuperscript{46} and their halves are equal to one another [i.e. $\text{BX} = \text{DN}$]. Therefore, by subtraction [of $\text{XH}$ from $\text{BX}$ and $\text{NH}$ from $\text{DN}$],
\[ \text{BH} = \text{DH}. \]
But $\text{HZ}$ is common [to triangles $\text{BHZ}$, $\text{DHZ}$] and $\angle \text{BHZ} = \angle \text{DHZ}$\textsuperscript{47}. Therefore base $\text{BZ} = \text{base DZ}$ and $\angle \text{HBZ} = \angle \text{HDZ}$.
But also $\text{BL} = \text{DM}$ (radii of the epicycle), and the angles at L and M are right.
\[ \therefore \angle \text{BZL} = \angle \text{DZM}. \]
\textbf{Q.E.D.}

Again, to represent the hypothesis for Mercury, let [Fig. 9.4] $\text{ABG}$ be the diameter through the centres and apogee of the [eccentric] circles, and let $\text{A}$ be taken as the centre of the ecliptic, $\text{B}$ as the centre of the eccentric producing the anomaly, and $\text{G}$ as the point about which rotates the centre of the eccentric carrying the epicycle. Draw, again on both sides [of the apogee], lines $\text{BD}$ and $\text{BE}$, representing the uniform motion of the epicycle towards the rear, and lines

\textbf{Fig. 9.4}

\textsuperscript{45} Because they are vertically opposite the equal angles $\text{AHB}$ and $\text{AHD}$.
\textsuperscript{46} Euclid III 14.
\textsuperscript{47} Excising ἦ ὑπὸ τὰς ἴσας πλευρὰς at H259.4-5. Heiberg emended to ἦ ὑπὸ τῶν ἴσων πλευρῶν (the normal expression). It would mean 'the angles enclosed by the equal sides', and was presumably interpolated to make explicit the condition of Euclid 14, 'If two triangles have two sides equal to two sides, and have the angles enclosed by the equal straight lines equal, they will also have the base equal to the base'. The reason for the equality of the angles is that they are the supplements of the equal angles $\text{AHB}$ and $\text{AHD}$.\textsuperscript{•}
GZ and GH representing the revolution of the eccentre in advance with a speed equal [to the epicycle's]. (So it is clear that the angles at G and B must be equal, and BD must be parallel to GZ, and BE to GH). On GZ and GH take the centres of the [moving] eccentres - let them be Θ and K - and let the eccentres drawn on those centres (on which the epicycles are located), pass through points D and E. On points D and E draw epicycles (again equal), join AD and AE, and draw AL and AM tangent to the epicycles on the same side [of the epicycles].

Then we must prove that in this situation too the angles of the equation\(^\text{48}\) of ecliptic anomaly

\[ \angle ADB = \angle AEB, \]
and that the angles of greatest elongation on the epicycle

\[ \angle DAL = \angle EAM. \]

[Proof:] Join BΘ, BK, ΘD and KE,
and drop perpendiculars GN and GX from G on to BD and BE,
perpendiculars DZ and EH from D and E on to GZ and GH,
and perpendiculars DL and EM from D and E on to AL and AM.

Then, since \( \angle GBN = \angle GBX \) [by hypothesis]
\[ GN = GX \]
i.e. \( DZ = EH.\)
And also \( \Theta D = KE \)
and the angles at Z and H are right.
So \( \angle D\Theta Z = \angle EKH. \)

And because [in triangles GΘB, GKB]
\[ \Theta G = GK \] (by hypothesis)
and GB is common
and \( \angle \Theta GB = \angle KGB, \)
hence \( \angle G\Theta B = \angle GKB. \)

Therefore, by subtraction, \( \angle B\Theta D = \angle BKE,\)
and base BD = base BE.

But again [in triangles BAD, BAE]
BA is common
and \( \angle DBA = \angle EBA \) [by hypothesis].
So base AD = base AE
and \( \angle ADB = \angle AEB. \)

By the same reasoning [as before]
since DL = EM [epicycle radii]
and the angles at L and M are right,
\[ \angle DAL = \angle EAM. \]

Q.E.D.

\(^{48}\) Reading τὸν παρὰ τὴν ζῳδιακὴν ἔνωμαλίαν διαφόροι at H260.8. Heiberg, following the Greek mss., omits the last word, which was restored by Halma (followed by Manitius), apparently without authority. It was in fact read by Is.

\(^{49}\) GZDN and GHEX are parallelograms.

\(^{50}\) Although one can see that this must be so by symmetry, the proof is quite intricate. For the radii of the deferent in its two positions are not ΘD and KE, but KD and KE. Cf. Manitius p. 435.

\(^{51}\) Therefore, \( \angle B\Theta D = 180° - (\angle D\Theta Z + \angle G\Theta B), \angle BKE = 180° - (\angle EKH + \angle GKB). \)

\(^{52}\) In the congruent triangles BΘD, BKE.
After establishing the above theory, we determined, first, in what part of the ecliptic Mercury's apogee lies, by the following method.

We sought out observations of greatest elongations in which the distance [of Mercury] as morning-star from the mean longitude of the sun (i.e. from the mean longitude of the planet) is equal to its distance as evening-star. For once we had found such a situation, it necessarily follows from our [above] demonstrations that the point on the ecliptic halfway between the two positions [of Mercury as morning-star and evening-star] occupies the apogee of the eccentric.

The observations which we used for this purpose are few in number, because precisely such combinations [of planet and sun positions] rarely occur; nevertheless they are sufficient to exhibit the desired result. The more recent of them are the following.

[1] In the sixteenth year of Hadrian, Phamenoth [VII] 16/17 in the Egyptian calendar [132 Feb. 2 '3], in the evening, we observed Mercury, by means of the astrolabe instrument, at its greatest distance from the mean longitude of the sun. Also, from a sighting with respect to the bright star in the Hyades, it was seen then to occupy a longitude of $\varpi 1^\circ$. At the time in question the sun's mean longitude was $= 9^\circ$. So the greatest elongation from the mean as evening-star comes out as $21^\circ$.

[2] And, in the eighteenth year of Hadrian, Epiphi [XI] 18/19 in the Egyptian calendar [134 June 3/4], at dawn, Mercury [was observed] at greatest elongation, appearing very small and dim; from a sighting with respect to the bright star in the Hyades it was seen then to occupy a longitude of $\varpi 1^\circ$. Now at that time the mean sun was in $\Pi 10^\circ$. Here too, then, the greatest elongation from the mean as morning-star was $21^\circ$, equal [to the elongation in [1]].

So, since the mean position of the planet was $= 9^\circ$ at one of the observations, and $\Pi 10^\circ$ at the other, and the point of the ecliptic halfway between these occupies $\varpi 9^\circ$, the diameter through the apogee must lie in that position at that time.

[3] Again, in the first year of Antoninus, Epiphi [XI] 20/21 in the Egyptian calendar [138 June 4/5], in the evening, we observed Mercury by means of the astrolabe at its greatest distance from the sun's mean longitude. From a sighting

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53 See *HAMA* 159-61, Pedersen 309-312. An acute critique of the method employed by Ptolemy for determining the apsidal line of the inner planets was made by Sawyer, 'Ptolemy's Determination of the Apsidal Line for Venus'. He shows that mere equality of mean maximum morning and evening elongations is an insufficient criterion for positing symmetry to the apsidal line, although the observations Ptolemy actually chose are in fact (grosso modo) symmetric. For other criticisms see Wilson, 'Inner Planets', 225 ff.

54 The star in question is a Tau, which has in the catalogue (XXIII 14) a longitude of $\varpi 12^\circ$. In order to find the result he does, Ptolemy should have observed on the instrument a longitudinal difference of $71^\circ$, which is so large as to cast doubt on the validity of the observation. But, by using the same star as reference-point in both observations [1] and [2], Ptolemy may have thought that he was minimizing any error resulting from faulty determination of the star's ecliptic position.

55 I.e. on this occasion the observed longitudinal difference was only $61^\circ$. (see n.54).
at that moment with respect to the star on the heart of Leo it was seen to occupy \(\simeq 7^\circ\). But at the time in question the mean sun was in \(\Pi 10^1\). Therefore the greatest elongation [of Mercury] as evening-star comes out as \(26_{10}^1\).

[4] Similarly, in the fourth year of Antoninus, Phamenoth [VII] 18/19 in the Egyptian calendar [141 Feb. 1/2], at dawn, [Mercury was observed], again, at greatest elongation: from a sighting with respect to the star called Antares it was seen to occupy \(\simeq 13_{10}^1\), while the mean sun was in \(\simeq 10^0\). Here too, then, the greatest elongation from the mean as morning-star was \(26_{10}^1\), equal [to the elongation in [3]].

So, since the mean position of the planet was \(\Pi 10^1\) at one of the observations and \(\simeq 10^0\) at the other, and the point of the ecliptic halfway between them occupies \(\simeq 10^1\), the diameter through the apogee must lie in that position at that time.

From these observations, then, we find that the apogee falls at about \(10^0\) of Aries or Libra, whereas from the ancient observations made near the greatest elongations we find it at about \(6^0\) of the same signs, as can be calculated from the following kind [of data].

[5] In the 23rd year in Dionysius' calendar, Hydron 21, at dawn, Stilbon was 3 moons to the north of the brightest star in the tail of Capricorn. At that time the star in question had a position, according to [the coordinate system defined by] our origin, namely that beginning with the solstitial or equinoctial points, of \(\simeq 22_{10}^1\). Mercury, obviously, had the same longitude, and the mean sun was in \(\simeq 18_{10}^1\); for that moment was in the 486th year from Nabonassar, Choiak [IV] 17/18 in the Egyptian calendar [-261 Feb. 11/12], dawn. Therefore the greatest elongation from the mean [of Mercury] as morning-star was \(25_{10}^1\).

Now we did not find a greatest elongation from the mean as evening-star which was precisely equal to that, at least in the observations which have reached us: but we calculated the [position with] equal [elongation] by means of two observations which were very close [to the required situation], in the following manner.

[6] [Firstly], in the same 23rd year in Dionysius' calendar, Tauron 4,
in the evening, [Mercury] was 3 moons behind [i.e. to the rear of] the straight line through the horns of Taurus, and it seemed as if it was going to be more than 3 moons to the south of that one common [to Auriga and Taurus] when it passed by it. Thus its position according to our coordinates was $8^\circ 23^\prime$. That moment was again in the 486th year from Nabonassar, [Mechir [VI]] 30/Phamenoth [VII] 1 in the Egyptian calendar [-261 Apr. 25/26], evening, at which time the longitude of the mean sun was $29^\circ 29^\prime$. So the greatest elongation from the mean as evening-star was $24^\circ 4^\prime$.

[7] [Secondly], in the 28th year in Dionysius' calendar, Didymon 7, in the evening, [Mercury] was practically on a straight line with [the stars in] the heads of Gemini, and lay to the south of the southern one by $\frac{1}{3}$ of a moon less than twice the distance between [the stars in] the heads. Thus at that time, according to our coordinates, Mercury was in $29^\circ 29^\prime$. This moment is in the 491st year from Nabonassar, Pharmouthi [VIII] 5/6 in the Egyptian calendar.

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The stars in question are, in the catalogue, XXIII 19 and 21 (ζ and β Tau). The latter is also counted as Auriga [XII] no. 11. Subtracting 4° from the catalogue longitudes for precession, we get the coordinates at the observation as: southern horn, $\lambda = 23^\circ 21^\prime, \beta = 21^\prime$; northern horn, $\lambda = 21^\circ 9^\prime, \beta = 2^\prime$. Ptolemy concludes that the longitude of Mercury was the same as that of the southern horn.

There is no doubt that this is what is intended. The Greek ms. have, at H265,16, Φαενῶθ $\lambda$ etc τὴν α', which seems hardly possible. Petavius, followed by Ideler and Böckh, emended to Μεξίρ $\lambda$ etc τὴν α' Φαενῶθ; Halma, followed by Manitius, to $\lambda$ etc τὴν α' Φαενῶθ. The Arabic translations suggest that one must read Φαενῶθ etc τὴν δ', i.e. simply excise $\lambda$'. For the expression cf. p. 456 n.84.

These are, in the catalogue, XXIV 1 and 2 (α and β Gem), with coordinates (corrected for precession): northern head, $\lambda = 19^\circ 19^\prime, \beta = 9^\circ 1^\prime$; southern head, $\lambda = 22^\circ 19^\prime, \beta = 6^\circ 1^\prime$. See Fig. O, which shows that Mercury's 'distance to the south' is measured along the line between the stars.
Now, when the mean position was in $\varpi$ 29°, the greatest elongation was 24°, and when the mean position was in $\pi$ 26°, the [greatest] elongation was 26°; and the [greatest elongation] as morning-star, to which we were seeking the corresponding [greatest elongation as evening-star], was 25°. So we derived the location of the mean position for a [greatest] evening elongation of 25° from the difference between the above two observations: the difference between the mean positions at the two observations is 33°, and the difference between the greatest elongations 2°. Thus to 1° (which is the amount by which 25° exceeds 24°) correspond approximately 24°. If we add this amount to $\varpi$ 29°, we shall get the mean position at which the greatest evening elongation is calculated to be equal to the greatest morning elongation of 25°. This point is $\varepsilon$ 23°. And the point halfway between $\varepsilon$ 18° and $\varepsilon$ 23° is at $\varpi$ 5°.

[8] Again, in the 24th year in Dionysius’ calendar. Leonton 28, in the evening, [Mercury] was a little more than 3° in advance of Spica, according to Hipparchus’ reckoning. Thus at that moment its longitude according to our coordinates was $m_2$ 19°. That moment is in the 486th year from Nabonassar, Payni [X] 30 in the Egyptian calendar [-261 Aug. 23], evening, at which time the longitude of the mean sun was $\Omega$ 27°. Therefore the greatest elongation from the mean as evening-star was 21°. We again calculated [the position of] the morning elongation precisely corresponding to that from two of the available [observations].

[9] In the 75th year in the Chaldaean calendar, Dios 14, at dawn, [Mercury] was half a cubit [ca. 1°] above [the star on] the southern scale [of Libra]. Thus at that time it was in $\simeq$ 14°, according to our coordinates. This moment is in the 512th year from Nabonassar, Thoth [I] 9/10 in the Egyptian calendar [-236 Oct. 29/30], dawn, at which time the longitude of the mean sun was $m_2$ 5°. Therefore the greatest morning elongation was 21°.

[10] In the 67th year in the Chaldaean calendar. Apellaios 5, at dawn, [Mercury] was a half a cubit [ca. 1°] above the northern [star in the] forehead of Scorpius. Thus at that time it was in $m_2$ 2°, according to our coordinates. This moment is in the 504th year from Nabonassar. Thoth [I] 27/28 in the Egyptian calendar [-244 Nov. 18/19], dawn, at which time the longitude of Spica (catalogue XXVII 14) was, according to Ptolemy, $m_2$ 22° in Dionysius’ time; thus he takes Mercury as being 3° in advance of Spica.

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64 This is a crudely rounded result. In fact $33 \times 1 \frac{1}{3} \approx 23 \frac{49}{60}$, so a reasonable approximation would have been 23°. However, linear interpolation is itself a crude procedure here.
65 This is proof that this observation (by Dionysius) was one of those which Hipparchus ‘arranged in a more useful way’ (see IX 2 p. 421, with n.11), and it is a plausible surmise that all of these Mercury observations were derived by Ptolemy from that compilation. The longitude of Spica (catalogue XXVII 14) was, according to Ptolemy, $m_2$ 22° in Dionysius’ time; thus he takes Mercury as being 3° in advance of Spica.
67 The star is catalogue XXVIII 1 (a Lib, there said to be on the ‘southern claw’) to which Ptolemy assigns the longitude $\simeq$ 18° in his own time. Here, then, he has subtracted 3° to account for the precession in 373 years (one would have expected 3°).
longitude of the mean sun was $\mu_7 24^\circ_0$. Therefore this [greatest morning]
elongation was $22^\circ_0$.\textsuperscript{70}

In these two observations again, then, since the difference between the two
mean positions is $19^\circ_0$, and the difference between the greatest elongations is
$1^\circ_0$, it follows that to $3^\circ_0$ (which is the amount by which the $21^\circ_0$ of the required
elongation exceeds the $21^\circ_0$ of the lesser [of these two]) corresponds about $9^\circ$.\textsuperscript{71} If
we add the latter to $\mu_7 5^\circ_0$, we get the mean position at which the greatest
morning elongation becomes equal to the greatest evening elongation of $21^\circ_0$:
this point is $\mu_7 14^\circ_0$. And the point halfway between $\mu_7 27^\circ_0$ and $\mu_7 14^\circ_0$ is,
again, about $6^\circ$.\textsuperscript{72}

From the above, and also because the phenomena associated with the other
planets individually fit [the assumption], we find it consistent [with the facts to
assume] that the diameters through the apogees and perigees of the five planets
shift about the centre of the ecliptic towards the rear through the signs, and that
this shift has the same speed as that of the sphere of the fixed stars. For the latter
moves about $1^\circ$ in 100 years, as we demonstrated [p. 328]; and here too the
interval from the ancient observations, in which \textsuperscript{73} the apogee of Mercury was in
about the 6th degree [of the signs in question],\textsuperscript{74} to the time of our
observations, during which it has moved about $4^\circ$ (since it [now] occupies the
10th degree), is found to comprise approximately 400 years.

\section*{8. \textit{That the planet Mercury, too, comes closest to the earth twice in one revolution}}\textsuperscript{75}

In accordance with the above we investigated the size of the greatest
elongations which occur when the mean longitude of the sun is exactly in the
apogee, and again, when it is diametrically opposite that point. We cannot
derive this from the ancient observations, but we can do so from our own
observations made with the astrolabe. For it is in this situation that one can best
appreciate the usefulness of this way of making observations, since, even if those
stars with previously determined positions which are visible are not near the
planet being observed (which is generally the case with Mercury, since, for the
majority of the fixed stars, it is rare that they are visible when they are [only] as

\textsuperscript{70} Observations [9] and [10] are proven to be Babylonian by several marks: use of the Seleucid era
(called by Ptolemy 'according to the Chaldaeans'); the use of the 'cubit' as an astronomical
measurement; and also the fact that both the stars used as markers belong to the small group used in
Babylonian texts for precisely this purpose and known as 'normal stars' (see \textit{HAMA} 543; Sachs [1]
46).

\textsuperscript{71} This linear interpolation, like the earlier one (see p. 452 n.65) is inaccurate. $81^\circ$ would be much
more reasonable.

\textsuperscript{72} On this occasion the half-way point is at precisely $6^\circ$.

\textsuperscript{73} One would expect, at H269,12, καθ' ἐζ, referring to τηρήσεων, rather than καθ' ὄν, referring
to χρόνων, since the latter means 'interval'. But apparently, since χρόνος can also mean 'epoch',
Ptolemy has somewhat illogically assimilated the relative pronoun to it (cf. τὸν [sc. χρόνον] in the
next line, where it certainly means 'epoch').

\textsuperscript{74} It has not yet been decided whether the apogee lies in Aries or Libra.

\textsuperscript{75} See \textit{HAMA} 161, Pedersen 314–15. 'too' refers to the moon (picking up Ptolemy's remark IX 5
p. 443). On the term περιγείωσκός as applied to Mercury see p. 461 n.94.
far from the sun as Mercury is), one can still determine positions of the planet in question accurately in latitude and longitude, by sighting stars which are at a considerable distance.

[Firstly] then, in the nineteenth year of Hadrian, Atyr [III] 14/15 in the Egyptian calendar [134 Oct. 2/3], at dawn, Mercury, which was around its greatest elongation, was sighted with respect to the star on the heart of Leo, and was seen to have a longitude of $201^\circ$. The mean sun was at about $94^\circ$, so the greatest elongation was $19^\circ$.

[Secondly], in the same year, Pachon [IX] 19 [135 Apr. 5], in the evening, Mercury, which was again around its greatest elongation, was sighted with respect to the bright star in the Hyades, and was seen to have a longitude of $4^\circ$. The mean sun had a longitude of $11^\circ$. Hence in this case one calculates the greatest elongation as $23^\circ$, and it is immediately obvious that the apogee of the eccentric is in Libra and not in Aries.

With these data, let the diameter through the apogee be ABG. Let B be taken as the centre of the ecliptic, at which the observer is, A as the point at $10^\circ$, and G as the point at $10^\circ$. Describe equal epicycles with points D and E.

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76 Since Mercury's maximum elongation from the sun is never much more than $20^\circ$, it is only visible for a short time after sunset or before dawn, when the sky in its region is too illuminated for any but very bright stars to be visible. The 'ancient observations' (i.e. those by Babylonians or earlier Greeks) were made by giving the position with respect to nearby stars; but in some regions of the ecliptic there is a scarcity of bright stars.

77 The star had a longitude of $21^\circ$ according to Ptolemy's catalogue (XXVI 8), so the observed interval was $47^\circ 42^\circ$.

78 The star had a longitude of $125^\circ$, according to the catalogue (XXIII 14), so the observed interval was only $81^\circ$.

79 Heiberg has made an error in the figure on p. 271: Z is on the wrong side of B. Corrected by Manitius.
IX 8. Centre of Mercury's eccentric moves on a circle

E [on their circumferences] about A and G [respectively], and draw from B tangents to them, BD and BE. Drop perpendiculars AD and GE from the centres to the points of tangency.

Now since the greatest elongation from the mean as morning-star in Libra was observed as $19\frac{3}{5}^\circ$,

$$\angle ABD = \begin{cases} 19;3^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\ 38;6^\circ \text{ where } 2 \text{ right angles } = 360^\circ \end{cases}$$

Therefore in the circle about right-angled triangle ABD

arc $AD = 38;6^\circ$

and its chord, $AD \approx 39;9^\circ$ where hypotenuse $AB = 120^\circ$.

Again, since the greatest elongation from the mean as evening-star in Aries was observed as $23\frac{1}{4}^\circ$,

$$\angle GBE = \begin{cases} 23;15^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\ 46;30^\circ \text{ where } 2 \text{ right angles } = 360^\circ \end{cases}$$

Therefore in the circle about right-angled triangle GBE

arc $GE = 46;30^\circ$

and its chord, $GE = 47;22^\circ$ where hypotenuse $BG = 120^\circ$.

Therefore where $GE = 39;9^\circ$ and $AB = 120^\circ$

(for $AD = GE$, radii of the epicycle),

$BG = 99;9^\circ$

and, by addition [of $AB$ to $BG$], $ABG = 219;9^\circ$.

So if it is bisected at point $Z$,

its half, $AZ = 109;34^\circ$

and the distance between points B and $Z = 10;25^\circ$] in the same units.

Now it is clear that either point $Z$ is the centre of the eccentric on which the centre of the epicycle is always located, or else the centre of that [eccentric] moves about point $Z$. For those are the only conditions under which the centre of the epicycle could be equidistant from $Z$ at both the above diametrically opposite situations, as demonstrated. But if $Z$ were the actual centre of the eccentric on which the epicycle centre is always located, that eccentric would be stationary, and the situation in Aries would be the closest to the earth of all situations [i.e. the perigee], since $BG$ is the shortest of [all] lines drawn from $B$ to the circle described on centre $Z$. However, we find that the situation in Aries is not the closest to the earth of all, but the situations in Gemini and Aquarius are even closer to the earth than that, and approximately equal to each other. Hence it is clear that the centre of the eccentric in question rotates about point $Z$, in the opposite sense to the revolution of the epicycle (i.e. in advance with respect to the signs), it too making one rotation in one revolution [of the epicycle]. For if this is so the epicycle centre will be closest to the earth twice [in one revolution] on the eccentric.

As for the fact that the epicycle is closer to the earth in Gemini and Aquarius than in the [above] situation in Aries, this is easily seen to be an immediate consequence of the observations already detailed. For in the observation of the 16th year of Hadrian, Phamenoth 16 [p. 449 no. 1], the greatest elongation from the mean as evening-star was $21\frac{1}{4}^\circ$, and in the observation of the 4th year of

80 Euclid III 7.
Antoninus, Phamenoth \(19^{st}\) [p. 450 no. 4], the greatest elongation from the mean as morning-star was \(26^{15}_{10}\), while in both observations the mean sun was near \(\equiv 10^{10}\). Again, in the observation of the 18th year of Hadrian, Epiphi 19 [p. 449 no.2], the greatest elongation from the mean as morning-star was \(21^{40}_{10}\), and in the observation of the 1st year of Antoninus, Epiphi 20 [p. 449 no. 3], the greatest elongation from the mean as evening-star was \(26^{15}_{10}\), the mean sun in both these observations being near \(\Pi 10^{10}\). Thus both in Aquarius and in Gemini the sum of the opposite greatest elongations comes to \(47^{15}_{10}\), while the sum of the two greatest elongations in Aries is [only] \(46^{10}_{10}\), since the evening elongation (which is equal to the morning elongation) was observed as \(23^{40}_{10}\).

9. \{On the ratio and the amount of the anomalies of Mercury\}\(^{82}\)

Having completed the above preliminary investigation, we have still to demonstrate the position of the point on line \(AB\) about which takes place the annual revolution of the epicycle in uniform motion towards the rear with respect to the signs, and the distance from \(Z\) of the centre of that eccentre which performs its revolution in advance in the same period [as the epicycle]. For this investigation we used two observations of greatest elongations, one as morning-star and one as evening-star, but in both of which the mean position was a quadrant from the apogee on the same side: that is the situation in which, approximately, the greatest equation of ecliptic anomaly occurs.

[1] In the fourteenth year of Hadrian, Mesore [XII] 18 in the Egyptian calendar [130 July 4], in the evening, as we found in the observations we got from Theon,\(^{83}\) he says that [Mercury] was at its greatest distance from the sun, \(3^{15}_{10}\) behind [i.e. to the rear of] the star on the heart of Leo. Thus, according to our coordinates, its longitude was about \(\Omega 6^{10}\), while the longitude of the mean sun at that moment was about \(\equiv 10^{15}_{10}\). Thus the greatest evening elongation was \(26^{15}_{10}\).

[2] In the second year of Antoninus, Mesore [XII] [20]/21\(^{84}\) in the Egyptian calendar [139 July 4/5], at dawn, we observed its greatest distance by means of the astrolabe: sighting it with respect to the bright star in the Hyades, we found its longitude as \(\Pi 20^{10}_{10}\). The mean sun was, again, near \(\equiv 10^{15}_{10}\). Thus the greatest morning elongation was \(20^{10}_{10}\).

With the above as data, let [Fig. 9.6] the diameter through \(\equiv 10^{10}\) and \(\Pi 10^{10}\) again be \(AZBG\), and, as in the previous figure [9.5], let \(A\) be taken as the point

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\(^{81}\) Reading \(10^9\) (with D, Ar) for \(10^9\) (18) at H273,19. Ptolemy gives a double date (18/19) in the passage in question. Since the observation was taken at dawn, the second date is preferable, and agrees with the practice just below (Epiphi 19, for the earlier 18/19 at dawn).

\(^{82}\) HAMA 161-2, Pedersen, 318-19.

\(^{83}\) Other observations by this man are used by Ptolemy in X 1 and X 2. There (p. 469) he says that they were 'given to us by the mathematician Theon', implying personal contact. He has often been identified with Theon of Smyrna. This is chronologically possible, but given the frequency of the name, especially in Roman Egypt, the identification is highly uncertain.

\(^{84}\) Reading \(\text{Meso}r\eta\ \varepsilon\iota\tau\nu\ \kappa\alpha'\) (with D, Ar) for \(\text{Meso}r\eta\ \varepsilon\iota\tau\nu\ \kappa\delta\) (24th) at H275,13. The date is determined by the longitude of the mean sun (computed for Nabonassar 886 XII 20/21, 6 a.m., as 100:19\(^{\circ}\)). Neugebauer (HAMA 162 n.3) suggests reading \(\text{Meso}r\eta\ (\kappa'\) \varepsilon\iota\tau\nu\ \kappa\alpha'\), but for the above form cf. p. 451 n.63.
at which the epicycle centre is found when its longitude is \(\pm 10^\circ\), \(G\) as the point at which it is found when its longitude is \(\mp 10^\circ\), \(B\) as the centre of the ecliptic, and \(Z\) as the point about which the centre of the eccentre rotates in advance.

Let the first problem be to find the distance from point \(B\) of the centre about which we say the uniform motion of the epicycle towards the rear takes place.

Let that centre be \(H\), and draw a straight line through \(H\) at right angles to \(AG\), so that its [angular] distance from the apogee is a quadrant. On this line take \(\Theta\), the centre of the epicycle at the above observations (for at those observations the mean longitude of the sun was a quadrant from the apogee, since it was near \(\pm 10^\circ\)). Draw the epicycle \(KL\) on centre \(\Theta\), and draw the tangents to it from \(B\), \(BK\) and \(BL\). Join \(\Theta K\), \(\Theta L\) and \(B\Theta\).

Then, since at the mean position in question the greatest morning elongation from the mean is given as \(20^\circ\), and the greatest evening elongation as \(26^\circ\),

\[
\angle KBL = [20^\circ + 26^\circ =] 46;30^\circ \text{ where } 4 \text{ right angles} = 360^\circ.
\]

Therefore its half, \(\angle K\Theta = 46;30^\circ\) where 2 right angles = \(360^\circ\).

Therefore in the circle about right-angled triangle \(B\Theta K\)

\[
\text{arc } \Theta K = 46;30^\circ
\]

and its chord, \(\Theta K = 47;22^\circ\) where hypotenuse \(B\Theta = 120^\circ\).

Therefore where \(\Theta K\), the radius of the epicycle, is \(39;9^\circ\)

and, as was shown, \(BZ = 10;25^\circ\),

\(B\Theta = 99;9^\circ\).

Again, the difference between the above greatest elongations, \(6^\circ\), comprises twice the equation of the ecliptic anomaly; and the latter is represented by \(\angle B\Theta H\), as we proved previously.\(^{66}\)

---

\(^{66}\)Note that this is exactly equal to \(\angle GBE\) in IX 8 (p. 455), which implies that the distance of the epicycle from the observer is the same at quadrature (here) and at \(180^\circ\) from apogee (there).

\(^{66}\)IX 6 p. 448. But it is assumed rather than 'proven' there.
Therefore \( \angle B\Theta H = \begin{cases} 3^\circ \text{ where 4 right angles } = 360^\circ \\ 6^\circ \text{ where 2 right angles } = 360^\circ . \end{cases} \)

Therefore in the circle about right-angled triangle \( BH\Theta \)

\[
\text{arc } BH = 6^\circ \\
\text{and } BH = 6;17'' \text{ where hypotenuse } B\Theta = 120^\circ .
\]

Therefore where \( B\Theta = 99;9'' \), and likewise \( BZ = 10;25'' \),

\[
BH = 5;12''.
\]

Therefore \( BH \) is approximately half \( BZ \),

\[
\text{and } BH \approx HZ \approx 5;12'', \text{ where the radius of the epicycle is } 39;9''.
\]

Again, in the same figure [Fig. 9.7], draw line \( ZMN \) through \( Z \) at right angles to \( AG \), but on the opposite side to \( H\Theta \). Because lines \( H\Theta \) and \( ZN \) perform their returns to the same point in the same period, but in opposite senses, the centre of

that eccentre on which the epicycle centre \( \Theta \) is located will, obviously, lie on \( ZMN \) at that moment. Let \( ZN \) be equal to \( ZA \): thus \( ZN \), like \( AZ \), is the sum of the radius of the eccentre and the distance between the centres ([i.e.] between the centre of the eccentre and point \( Z \)). Take on \( ZN \) the centre of the eccentre, \( M \), and join \( Z\Theta \).

Now \( \angle MZH \) is right, and \( \angle \Theta ZH \) is practically a right angle (hence \( NZ\Theta \), too, is practically a straight line);\(^{87}\)

and it has been demonstrated that where the epicycle radius is \( 39;9'' \)

\[
NZ = AZ = 109;34''
\]

and \( Z\Theta = B\Theta = 99;9''.\(^88\)

\(^{87}\) This simplification is necessary in order to solve the problem at all: for one does not know \textit{a priori} where on \( ZM \) the point \( M \) lies, only that it lies on a circle with center \( Z \).

\(^{88}\) See p. 455.
Therefore, by addition, \( \text{NZ}\theta = 208;43' \)
and its half, \( \text{NM} \), the radius of the eccentre, is about \( 104;22' \),
and by subtraction [of \( \text{NM} \) from \( \text{NZ} \)],
\( \text{ZM} \), the distance between the centres, is \( 5;12'' \).
But we showed that both \( \text{BH} \) and \( \text{HZ} \) were the same amount, \( 5;12'' \).
Thus we have computed that
where the radius of the eccentre is \( 104;22' \)
each of the distances between the centres \([\text{BH}, \text{HZ}, \text{ZM}]\) is \( 5;12'' \)
and the radius of the epicycle is \( 39;9'' \).
Therefore where the radius of the eccentre is \( 60'' \),
each of the distances between the centres is \( 3;0'' \)
and the radius of the epicycle is \( 22;30'' \).
Q.E.D.

With the above [elements] given, the [computed] greatest elongations at the
points closest to the earth are in agreement with those observed (i.e. when the
mean position is at \( \equiv 10^\circ \) or \( \Pi 10^\circ \), and [thus] its distance from the apogee is the
side of the [inscribed] triangle [i.e. \( 120^\circ \)], the angle subtended by the epicycle at
the eye is about \( 47;1'' \), as we can deduce by the following.

Let [Fig. 9.8] the diameter through the apogee be \( \text{ABGDE} \), on which point \( A \)
is taken as the apogee, \( B \) as the point about which the centre of the eccentre
performs its motion in advance, \( G \) as the point about which the epicycle centre-
performs its [uniform] motion towards the rear, and \( D \) as the centre of the
ecliptic. Let each of the [above] motions have gone through the side of the
[inscribed] triangle [i.e. \( 120^\circ \)] (performed uniformly and with equal speed
about its own centre) from the apogee $A$ on opposite sides of it. Let the straight line rotating the epicycle be $GZ$, and that rotating the centre of the eccentre be $BH$, and let the centre of the eccentre be $H$ and the centre of the epicycle, $Z$.

With the latter as centre describe the epicycle, draw tangents to the epicycle, $D\Theta$ and $DK$, join $GH$, $DZ$, $Z\Theta$ and $ZK$, and drop perpendicular $DL$ from $D$ on to $GZ$.

We have to show that

$$\angle \Theta DK = 47^{1}\frac{1}{4} \text{ where } 4 \text{ right angles } = 360^\circ.$$

Now both $\angle ABH$ and $\angle AGL$ subtend the side of the [inscribed] triangle and are equal to $120^\circ$ where $2$ right angles $= 180^\circ$;

so $\angle GBH = \angle DGL = 60^\circ$;

and $\angle BHG = \angle BGH$ ($BG = BH$, by hypothesis).

But $\angle BHG + \angle BGH = 120^\circ$ (supplement [to $\angle GBH = 60^\circ$]).

$\therefore \angle BHG = \angle BGH = 60^\circ$.

So triangle $BGH$ is equiangular and equilateral.

And $\angle DGL = \angle BGH$.

So points $H$, $G$ and $Z$ lie on a straight line.

Hence $HZ$, the radius of the eccentre $= 60^\circ$ where $GH$ (which equals $GD$) $= 3^\circ$, the distance between the centres.

Therefore, by subtraction [of $GH$ from $HZ$], $GZ = 57^\circ$ in the same units.

Again, since

$$\angle DGL = \begin{cases} 
60^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\
120^\circ \text{ where } 2 \text{ right angles } = 360^\circ 
\end{cases}$$

in the circle about right-angle triangle $GDL$

arc $DL = 120^\circ$

and arc $GL = 60^\circ$ (supplement).

Therefore the corresponding chords

$DL = 103;55^\circ$

and $GL = 60^\circ$ (where hypotenuse $GD = 120^\circ$).

Therefore where $DG = 3^\circ$ and $GZ = 57^\circ$

$DL = 2;36^\circ$

and $GL = 1;30^\circ$;

and, by subtraction [of $GL$ from $GZ$], $LZ = 55;33^\circ$.

And since $LZ^2 + DL^2 = DZ^2$,

$DZ = 55;34^{989}$

where the radius of the epicycle (i.e. $Z\Theta$ and $ZK$) $= 22;30^\circ$, by hypothesis.

Therefore where hypotenuse $DZ = 120^\circ$

$\Theta Z = ZK = 48;35^\circ$;

and $\angle Z\Theta = \angle ZDK = 47;46^\circ$ where $2$ right angles $= 360^\circ$.

Therefore, by addition [of $\angle Z\Theta$ to $\angle ZDK$], $\angle \Theta DK = 47;46^\circ$ where $4$ right angles $= 360^\circ$.

Q.E.D.

**This is, according to Ptolemy, the least distance of the center of Mercury's epicycle (cf. XI 10 p. 546). It was shown by Hartner, 'Mercury Horoscope' 109-17 (cf. Pedersen 321-4) that, with the parameters of Ptolemy's model, the least distance actually occurs at about $120^{10}$ from apogee, and is less than $55;34$ (about $55;33;38$). These differences are utterly negligible for practical purposes.**
The sequel to the above is the establishment of the periodic motions of Mercury and their epochs. Now the [motion and epoch] in longitude, that is, of the epicycle in its uniform motion about point G, are given immediately from those of the sun. As for the [motion and epoch] in anomaly, that is, of the planet in its [uniform] motion on the epicycle about the epicycle centre, we have derived it from two reliable observations, one from among those recorded in our time, and the other from the ancient observations.

[Firstly], we observed the planet Mercury in the second year of Antoninus (which was the 886th year from Nabonassar), Epiphi [XI] 2/3 in the Egyptian calendar [139 May 17/18], by means of the astrolabe instrument. It had not yet reached its greatest elongation as evening-star. When sighted with respect to the star on the heart of Leo it was observed at a longitude of $\Pi 17;^0$; and at that moment it was also $1;^0$ to the rear of the moon’s centre. The time at Alexandria was $4;^1$ equinoctial hours before midnight of [Epiphi 2/3], since, according to the astrolabe, the 12th degree of Virgo [i.e. $\mu 11^0-12^0$] was culminating, while the sun was in about $8; 23^0$. Now at that moment, the positions according to the hypotheses we have demonstrated were as follows:

- mean longitude of the sun $8; 22;34^0$
- mean longitude of the moon $\Pi 12;14^0$
- anomaly of the moon from the apogee of the epicycle $281;20^0$
- hence, by computation, true position of the moon’s centre $\Pi 17;10^0$
- apparent position of the moon’s centre $\Pi 16;20^0$.

Thus from this [computation] too we find that Mercury’s longitude was $\Pi 17;10^0$ (since it was $1;^0$ to the rear of the moon’s centre).

With this as datum, let [Fig. 9.9] the diameter through the apogee and perigee be ABGDE, on which point A is taken as the apogee, B as the point about which the centre of the eccentre performs its motion in advance, G the point about which the centre of the epicycle performs its [uniform] motion towards the rear, and D the centre of the ecliptic. Let the epicycle centre, Z, have been carried by the line GZ about point G through the angle AGZ, and let the centre of the eccentre, H, have been carried by line BH about point B through the angle ABH, which will, obviously, be equal to $\angle AGZ$ because of the equal speed of the motions. Draw the epicycle, $\Theta K L$, on centre Z, and let the planet be situated at L. Join GH, HZ, DZ, ZL and DL, extend GZ0 and drop perpendiculars HM and DN on to it from H and D, and drop perpendicular $ZX$ from Z on to DL.

---

90 See H.A.M.4 165–8.
91 Reading αὐτοῦ (with D,L) for αὐτοῦ (‘its epochs’) at H283,4.
92 Literally ‘of the midnight towards the 3rd’.
93 These positions are computed for 7:7 p.m. Alexandria, i.e. Ptolemy has applied the equation of time (I find −25 mins. with respect to era Nabonassar). For this moment the computations are accurate (I find a longitudinal parallax of −53’ where Ptolemy applies −50’).
94 ‘perigee’ (τὸ περίγειον) here and at H285.12 and 14 is taken, somewhat loosely, as the point $180^0$ from the apogee, and not the point where Mercury’s center is closest to the earth. For the latter Ptolemy always uses the superlative form τὸ περίγειοτατον (H273.11, al.)
Let us consider the problem, to find the arc of the epicycle between $\Theta$, the apogee [of the epicycle], and the planet at $L$.

Now at that moment the longitude of the mean sun was $82234^\circ$, and the perigee of the planet was at about $100^\circ$. Thus its distance from the perigee in mean longitude was $4234^\circ$.

\[
\therefore \angle GBH = \begin{cases} 4234^\circ & \text{where 4 right angles} = 360^\circ \\ 858^\circ & \text{where 2 right angles} = 360^\circ \end{cases}
\]

And since $BG$ always equals $BH$

\[
\angle BHG = \angle BGH = 13726^\circ \text{ in the same units.}
\]

So, in the circle about triangle $BGH$\begin{enumerate}
\item arc $HG = 858^\circ$
\item and arc $BG = 13726^\circ$.
\end{enumerate}

Therefore the corresponding chords

\[
\begin{align*}
\text{GH} &= 8110^\circ \\
\text{and } \text{BG} &= 11149^\circ
\end{align*}
\]

where the diameter of the circle is $120^\circ$.

Therefore where $BG = 3^\circ$, $GH = 211^\circ$.

Again, since $\angle BGH = 13726^\circ$

\[
\begin{align*}
\text{and } \angle BGM &= 858^\circ \\
\text{where 2 right angles} &= 360^\circ
\end{align*}
\]

by subtraction, $\angle HGM = 5218^\circ$ in the same units.

Therefore in the circle about right-angled triangle $GHM$

\begin{enumerate}
\item [95] Cf. IX 7 p. 450 and IX 8 p. 454.
\item [96] This is one of the rare cases where Ptolemy applies the equivalent of the sine theorem in a triangle which is not right-angled. See Introduction p. 7 n.10.
\end{enumerate}
arc $HM = 52;18^\circ$
and arc $GM = 127;42^\circ$ (supplement).

Therefore the corresponding chords

\[
\begin{align*}
HM &= 52;53''; \quad \text{where hypotenuse } GH = 120^\circ. \\
GM &= 107;43'';
\end{align*}
\]

Therefore where $GH = 2;11^\circ$.

and $HZ$, the radius of the eccentrc carrying the epicycle, is $60^\circ$,

\[
\begin{align*}
HM &= 0;58''; \\
GM &= 1;58''.
\end{align*}
\]

Hence $MZ$, being a negligible amount less than $HZ$, the hypotenuse [of triangle $HMZ$], is the same, $60^\circ$,

and, by subtraction [of $GM$ from $MZ$], $GZ = 58;2^\circ$.

Similarly, since $\angle DGN = 85;8^\circ$ where $2$ right angles $= 360^\circ$,

in the circle about right-angled triangle $GDN$

\[
\begin{align*}
arc DN &= 85;8^\circ; \\
arc GN &= 94;52^\circ \text{ (supplement).}
\end{align*}
\]

Therefore the corresponding chords

\[
\begin{align*}
DN &= 81;10^\circ; \quad \text{where hypotenuse } GD = 120^\circ. \\
GN &= 88;23'';
\end{align*}
\]

Therefore where $GD = 3^\circ$ and, as was demonstrated, $GZ = 58;2^\circ$.

\[
\begin{align*}
DN &= 2;26''; \\
and GN &= 2;13''.
\end{align*}
\]

and, by subtraction [of $GN$ from $GZ$], $NZ = 55;49''$.

\[
\begin{align*}
\text{Hence hypotenuse } DZ &= \sqrt{DN^2 + NZ^2} \approx 55;51^\circ; \\
\text{where the radius of the epicycle} &= 22;30^\circ.
\end{align*}
\]

Therefore in the circle about right-angled triangle $DZN$,

where hypotenuse $DZ = 120^\circ$,

\[
\begin{align*}
DN &= 4;22''; \\
and arc DN &= 4;11''.
\end{align*}
\]

\[
\therefore \angle DZN = 4;11^\circ \text{ where } 2 \text{ right angles} = 360^\circ,
\]

and, by addition [of $\angle DZN$ and $\angle DGN$, $\angle EDZ = 89;19^\circ$.

And the whole angle $EDL = 135^\circ$ in the same units, since the planet was observed at $67;30^\circ$ from the perigee.

Therefore by subtraction [of $\angle EDZ$ from $\angle EDL$, $\angle ZDL = 45;41^\circ$.

Therefore in the circle about right-angled triangle $DZX$,

\[
\begin{align*}
arc ZX &= 45;41^\circ; \\
\text{and } ZX &= 46;35^\circ \text{ where hypotenuse } DZ = 120^\circ.
\end{align*}
\]

Therefore where hypotenuse $DZ = 55;51^\circ$ and the radius of the epicycle,

\[
\begin{align*}
ZL &= 22;30^\circ; \\
ZX &= 21;41^\circ.
\end{align*}
\]

And, in the circle about right-angled triangle $ZLX$,

\[
\begin{align*}
\text{where hypotenuse } ZL &= 120^\circ, \\
ZX &= 115;39^\circ.
\end{align*}
\]

\[
\therefore \text{arc } ZX = 149;2^\circ
\]

\[97\text{The arc corresponding to } 115;39^\circ \text{ is in fact } 149;3^\circ. \text{ But if one takes the chord as } 115;38;40\text{ (which is an accurate transformation of } 46;35 \times 55;51/120\text{), one finds as arc } 149;1;56^\circ. \text{ As often, Ptolemy computes with more accuracy than he displays.}\]
and $\angle ZLX = 149;2^\circ$ where 2 right angles $= 360^\circ$.

But we showed that $\angle ZDL = 45;41^\circ$ in the same units.

$\therefore \angle LZK = \angle ZLX + \angle ZDL = 194;43^\circ$.

And $\angle DZN = 4;11^\circ$ likewise.

Therefore, by addition of $\angle DZN + \angle LZK$,

$\angle DZL = \begin{cases} 198;54^\circ \text{ where 2 right angles } = 360^\circ \\ 99;27^\circ \text{ where 4 right angles } = 360^\circ. \end{cases}$

Therefore arc $\Theta KL$ of the epicycle, which was the distance of the planet Mercury from the apogee $\Theta$ at the observation, is $99;27^\circ$.

Q.E.D.

Secondly, in the 21st year of Dionysius' calendar (which was in the 484th year from Nabonassar), Scorpion 22, [which is] Thoth [I] 18/19 in the Egyptian calendar [-264 Nov. 14/15], at dawn, Stilbon [i.e. Mercury] was 1 moon to the rear of the straight line through the northern [star in the] forehead of Scorpius and the middle [star in the forehead], and was 2 moons to the north of the northern [star in the] forehead. Now according to our coordinates at that time the midmost of the stars in the forehead of Scorpius had a longitude of $m L 11^\circ$, and is the same amount [11°] south of the ecliptic, while the northernmost star had a longitude of $m L 21^\circ$ and is 11° north of the ecliptic. So the planet Mercury had a longitude of about $m L 31\frac{1}{2}^\circ$. Furthermore it is clear that it had not yet reached its greatest elongation as morning-star, since 4 days later, on Scorpion 26, it is recorded that its distance from the same straight line towards the rear was 11 moons: for [by that time] the elongation had become greater, the sun having moved about 4 degrees, but the planet [only] half a moon. And on Thoth 19 at dawn the longitude of the mean sun, according to our tables, was $m L 20;5^\circ$, while the longitude of the apogee of the planet was about $= 6^\circ$, since the 400 or so years between the observations produce a displacement of the apogee of about 4°.

With the above as data, then, let us draw a figure [Fig. 9.10] similar to the one preceding [Fig. 9.9], but in which, because of the difference in the positions, the angles towards the apogee $A$ [i.e. $\angle AGZ$, $\angle ABH$] are to be drawn as acute, the straight lines joining [points] to the planet [i.e. $ZL$, $DL$], as in advance of the epicycle [centre], and perpendicular $ZX$ as beyond $ZL$, the radius of the epicycle. Then, since the mean position of the planet was $m L 20;5^\circ = 6^\circ = 44;50^\circ$ from the apogee.

$\angle ABH = \begin{cases} 44;50^\circ \text{ where 4 right angles } = 360^\circ \\ 89;40^\circ \text{ where 2 right angles } = 360^\circ. \end{cases}$

Therefore its supplement, $\angle GBH = 270;20^\circ$.

and $\angle BGH = \angle BHG = 44;50^\circ$ in the same units.

98 See catalogue nos. XXIX 2 and 1. Ptolemy has subtracted 4° from the longitudes there to account for precession.

99 It is difficult to see how Ptolemy arrives at this position from his data: see the discussion HAMA 166, with Fig. 151. This was an observation of a station. Cf. Ptolemy's remark about ancient observations IX 2 pp. 420-1.

100 There is the additional difference (as noted by Manitius) that the significations of points $\Theta$ and $K$ has been interchanged: in Fig. 9.9 $\Theta$ was the mean apogee and $K$ the true. while in Fig. 9.10 $K$ is the mean perigee and $\Theta$ the true.
And, by the same reasoning [as before] in the circle about triangle BGH the corresponding chords
\[ GH = 84;36'' \]
and \[ BG = BH = 45;46'' \]
Therefore where \[ BG = BH = 5;33'' \]
Again, by hypothesis,
\[ \angle AGZ = 89;40'' \] where 2 right angles = 360°,
and \[ \angle BGH = 44;50'' \] in the same units,
so, by addition, \[ \angle ZGH = 134;30'' \]
and, in the circle about right-angled triangle GHM
\[ \text{arc } HM = 134;30'' \]
and \[ \text{arc } GM = 45;30'' \] (supplement).
Therefore the corresponding chords
\[ MH = 110;40'' \]
and \[ GM = 46;24'' \] where hypotenuse \[ GH = 120'' \].
Therefore where \[ GH = 5;33'' \] (i.e. where ZH, the radius of the eccentre, is 60°),
\[ \text{arc } HM = 5;7'' \]
and \[ GM = 2;10'' \].

\footnote{2;9'' would be more accurate by any method of computation}
IX.10. Derivation of Mercury's mean motion from observations

Hence we compute $ZM = \sqrt{ZH^2 - HM^2}$ as 59;47°,
and, by addition [of $MG$ to $ZM$], $ZMG$ as 61;57° in the same units.
Similarly, since $\angle DGN = \angle AGZ = 89;40°°$ where 2 right angles = 360°°,
in the circle about right-angled triangle $GDN$,
arc $DN = 89;40°$
and arc $GN = 90;20°$ (supplement).

So the corresponding chords
\[
\begin{align*}
DN &= 84;36° \\
and GN &= 85;6°
\end{align*}
\]
Therefore where $GD = 3°$,
\[
\begin{align*}
DN &= 2;7° \\
and GN &= 2;8°,
\end{align*}
\]
and, by addition [of $ZG$ to $GN$], $ZGN = 64;5°$
Hence hypotenuse $ZD = \sqrt{ZN^2 + DN^2}$ = 64;7° in the same units.
Therefore, in the circle about right-angled triangle $ZDN$,
where $ZD = 120°$
\[
\begin{align*}
DN &= 3;58° \\
and arc DN &= 3;48°.^{102}
\end{align*}
\]
\[
\therefore \angle DZN = 3;48°° \text{ where 2 right angles = 360°°},
\]
and, by subtraction [of $\angle DZN$ from $\angle AGZ$],
\[
\angle ADZ = 86;52°° \text{ in the same units}.
\]

But $\angle ADL$ is given as 54;40°° in the same units
(for the planet was $m_L 3\frac{1}{2} = 6° \Rightarrow 27;20°$ from the apogee at the observation).

Hence, by subtraction, $\angle ZDL = 31;12°° \text{ where 2 right angles = 360°°}$.

Therefore in the circle about right-angled triangle $ZDX$,
arc $ZX = 31;12°$
\[
\begin{align*}
and ZX &= 32;16° \text{ where hypotenuse $DZ = 120°$},
\end{align*}
\]
Therefore where $DZ = 64;7°$ (i.e. where $ZL$, the radius of the epicycle, is 22;30°),
\[
XZ = 17;15°.\]

And, in the circle about right-angled triangle $ZLX$,
where hypotenuse $ZL = 120°$,
\[
\begin{align*}
ZX &= 92°. \\
\therefore \text{arc } ZX &= 100;8°.^{103}
\end{align*}
\]
and $\angle ZLX = 100;8°° \text{ where 2 right angles = 360°°}$.
And we showed that, in the same units, $\angle ZDL = 31;12°°$.
\[
\begin{align*}
\text{[hence } \angle ZLX - \angle ZDL &= 68;56°°], \\
\text{and that } \angle ZLK &= 3;48°°.
\end{align*}
\]
Therefore, by subtraction [of $\angle ZLK$ from $\angle ZLX$],
\[
\angle KZL = \begin{cases} 65;8°° \text{ where 2 right angles = 360°°} \\
32;34° \text{ where 4 right angles = 360°}. \end{cases}
\]
At this observation, then, the planet was 32;34° from the epicycle perigee $K$,
and, obviously, 212;34° from the apogee. But we showed that at the moment of

\[103\] 3;47° would be more accurate by any method of computation.
\[103\] The nearest one can get to this by any method of computation is 100;7°. More accurate calculation would give 100;4°.
our observation it was 99;27° from the apogee of the epicycle. Now the interval between the two observations is approximately 402 Egyptian years 283 days 13½ hours. This interval contains 1268 complete returns of the planet in anomaly (for 20 Egyptian years produce very nearly 63 returns, so 400 years produce 1260, and the remaining 2 years plus the additional days another 8 complete returns). Thus we have shown that in 402 Egyptian years 283 days 13½ hours the planet Mercury moved in anomaly, beyond 1268 complete revolutions, 246;53°, which is the amount by which the position at our observation is beyond the previous one. And just about the same increment [in anomaly] results from the tables we set out before: for it was on the basis of these very same calculations that we made our correction to the periodic motions of Mercury, by reducing the above interval to days, and the above revolutions in anomaly plus the increment to degrees. For when the total of degrees is divided by the total of days, there results the mean daily motion in anomaly which we set out for Mercury in our previous discussion [IX 3].

11. (On the epoch of its [Mercury's] periodic motions)

Then in order to establish the epochs of the five planets, as we did for the sun and moon, for the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, we took the interval between that moment and the more ancient of the observations, which is closer to it: this is very nearly 483 Egyptian years 17 days 18 hours. The increment in mean anomaly corresponding to that interval is 190;39°. If we subtract the latter from the 212;34° (counted from the apogee) derived from the observation, we get the following epoch positions for Nabonassar 1, Thoth 1 in the Egyptian calendar, noon:

- anomaly counted from the apogee of the epicycle 21;55°
- [mean] longitude the same as the sun’s, i.e. Χ 0:45°
- apogee of the eccentric in about $\simeq 1°$

(for $\simeq \frac{1}{10}$ of a degree for each of the above years comes to about 45°, which, subtracted from the [longitude] $\simeq 6°$ at the observation, gives $\{\simeq\} 1\°$).

For the actual derivation of the mean motion in anomaly see Appendix C. In the derivation of the two positions in anomaly on which the mean motion is allegedly based Ptolemy has committed a number of small computational and rounding errors. These result in a compounded error which is not negligible, as accurate computation from his initial values reveals:

<table>
<thead>
<tr>
<th>Observations</th>
<th>Ptolemy</th>
<th>Computed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs. I</td>
<td>212;34°</td>
<td>212;29,18°</td>
</tr>
<tr>
<td>Obs. II</td>
<td>99;27°</td>
<td>99;33,31°</td>
</tr>
<tr>
<td>Increment</td>
<td>246;53°</td>
<td>247; .413°</td>
</tr>
</tbody>
</table>

The difference of +11°, distributed over about 400 years, leads to +0;0,0,0,16% in the mean motion.

Reading $\varpi$ (with Ar) for $\gamma$ (181) at H294.5. 18° is shown to be correct both by the increment in mean motion below (18° would give 190;42° instead) and by the interval between the two observations given above. Corrected by Manitius.
Such, then, was the method by which we found the hypotheses for the planet Mercury, the sizes of its anomalies, and also the precise amounts of its periodic motions, and their epochs. For the planet Venus, again, we first investigated the position in the ecliptic of the apogee and perigee of the eccentric by finding greatest elongations which are equal and in the same direction. The available ancient observations did not supply us with exact pairs of positions suitable for this purpose, but we used contemporary observations for our approach, as follows.

[1] Among the observations given to us by the mathematician Theon, we found one recorded in the sixteenth year of Hadrian, on Pharmouthi [VIII] 21/22 in the Egyptian calendar [132 Mar. 8/9], at which, he says, the planet Venus was at its greatest elongation as evening-star from the sun, and was the length of the Pleiades in advance of the middle of the Pleiades; and it seemed to be passing it a little to the south. Now, according to our coordinates, the longitude of the middle of the Pleiades at that time was $8^\circ 3^\prime$, and its length is about $1^\circ$.

So clearly Venus' longitude at that moment was $8^\circ 1^\prime 4^\prime$. So, since the longitude of the mean sun at that moment was $8^\circ 14^\prime$, the greatest distance from the mean as evening-star was $47^\prime 10^\prime$.

[2] In the fourth year of Antoninus, Thoth [I] 11/12 in the Egyptian calendar [140 July 29/30], we observed Venus at its greatest elongation from the sun as morning-star. It was the breadth of half a full moon to the north-east of the star in the middle knee of Gemini. At that moment the longitude of the fixed star, according to us, was $11^\circ 18^\prime$, so Venus was in about $18^\circ$. And the

---


2 See p. 446 n.43. Many of the dates of greatest elongations of Venus given here by Ptolemy are in error, some by as much as three weeks (see H.A.M. 153 n.1). We cannot doubt that he was aware of this, but he was forced by the lack of suitable observations during the limited period available to take those positions of Venus close to greatest elongation which gave the required positions of the mean sun with respect to Venus' apsidal line. The point is discussed in detail by Swerdlow and Neugebauer, Ch.5.

3 In the catalogue (XXIII 30–32) the group of the Pleiades has longitudes between $8^\circ 2^\circ$ and $8^\circ 3^\circ$. The length of this is indeed $1^\circ$, but its midpoint is $8^\circ 25^\prime$, which Ptolemy has rounded to $3^\circ$ (a correction for precession would make it even less than $2^\circ 55^\prime$).

4 Reading $6^\prime$ (with D,Ar) for $6^\prime$ (14th) at H297.5. The date is confirmed by the computations below. Corrected by Manitius.

5 Catalogue XXIV 11, where the description is somewhat different. Of the three knees mentioned (nos. 10, 11 and 13) this is the middle one.
mean sun was in \( \Omega \, 5^\circ \). So the greatest distance as morning-star was the same
amount as before, \( 47^\circ \).

Therefore, since the mean position was \( \kappa \, 14^\circ \) at the first observation, and \( \Omega \, 5^\circ \) at
the second, and the point on the ecliptic halfway between these falls in [either] \( 8 \, 25^\circ \) [or] \( m, \, 25^\circ \), the
diameter through apogee and perigee must go through the latter [points].

[3] Similarly, in the [observations we got] from Theon, we found that in the
twelfth year of Hadrian, Athyr [III] 21/22 in the Egyptian calendar [127 Oct. 11/12], Venus as morning-star had its greatest elongation from the sun when it
was to the rear of the star on the tip of the southern wing of Virgo by the length
of the Pleiades, or less than that amount by its own diameter; and it seemed to
be passing the star one moon to the north. Now the longitude of the fixed star at
that time, according to us, was \( \Omega \, 28^{10}_3^\circ \); hence the longitude of Venus was about
\( m, \, 0^{10}_1^\circ \). And the mean sun was in \( \kappa \, 17^{30}_1^\circ \). So the greatest elongation from
the mean as morning-star was \( 47^{30}_1^\circ \).

[4] In the twenty-first year of Hadrian, Mechir [VI] 9/10 in the Egyptian
calendar [136 Dec. 25/26], in the evening, we observed Venus at its greatest
elongation from the sun. It was in advance of the northernmost star of the four
which almost form a quadrilateral (behind the star to the rear of and on a
straight line with the [two] in the groin of Aquarius);\(^7\) [its distance from the star
was] about two-thirds of a full moon, and it seemed about to obscure the star
with its light.\(^8\) Now the longitude of the fixed star at that time, according to us,
was \( \kappa \, 20^\circ \); hence Venus was in about \( \kappa \, 19^{10}_3^\circ \), and the mean sun’s longitude
was \( \kappa \, 2^{10}_3^\circ \).

Here too, then, the greatest elongation as evening-star was the same [as in [3]
as morning-star], \( 47^{30}_1^\circ \). And the points on the ecliptic halfway between the \( \kappa \, 17^{30}_1^\circ \)
of the first observation and the \( \kappa \, 2^{10}_3^\circ \) of the second are again about \( m, \, 25^\circ \)
and \( 8 \, 25^\circ \).

2. {On the size of [Venus’] epicycle}

By these means, then, we determined that in our time the apogee and perigee
of [Venus’] eccentre lie in \( 8 \, 25^\circ \) and \( m, \, 25^\circ \). Accordingly, we again looked for
greatest elongations from the mean which occur when the sun is near \( 8 \, 25^\circ \)
and \( m, \, 25^\circ \).

\(^{a}\) Literally ‘a third of the first degree of Virgo’. The longitude in the catalogue (XXVII 5) is \( \Omega \, 29^\circ \). Ptolemy subtracts 5’ for 11 years’ precession, adds 14’ for the length of the Pleiades, and then
subtracts 5’ for the diameter of Venus. (In the *Planetary Hypotheses*, ed. Goldstein p. 8 § 5, he
estimates the apparent diameter of Venus as 9th of the sun’s, i.e. 3’).

\(^{7}\) The stars in question are (according to Manitius’ identification): the quadrilateral, catalogue
nos. XXXII 26–9; the two in the groin, nos. 15 and 16. The differences in the description here from
the catalogue are so great that we must assume that this was originally written before the catalogue
existed (as the date of the observation suggests).

\(^{8}\) Reading καταλυμένη (with GD) for καταλύμενη (‘seemed to be obscuring’) at H298,14–15.
The word is a technical term for one bright body (the sun, as at VIII 6, H201,1, cf. καταλύμενης at
XIII 7, H591,11, or the moon, as here) coming so close to another that it ‘outshines’ it and makes it
no longer visible.

\(^{9}\) ‘Two-thirds of a moon’ is only 20’, whereas Ptolemy subtracts 24’. Is the difference to account
for the diameter of Venus?
[1] In the observations given to us by Theon we find that in the thirteenth year of Hadrian, Epiphi [XI] 2/3 in the Egyptian calendar [129 May 19/20], Venus was at its greatest elongation from the sun as morning-star, and was 1° in advance of the straight line through the foremost of the 3 stars in the head of Aries and the star on the hind leg, while its distance from the foremost star of those in the head was approximately double its distance from the star on the leg. Now at that time, according to us, the foremost star of the 3 in the head of Aries had a longitude of 63° and is 71° north of the ecliptic, while the star in the hind leg of Aries had a longitude of 14°, and is 51° south of the ecliptic. Therefore the longitude of Venus was 10° south and it was 1° south of the ecliptic. Hence, since the longitude of the mean sun at that time was 25°, the greatest elongation from the mean was 44°.

[2] In the twenty-first year of Hadrian, Tybi [V] 2/3 in the Egyptian calendar [136 Nov. 18/19], in the evening, we observed Venus at its greatest distance from the sun: when sighted with respect to the stars in the horns of Capricorn it was seen to occupy 2°, while the longitude of the mean sun was 25°. Hence in this position the greatest elongation from the mean comes out as 47°.

Hence it is clear that the apogee lies in 25°, and the perigee in 25°. Furthermore, it has also become plain to us that the eccentre of Venus carrying the epicycle is fixed, since nowhere on the ecliptic do we find the sum of the greatest elongations from the mean on both sides to be less than the sum of both in Taurus, or greater than the sum of both in Scorpius.

With the above as data, let [Fig. 10.1] the eccentric circle, on which Venus' epicycle is always carried, be ABG on diameter AG, on which D is taken as the centre of the eccentric, E as the centre of the ecliptic, and A as the point at 25°. About points A and G let there be drawn equal epicycles, on which lie points Z and H respectively. Draw the tangents EZ and EH, and join AZ. GH.

Then, since \( \angle AEZ \), which is at the centre of the ecliptic, subtends the greatest elongation of the planet at the apogee, which is, by hypothesis, 44°,

\[
\angle AEZ = \begin{cases} 
44;48" \text{ where 4 right angles = 360°} \\
89;36" \text{ where 2 right angles = 360°}
\end{cases}
\]

Therefore in the circle about right-angled triangle AEZ

arc \( AZ = 89;36" \)

and its chord \( AZ \approx 88;33" \) where hypotenuse \( AE = 120" \).

Similarly, since \( \angle GEH \) subtends the greatest elongation at the perigee, which is, by hypothesis, 47°,

\[
\angle GEH = \begin{cases} 
47;20" \text{ where 4 right angles = 360°} \\
94;40" \text{ where 2 right angles = 360°}
\end{cases}
\]

Therefore in the circle about right-angled triangle GEH

arc \( GH = 94;40" \)

and its chord \( GH \approx 88;13" \) where hypotenuse \( EG = 120" \).

The stars in question are catalogue XXII 1 and 13 (note the different descriptions there), with longitudes of 6° and 15°. The difference in the longitudes given here is -4° and -15° respectively. One would expect about -5° for the precession in 8 years. Hence Manitius emended 141 to 14I; but it is implausible to change, as he does. \( \angle 8° \) to \( 5°, 3' \) \((6 + 1)\); for \( 14° \) is written \( \angle \gamma' \mu' \beta' \((1 + 1 + 1)\), e.g. H303.7. The stars in the alignment are too far apart to allow us to use it to check the text, so in the absence of any ms. variation I merely note the possibility of some corruption.
Therefore where $GH (= AZ)$, the radius of the epicycle, is $84;33^\circ$, and $AE = 120^\circ$,

$$EG = 115;1^\circ,$$

and obviously, by addition, $AG = 235;1^\circ$

and its half, $AD \approx 117;30^\circ$,

and, by subtraction, the distance between the centres, $DE = 2;29^\circ$.

Therefore where the radius of the eccentre, $AD = 60^\circ$, the distance between the centres, $DE \approx 1;4^\circ$, and the radius of the epicycle, $AZ = 43;4^\circ$.

3. [On the ratios of the eccentricities of the planet [Venus]]

But since it is not clear whether the uniform motion of the epicycle takes place about point $D$, here too we took two greatest elongations, in opposite directions [i.e. one as evening-star and the other as morning-star], in each of which\(^{11}\) the mean motion of the sun was a quadrant from the apogee.

\(^{11}\)Reading ἐφ' ἑκάτερα (with CDG,Is) at H303,2 for ἐφ' ἑκάτερα ('in both directions'). Corrected by Manitius.
greatest elongation from the sun as morning-star, and when it was sighted with respect to the star called Antares [catalogue XXIX 8], its longitude was \( \phi \ 11^{11}_1^0 \), at which time the longitude of the mean sun was \( \approx 25^1_0^0 \). So the greatest elongation from the mean as morning-star was \( 43^1_2^0 \).

[2] We observed the second in the third year of Antoninus, Pharmouthi [VIII] 4/5 in the Egyptian calendar [140 Feb. 18/19], in the evening. In this Venus was at its greatest elongation from the sun, and when it was sighted with respect to the bright star in the Hyades [catalogue XXIII 14], its longitude was \( \varpi \ 13^{5}_1^0 \), while the longitude of the mean sun was again \( \approx 25^1_0^0 \). Hence in this case the greatest elongation from the mean as evening-star was \( 48^1_0^0 \).

With the above as data, let [Fig. 10.2] the diameter through the apogee and perigee of the eccentric be \( ABG \); let \( A \) represent the point at \( \beta \ 25^0 \), and let \( B \) represent the centre of the ecliptic. Let our task be to find the centre about which we say that the uniform motion of the epicycle takes place. Let that centre be point \( D \), and draw \( DE \) through \( D \) perpendicular to \( AG \), in order for the mean position of the epicycle to be a quadrant from the apogee, as in the observations. On \( DE \) take \( E \) to represent the centre of the epicycle at the observations in question, draw the epicycle \( ZH \) on it as centre, draw the tangents to it from \( B, BZ \) and \( BH \), and join \( BE, EZ \) and \( EH \).

Then since, at the mean position in question, the greatest elongation from the mean as morning-star is, by hypothesis, \( 43^1_2^0 \), and the greatest as evening-star \( 48^1_0^0 \),

by addition, \( \angle ZBH = 91;55^0 \) where 4 right angles = 360\(^0\).

Therefore its half, \( \angle ZBE = 91;55^0 \) where 2 right angles = 360\(^0\).

Therefore in the circle about right-angled triangle \( BEZ \)

\[
\text{arc } EZ = 91;55^0 \\
\text{and } EZ = 86;16^0 \text{ where hypotenuse } BE = 120^0.
\]
Therefore where the radius of the epicycle, $EZ = 43;10^\circ$

$$BE = 60;3^\circ.$$  

Again, since the difference between the above greatest elongations (which is $4;45^\circ$) comprises twice the equation of the ecliptic anomaly at that point, which is represented by $\angle BED$,

$$\angle BED = \begin{cases} 2;22,30^\circ \text{ where } 4 \text{ right angles } = 360^\circ \\ 4;45^\circ \text{ where } 2 \text{ right angles } = 360^\circ. \end{cases}$$

Therefore in the circle about right-angled triangle $BDE$

$$\text{arc } BD = 4;45^\circ$$

and $BD \approx 4;59^\circ$ where hypotenuse $BE = 120^\circ$.

Therefore where $BE = 60;3^\circ$ and the radius of the epicycle is $43;10^\circ$, $BD \approx 24^\circ$.

But we showed [p. 472] that the distance between $B$, the centre of the ecliptic, and the centre of the eccentric on which the epicycle centre is always carried, is $1\frac{1}{4}$ in the same units; thus it is half of $BD$.

Therefore, if we bisect $BD$ at $\Theta$, we have demonstrated\(^{12}\) that

where $\Theta A$, the radius of the eccentric carrying the epicycle, is $60^\circ$,

each of the distances between the centres, $B\Theta$ and $\Theta D = 1\frac{1}{4}^\circ$,

and $EZ$, the radius of the epicycle, is $43;10^\circ$.

Q.E.D.

4. [On the correction of the periodic motions of the planet [Venus]]\(^{13}\)

Such, then, is the method by which we determined the type of [Venus'] hypothesis and the ratios of its anomalies. For the periodic motions and epochs of the planet, once again [as for Mercury], we took two reliable observations, [one] from among ours, and [one] of the ancient ones.

[1] In the second year of Antoninus, Tybi [V] 29/30 in the Egyptian calendar [138 Dec. 15/16], we observed the planet Venus, after its greatest elongation as morning-star, using the astrolabe and sighting it with respect to Spica: its apparent longitude was $\mu 61^\circ$. At that moment it was also between and on a straight line with the northernmost of the stars in the forehead of Scorpius and the apparent centre of the moon, and was in advance of the moon's centre $1\frac{1}{4}$ times the amount it was to the rear of the northernmost of the stars in the forehead. Now the [latter] fixed star had at that time, according to our coordinates, a longitude of $\mu 6;20^\circ$, and is $1;20^\circ$ north of the ecliptic.\(^{14}\) The time was $4\frac{1}{2}$ equinoctial hours after midnight, since the sun was in about $\tau$ 23,
and the second degree of Virgo [i.e. Μ 1°-2°] was culminating according to the astrolabe. At that moment the positions were as follows:

- mean longitude of the sun ⋆ 22°9\°
- mean longitude of the moon ⋆ 11°24\°
- anomaly of the moon, counted from apogee 87°30\°
- [argument of] latitude of the moon, from the northern limit 12°22\°
- hence, true position of the moon's centre ⋆ 5°45\°
- [moon's latitude] 5° north of the ecliptic
- apparent position [of the moon] at Alexandria in longitude ⋆ 6°45\°
- [apparent position of the moon in latitude] 4°40\° north of the ecliptic.

From these considerations too, then, Venus' longitude was ⋆ 6°30\°, and it was 2°40\° north of the ecliptic.

With the above as data, let [Fig. 10.3] the diameter through the apogee be ABGDE. Let A represent the point at 8° 25\°, B the point about which the epicycle moves uniformly, G the centre of the eccentre carrying the epicycle centre, and D the centre of the ecliptic. Since the mean sun had a longitude of ⋆ 22°9\° at the observation, the mean position of the epicycle is [⋆ 22°9\° - ⋆ 25\° =] 27°9\° towards the rear from the perigee at E. So let the epicycle centre be located at Z, and draw the epicycle HK on Z as centre. Join DZH, GZ and BZ0, and drop perpendiculars GL and DM from G and D on to BZ. Let the

---

15The following data are calculated, accurately, not for 4;45 a.m., but for 4;30 a.m. Since the equation of time for a solar longitude of ⋆ 23° is about −17 mins., Ptolemy's (silent) correction is justified. For 4;45 a.m. local time I find the culminating point as a little past Μ 1°, in agreement with the text.
planet be located at point K, join DK and ZK, and drop perpendicular ZN [on to DK]. Let the problem be, to find the arc ΘK, which is the distance of the planet from the epicycle apogee Θ [at the observation].

Now since

\[ \angle EBZ = \begin{cases} 27:9^\circ \text{ where 4 right angles} = 360^\circ \\ 54:18^\circ \text{ where 2 right angles} = 360^\circ \end{cases}, \]

in the circle about right-angled triangle BGL

arc GL = 54:18^\circ

and arc BL = 125:42^\circ (supplement).

Therefore the corresponding chords

\[ \begin{align*} GL &= 54:46^\circ \\ BL &= 106:47^\circ \end{align*} \]

where hypotenuse BG = 120^\circ.

Therefore where BG = 1:15^p and GZ, the radius of the eccentric, is 60^p,

\[ \begin{align*} GL &= 0:34^p \\ BL &= 1:7^p. \end{align*} \]

And since \( ZG^2 - GL^2 = ZL^2, \)

\[ ZL \approx 60^p \text{ in the same units.} \]

And since BG = GD

\[ \begin{align*} ML &= LB \left[= 1:7^p \right] \\ DM &= 2GL. \end{align*} \]

Therefore, by subtraction [of ML from ZL], ZM = 58:53^p

and DM = 1:8^p in the same units.

Hence hypotenuse ZD = \( \sqrt{ZM^2 + DM^2} \) = 58:54^p.

Therefore, where ZD = 120^p, DM = 2:18^p,

and, in the circle about right-angled triangle DZM,

arc DM = 2:12^p

\[ \therefore \angle BZD = 2:12^\circ \text{ where 2 right angles} = 360^\circ. \]

And, by addition [of \( \angle EBZ \) and \( \angle BZD \)], \( \angle EDZ = 56:30^\circ \) in the same units.

And, since the planet was 18:30^p in advance of the perigee at E (i.e. \( \Pi, 25^\circ \)) at the observation,

\[ \angle EDK = \begin{cases} 18:30^p \text{ where 4 right angles} = 360^p \\ 37^\circ \text{ where 2 right angles} = 360^\circ \end{cases}. \]

Therefore, by addition [of \( \angle EDK \) to \( \angle EDZ \)],

\[ \angle KDZ = 93:30^\circ \text{ where 2 right angles} = 360^\circ, \]

and, in the circle about right-angled triangle DZN,

arc ZN = 93:30^p.

Therefore its chord, ZN = 87:25^p where ZD = 120^p.

So where ZD = 58:54^p, i.e. where the epicycle radius ZK is 43:10^p,

\[ ZN = 42:54^p. \]

\[ \therefore ZN = 119:18^p \text{ where hypotenuse ZK = 120^p,} \]

and, in the circle about right angled triangle ZKN,

\[ \text{arc ZN} = 167:38^p. \]

\[ \therefore \angle ZKD = 167:38^\circ \text{ where } \angle ZDK \text{ has already been found as 93:30^\circ}. \]

\[ ^{16} \text{The accumulated rounding error here is considerable. } ZN \text{ should be about 119:18^p rather than } 119:18^p. \text{ Since this chord is so close to the maximum of } 120^p, \text{ the resulting error in the arc is great: } \text{accurate computation would give } ZN = 167:22^p, \text{ resulting in a not negligible change of 8^p in the final result (230:40^p).} \]
So, by addition, \( \angle KZH = 261;8^\circ \).

And we showed that \( \angle BZD (= \angle HZ\Theta) = 2;12^\circ \) in the same units.

Therefore, by subtraction, \( \angle \Theta ZK = \begin{cases} 258;56^\circ & \text{where 2 right angles} = 360^\circ \\ 129;28^\circ & \text{where 4 right angles} = 360^\circ. \end{cases} \)

So the planet Venus, at the time in question, was the above distance, 129;28\(^\circ\), in advance of the epicycle apogee \( \Theta \), and, [therefore], in the motion [on the epicycle] assigned to it in the hypothesis, [namely] towards the rear, it was the difference of the above from one revolution, 230;32\(^\circ\), which was what we had to determine.

[2] From the ancient observations we selected one which is recorded by Timocharis as follows. In the thirteenth year of Philadelphos, Mesore \([XII]\) 17/18 in the Egyptian calendar \([-271\ \text{Oct.} \ 11/12]\), at the twelfth hour, Venus was seen to have exactly overtaken\(^1\) the star opposite Vindemiatrix. That is the star which, in our descriptions [catalogue XXVII 6], is the one following the \( H311 \) star on the tip of the southern wing of Virgo, and which had a longitude of \( m^\prime \ 81^\circ \) in the first year of Antoninus. Now the year of the observation is the 476th from Nabonassar, while the first year of Antoninus is 884 [years] from Nabonassar;\(^8\) to the 408 years of the interval corresponds a motion of the fixed stars and the apogees of about \( 4\frac{1}{4}^\circ \). Hence it is clear that the longitude of Venus was \( m^\prime \ 4\frac{1}{4}^\circ \), and the longitude of the perigee of its eccentric \( m^\prime \ 20\frac{1}{6}^\circ \). And here too Venus was past its greatest elongation as morning-star; for 4 days after the above observation, on Mesore 21/22, as one can deduce from what Timocharis says, its longitude was \( m^\prime \ 8^\circ \) according to our coordinates; and the mean position of the sun was \( \pm 17;3^\circ \) at the first observation and \( \pm 20;59^\circ \) at the next: thus its elongation at the first observation comes to 42;53\(^\circ\) and at the next 42;9\(^\circ\).

With the above as data, let there be drawn [Fig. 10.4] a figure similar [to the preceding], but which has the epicycle in advance of the perigee, since the mean longitude of the epicycle is \( \pm 17;3^\circ \), while the longitude of the perigee is \( m^\prime \ 20;55^\circ \). Now for that reason

\[ \angle EBZ [=m^\prime \ 20;55^\circ - \pm 17;3^\circ] = \begin{cases} 33;52^\circ & \text{where 4 right angles} = 360^\circ \\ 67;44^\circ & \text{where 2 right angles} = 360^\circ. \end{cases} \]

Therefore, in the circle about right-angled triangle BGL,

\[ \text{arc } GL = 67;44^\circ \]

and arc \( BL = 112;16^\circ \) (supplement).

Therefore the corresponding chords

\[ \text{GL} = 66;52^p \]

and \( BL = 99;38^p \) \( \{ \text{where hypotenuse } BG = 120^p. \} \)

Therefore where \( BG = 1;15^p \) and the radius of the eccentric, \( GZ = 60^p, \)

\[ \text{GL} = 0;42^p \]

and \( BL = 1;2^p. \)

\(^1\) Most translations interpret this word \((κατεταληφώς)\) as 'occulted'. Modern calculations show that no occultation occurred, since Venus passed about 12' to the south of \( \eta \) Vir. Nevertheless, since another observation where no occultation could have occurred is unambiguously described as an occultation (see p. 522 n.16), and \( καταληφών \) denotes occultations by the moon at \( H28,15, H31,5, H32,7 \) and \( H33,9 \), the same is probably intended here.

\(^8\) Reading \( το \ δε α' \ ετος της 'Αντωνίνου βασιλείας μεν έστιν ἀπὸ Ναβονασσάρου with DG.Ar) at H311, 4-5, for το \ δε μεχρι της 'Αντωνίνου βασιλείας μεν' of the other ms. The first year of Antoninus is the 885th in the era Nabonassar, but since this observation is towards the end of the Egyptian year. Ptolemy correctly counts to the end of Nabonassar 884.
And since $ZG^2 - GL^2 = ZL^2$,
$ZL \approx 60^\circ$.

And by the same reasoning [as before]

$BL = LM$
and $DM = 2GL$.

Therefore, by subtraction [of $LM$ from $ZL$], $ZM = 58;58^\circ$
and $DM = 1;24^\circ$ in the same units.

Hence hypotenuse $ZD = \sqrt{ZM^2 + DM^2} \approx 58;59^\circ$.

Therefore, where $ZD = 120^\circ$, $DM = 2;51^\circ$.

and, in the circle about right-angled triangle $ZDM$,

\[
\text{arc } DM = 2;44^\circ
\]

$\therefore \angle BZD = 2;44^\circ$ where 2 right angles $= 360^\circ$.

And, by addition [of $\angle BZD$ and $\angle EBZ$], $\angle EDZ = 70;28^\circ$ in the same units.

And the distance of the planet in advance from the perigee,

$\angle EDK = m 20;55^\circ - m 4;10^\circ$ = \begin{cases} 76;45^\circ & \text{where 4 right angles} = 360^\circ \\ 153;30^\circ & \text{where 2 right angles} = 360^\circ \end{cases}$

Therefore, by subtraction, $\angle ZDK = 83;2^\circ$ in the same units,

and, in the circle about right-angled triangle $DZN$,

\[
\text{arc } ZN = 83;2^\circ.
\]

So its chord $ZN = 79;33^\circ$ where hypotenuse $DZ = 120^\circ$.

and where $DZ = 58;59^\circ$, i.e. where the epicycle radius $ZK = 43;10^\circ$,

$ZN = 39;7^\circ$.

Therefore, in the circle about right-angled triangle $ZKN$,

where hypotenuse $ZK = 120^\circ$.

$ZN = 108;45^\circ$
5. Epoch positions of Venus in mean motion

and arc $ZN \approx 130^\circ$.

$\therefore \angle DKZ = 130^\circ$ where $\angle ZDK$ has already been found as $83;2^\circ$.

And, by addition, $\angle \Theta ZK = 213;2^\circ$ in the same units.

But we showed that $\angle BZD (= \angle HZ\Theta) = 2;44^\circ$ in the same units.

Therefore, by addition, $\angle HZK = \begin{cases} 215;46^\circ & \text{where 2 right angles} = 360^\circ \\ 107;53^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}$

At that moment, then, the distance of the planet Venus, [in the sense of rotation] towards the rear, from the epicycle apogee $H$ was the difference from one revolution, $252;7^\circ$, which was what we had to determine.

Now its distance from the apogee of the epicycle, in the same sense, at the moment of our observation was $230;32^\circ$. And the interval between the two observations comprises 409 Egyptian years and about 167 days, and 255 complete revolutions in anomaly (for 8 Egyptian years produce approximately 5 revolutions, so the 408 years produce 255 revolutions, while the remaining year plus the additional days do not complete the period of one revolution). So we have demonstrated that in 409 Egyptian years 167 days the planet Venus travels on the epicycle, beyond 255 complete revolutions in anomaly, $338;25^\circ$, which is the amount by which the position at our observation exceeded the earlier one. And approximately the same increment results from the mean motion tables which we set out above. For our correction of the mean motions was derived from the increment over complete revolutions we have found [above]: the time-interval was reduced to days, and the revolutions plus the increment to degrees. For then, when the total in degrees is divided by the total in days, there results the mean daily motion of Venus in anomaly which we set out previously.\[^{21}\]

\[^{15}\]The accumulated rounding error here amounts to $4'$ (one finds $107;49^\circ$).

\[^{19}\]Reading $\alpha_\nu\omicron\mu\omicron\alpha\lambda\lambda\iota\zeta$ (with DG) for $\alpha\omicron\nu\omicron\mu\omicron\alpha\lambda\lambda\iota\zeta$ at H314,22. Corrected by Manitius.

\[^{21}\]On the actual derivation of the mean motion for Venus see Appendix C. Ptolemy's increment in mean motion, $338;25^\circ$, is the motion from $252;7^\circ$ (above) to $230;32^\circ$ (p. 477). The accumulated rounding errors in those figures (see p. 476 n.16 and above n.19) lead to a difference in the increment of $+4'$, which would have an effect on the resulting mean motion. Furthermore it is unclear what interval in days Ptolemy is actually using. He gives the round number $409;346^3$ days approximately.\[^{22}\]

The increment in mean motion corresponding to that interval in the columns

15 The accumulated rounding error here amounts to 4' (one finds 107;49').
19 Reading ανομαλίας (with DG) for ανομαλίαν at H314,22. Corrected by Manitius.
21 On the actual derivation of the mean motion for Venus see Appendix C. Ptolemy's increment in mean motion, 338;25°, is the motion from 252;7° (above) to 230;32° (p. 477). The accumulated rounding errors in those figures (see p. 476 n.16 and above n.19) lead to a difference in the increment of +4', which would have an effect on the resulting mean motion. Furthermore it is unclear what interval in days Ptolemy is actually using. He gives the round number 409° 167°. But the time of Ptolemy's observation is given as 4:45 a.m., and of Timocharis' as 'at the 12th hour' (interpreted as 6 a.m. in X 5, see below n.22). So the interval should be 1½ hours less than the above, or, if one corrects for the equation of time at Ptolemy's observation, (cf. p. 475 n.15) 1½ hours less.
22 If one assumes that the observation of Timocharis (p. 477) was made just at dawn, and applies the equation of time with respect to the epoch of era Nabonassar (about – ½ hour), the interval given is approximately correct. But see n.23.
for anomaly is approximately 181°. Subtracting the latter from the 252;7° of the position at the observation, we get for the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon:

epoch in anomaly: 71;7° from the apogee of the epicycle.

The mean position in longitude is again, by hypothesis, the same as the sun's namely

longitude: 0;45°.

And it is obvious that, since the apogee [of the eccenter] was at about 8 20;55° at the observation, and to the intervening 476 years correspond approximately 44° [of motion of the apogee], at the moment of epoch the apogee will be in about 8 16;10°.

6. {Preliminaries for the demonstrations concerning the other [3 outer] planets}

Such, then, were the methods which we successfully used for these two planets, Mercury and Venus, to establish the hypotheses and demonstrate [the sizes of] the anomalies. For the other three, Mars, Jupiter and Saturn, the hypothesis which we find for their motion is the same [for all] and like that established for the planet Venus, namely one in which the eccenter on which the epicycle centre is always carried is described on a centre which is the point bisecting the line joining the centre of the ecliptic and the point about which the epicycle has its uniform motion; for in the case of each of these planets too, using rough estimation, the eccentricity one finds from the greatest equation of ecliptic anomaly turns out to be about twice that derived from the size of the retrograde arcs at greatest and least distances of the epicycle. However, the demonstrations by which we calculate the amounts of both anomalies and [the positions of] the apogees cannot proceed along the same lines for these planets as for the previous two, since these reach every possible elongation from the sun, and it is not obvious from observation, as it was from the greatest elongations for Mercury and Venus, when the planet is at the point where the line of our sight is tangent to the epicycle. So, since that approach is not available, we have used observations of their oppositions to the mean position of the sun to demonstrate, first of all, the ratios of their eccentricities and [the positions of] their apogees. For only in such positions [of the planet], considered from a theoretical point of view, do we find the ecliptic anomaly isolated, with no effect from the anomaly related to the sun.

For let [Fig. 10.5] the planet's eccenter, on which the epicycle centre is carried, be ABG on centre D, and let the diameter through the apogee be AG, on which point E is the centre of the ecliptic, and Z the centre of that eccenter with respect to which the epicycle's mean motion in longitude is taken. Draw the epicycle H0KL on centre B, and join ZLB0 and HBKEM.

I say, first, that when the planet is seen along line EH through the epicycle

23 Computing from the table (IX 4) one finds for the stated interval 180:58,34°. Ptolemy has either rounded unjustifiably, or computed for a slightly longer interval. A motion of half an hour more (i.e. neglecting the equation of time, cf. n.22) produces 180:59:20°.

24 See H.A.M.1 172. An ingenious analysis of the way in which Ptolemy arrived at the notion of the equant for the outer planets was made by Swerdlow. 'The Origin of Ptolemaic Planetary Theory'.
centre B, then the mean position of the sun, too, will always be on the same line, and that when the planet is at H it will be in conjunction \(^{25}\) with the mean sun (which will also, in theory, be seen towards H), and when the planet is at K it will be in opposition to the mean sun (which will be seen, in theory, towards M).

[Proof:] For each of these [outer] planets, the sum of the mean motions in longitude and anomaly, counted from the apogee [of eccentrc and epicycle respectively], equals the mean motion of the sun counted from the same starting-point. And the difference between the angle at centre Z (which comprises the mean motion of the planet in longitude), and the angle at E (which comprises the apparent motion in longitude), \(^{26}\) is always the angle at B (which comprises the mean motion on the epicycle). Hence it is clear that when the planet is at H, it will fall short of a return to the apogee \(\Theta\) by \(\angle \text{HB}\Theta\); but \(\angle \text{HB}\Theta\) added to \(\angle \text{AZB}\) produces the angle comprising the sun's mean motion, namely \(\angle \text{AEH}\), which is the same as the apparent motion of the planet. \(^{27}\) And when the planet is at K, its motion on the epicycle, again, will be \(\angle \Theta \text{BK}\), and \(\angle \Theta \text{BK} + \angle \text{AZB}\) equal the mean motion of the sun counted from the apogee A.

\(^{25}\) Reading συνοδεύει (with G. and possibly Ar, but the translations are ambiguous) for συνοδεύεται ('is in conjunction') at H318,18.

\(^{26}\) By this expression (ή φαίνομένη κατά μήκος κίνησις) Ptolemy means, not the true position of the planet, but the position of the epicycle center as seen from the earth. Compare the expression ἡ φαινομένη ἐξί τοῦ ἐπικύκλου πάροδος at XII 2 (H470,11) to denote the 'true anomaly' (i.e. as counted from true and not mean perigee of the epicycle).

\(^{27}\) In fact \(\angle \text{AZB} - \angle \text{HB}\Theta = \angle \text{AEH}\). But what Ptolemy means is illustrated by Figs. P1 and P2: in Fig. P1 planet and mean sun are in conjunction. In Fig. P2 (= Fig. 10.5) they are again in conjunction. The epicycle has travelled through the angle \(\kappa\) (\(\angle \text{AZB}\)), the planet on the epicycle has travelled through \(\bar{\alpha}\), and the mean sun through \(\kappa + 360^\circ\). Then (from the figure) \(\kappa = \kappa - (360^\circ - \bar{\alpha}) = \kappa + \bar{\alpha} - 360^\circ\). Hence the mean sun's motion \(\kappa + 360^\circ = \kappa + \bar{\alpha}\). Failing to understand this, an interpolator has inserted τοιτάτον λειτουργία να' αύθας at H319,8, producing the strange result \(\angle \text{HB}\Theta\) added to \(\angle \text{AZB}\), i.e. subtracted from it.
X 6. Relation between mean motions of outer planet and sun

Fig. P1

Fig. P2
Thus the latter comprises $180^\circ + (\angle AZB - \angle LBK) = 180^\circ + \angle GEM$, i.e. the mean position of the sun will be opposite the apparent position of the planet.

Hence, furthermore, in such configurations [i.e. mean conjunctions and oppositions], the line joining the epicycle centre $B$ to the planet, and the line from $E$, our point of view, to the mean sun, will coincide in one straight line, but at all other [sun-planet] elongations [those vectors] will always be parallel to each other, although the direction in which they point will vary.

Fig. 10.6

[Proof:] In the above figure [see Fig. 10.6], if we draw the line $BN$ from $B$ to the planet in any situation, and the line $EX$ from $E$ to the mean sun, for the reasons stated above

$$\angle AEX = \angle AZ\Theta + \angle NB\Theta,^{28}$$

and $\angle AZ\Theta = \angle AEH + \angle HB\Theta$.

$$[\therefore \angle AEX = \angle AEH + \angle NB\Theta + \angle HB\Theta.]$$

If we subtract $\angle AEH$ from both sides,

$$\angle HEX = \angle HBN.$$ Therefore line $EX$ is parallel to line $BN$.

Thus we find that in the above configurations of conjunction and opposition with respect to the mean sun, the planet is viewed, in theory, [along the line through the centre of the epicycle, just as if its motion on the epicycle did not exist, but instead it were itself situated on circle $ABG$ and were carried in uniform motion by the line $ZB$, in the same way as the epicycle centre is. Hence it is clear that it is possible to isolate and demonstrate the ratio of the ecliptic eccentricity by [both] such types of [planetary] positions, but since the

$^{28}$ I.e. the mean motion of the sun equals the mean longitudinal motion of the planet plus the mean anomaly of the planet.
conjunctions are not visible, we are left with the oppositions on which to build our demonstrations.

7. {Demonstration of the eccentricity and apogee [position] of Mars}

In the case of the moon we took the positions and times of three lunar eclipses, and demonstrated the ratio of the anomaly and the position of the apogee geometrically. So too, here, in the same way, for each of these [outer] planets, we observed the positions of three oppositions to the mean sun, as accurately as possible, using the astrolabe instruments, computed, too, the time and position for the precise 180° elongation from the position of the mean sun at [each of] the observations, and thence demonstrate the ratio of the eccentricity and [the position of] the apogee.

First, then, for Mars, we took three oppositions, which we observed as follows.


The intervals between the above are as follows:

From oppositions [1] to [2] 4 Egyptian years 69 days 20 equinoctial hours.


For the first interval we compute a [mean] motion in longitude, beyond complete revolutions, of 81;44°.

And for the second interval, 95;28°.

Even if we used the crude periods of return, which we listed above, to compute the mean motions, it would make no significant difference over such a short interval.

29 ἀκρωτοι σχηματισμοί, literally 'configurations [at which the planet rises and sets] at the beginning and end of night'.

30 On the method used to find the eccentricities of the outer planets see H.4.M.1 172–7, Pedersen 273–83.

31 Reading διαμέτρου στάσεως (with DG, Ar) for διαστάσεως 'elongation' at H322.1.

32 The times are arrived at by computing the position of the mean sun. Therefore the computed position of the mean sun at the time stated ought to be exactly 180° different from the longitudes given. I find, from the solar mean motion tables, 260;58,55° (instead of 261°), 328,50,22° (for 328;50°) and 62,31,45° (for 62,34°). The latter discrepancy represents about half an hour in solar motion. Could Ptolemy have applied the equation of time (which is about -25° mins. compared with epoch) here? If so, he was mistaken, since all the computations are in terms of mean solar days.

33 Ptolemy is referring to the crude periods of IX 3. Thus for Mars (cf. p. 424) in 79 solar years occur 37 returns in anomaly and 42 returns in longitude. Assuming Ptolemy's year-length of 365,14,48°, one finds from this, for 4° 69° 20°, a longitudinal increment of 81,39°, and, for 4° 96° 1°, 95;23°. Using Ptolemy's procedure, and carrying out three iterations, I find from the above data 2r ≈ 11,57°, distance of 3rd opposition from perigee ≈ 44°. Comparison with Ptolemy's results from the more accurate data, 12° and 44,21°, shows that the differences are indeed negligible.
It is obvious that the apparent motion of the planet, beyond complete revolutions, is
for the first interval \(67;50^\circ\)
and for the second interval \(93;44^\circ\).

Then [see Fig. 10.7] let there be drawn in the plane of the ecliptic three equal circles: let the circle carrying the epicycle centre of Mars be \(ABG\) on centre \(D\), the eccentre of uniform motion \(EZH\) on centre \(\Theta\), and the circle concentric with the ecliptic \(KLM\) on centre \(N\), and let the diameter through all [three] centres be \(XOPR\). Let \(A\) be the point at which the epicycle centre was at the first opposition, \(B\) the point where it was at the second opposition, and \(G\) the point where it was at the third opposition. Join \(\Theta AE\), \(\Theta BZ\), \(\Theta HG\), \(NKA\), \(NLB\) and \(NGM\). Then arc \(EZ\) of the eccentric [equant] is \(81;44^\circ\), the amount of the first interval of mean motion, and arc \(ZH\) is \(95;28^\circ\), the amount of the second...
interval. Furthermore arc KL of the ecliptic is 67;50°, the amount of the first interval of apparent motion, while arc LM is 93;44°, the amount of the second interval.

Now if arcs EZ and ZH of the eccentric [equant] were subtended by arcs KL and LM of the ecliptic, that would be all we would need in order to demonstrate the eccentricity. However, as it is, they [arc KL and arc LM] subtend arcs AB and BG of the middle eccentric, which are not given; and if we join NSE, NTZ, NHY, we again find that arcs EZ and ZH of the eccentric [equant] are subtended by arcs ST and TY of the ecliptic, which are, obviously, not given either. Hence the difference arcs, KS, LT and MY, must first be given, in order to carry out a rigorous demonstration of the ratio of the eccentricity starting from the corresponding arcs, EZ, ZH, and ST, TY. But the latter [arcs ST and TY] cannot be precisely determined until we have found the ratio of the eccentricity and [the position of] the apogee; however, even without the previous precise determination of eccentricity and apogee, the arcs are given approximately, since the difference arcs are not large. Therefore we shall first carry out the calculation as if the arcs ST, TY did not differ significantly from the arcs KL, LM.

For [see Fig. 10.8] let the eccentre of mean motion of Mars be ABG, on which A is taken as the point of the first opposition, B of the second, and G of the third. Inside the eccentre take D as the centre of the ecliptic, which is our point of view, draw in every case [where one has to carry out this kind of calculation] the lines joining the points of the three oppositions to the observer (as here AD, BD

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Fig. 10.8

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34 For the situation would be identical with that of the lunar hypothesis (IV 6).
35 Reading αὐτοί (with Α,Β [not reported by Heiberg]. AR) for αὐταί at H324.8.
36 The arcs forming the differences between arc KL and arc TS, and between arc LM and arc TY.
37 Reading παρά τὰς ΚΑΜ τῶν ΣΤΥ περιφερεῖν, at H324.22, for παρά τὰς ΚΑΜ, ΣΤΥ περιφερεῖν ("as if arcs did not differ significantly from [arcs] KLM and STY", which is senseless). My text is the reading of all mss., Greek and Arabic. Heiberg omitted τῷ through a slip or a misprint. Because Manitius did not realize this, his translation here is badly flawed.
and GD), and, as a universal rule, produce one of the three lines so drawn to meet the circumference of the eccentre on the other side (as here GDE), and draw the line joining the other two opposition points (as in this case AB). Then, from the point where the straight line produced intersects the eccentre (as E), draw the lines joining it to the other two opposition points (as here EA and EB), and drop perpendiculars [from the point corresponding to E] on to the lines joining the above-mentioned two points to the centre of the ecliptic (in this case, drop EZ on to AD, and EH on to BD). Also, drop a perpendicular from one of those two points on to the line joining the other with the extra point generated on the eccentre (as here, perpendicular A0 on to line BE). If we always observe the above rules when drawing this type of figure, we will find that the same numerical ratios result however we decide to draw it. The remainder of the demonstration will become clear as follows, on the basis of the above arcs for Mars.

Since arc BG of the eccentre is given as subtending 93;44° of the ecliptic, the angle at the centre of the ecliptic

\[ \angle BDG = \begin{cases} 93;44^\circ & \text{where 4 right angles = 360}^\circ \\ 187;28^\circ & \text{where 2 right angles = 360}^\circ \end{cases} \]

and its supplement, \( \angle EDH = 172;32^\circ \) in the same units.

Therefore, in the circle about right-angled triangle DEH,

arc EH = 172;32^\circ
and EH = 119;45^\circ where hypotenuse DE = 120^\circ.
Similarly, since arc BG = 95;28^\circ
the angle at the circumference, \( \angle BEG = 95;28^\circ \) where 2 right angles = 360^\circ.
But we found that \( \angle BDE = 172;32^\circ \) in the same units.

Therefore the remaining angle [in triangle BDE],

\( \angle EBH = 92^\circ \) in the same units.

Therefore, in the circle about right-angled triangle BEH,

arc EH = 92^\circ
and EH = 86;19^\circ where hypotenuse BE = 120^\circ.
Therefore where EH, as we showed, is 119;45^\circ, and ED = 120^\circ,
BE = 166;29^\circ.

Again, since the whole arc ABG of the eccentre is given as subtending [93;44° + 67;50° =] 161;34° of the ecliptic (the sum of both intervals), \( \angle ADG = 161;34^\circ \) where 4 right angles = 360^\circ,
and, by subtraction [from 180°],

\( \angle ADE = \begin{cases} 18;26^\circ & \text{where 4 right angles = 360}^\circ \\ 36;52^\circ & \text{where 2 right angles = 360}^\circ \end{cases} \)

Therefore, in the circle about right-angled triangle DEZ,

arc EZ = 36;52^\circ
and EZ = 37;57^\circ where hypotenuse DE = 120^\circ.
Similarly, since arc ABG of the eccentre is, by addition [of 81;44° to 95;28°], 177;12°.

\( \angle AEG = 177;12^\circ \) where 2 right angles = 360^\circ.
But we found that \( \angle ADE = 36;52^\circ \) in the same units.

\(^{38}\) I.e. whichever of the lines AD, BD, GD we decide to produce.
Therefore the remaining angle [in triangle $\triangle ADE$],

$\angle DAE = 145;56^\circ$ in the same units.

Therefore, in the circle about right-angled triangle $\triangle AEZ$,

arc $EZ = 145;56^\circ$

and $EZ = 114;44^\circ$ where hypotenuse $AE = 120^\circ$.

Therefore, where $EZ$, as was shown = $37;57^\circ$, and $ED = 120^\circ$,

$AE = 39;42^\circ$.

Again, since arc $AB$ of the eccentricity = $81;44^\circ$,

$\angle AEB = 81;44^\circ$ where 2 right angles = $360^\circ$.

Therefore, in the circle about right-angled triangle $\triangle AE\Theta$,

arc $\Theta \Theta = 81;44^\circ$

and arc $E\Theta = 98;16^\circ$ (supplement).

Therefore the corresponding chords

$\Theta \Theta = 78;31^\circ$

and $E\Theta = 90;45^\circ$ where hypotenuse $AE = 120^\circ$.

Therefore where $AE$, as was shown, is $39;42^\circ$, and $DE$ is given as $120^\circ$,

$\Theta A = 25;58^\circ$

and $E\Theta = 30;2^\circ$.

But the whole line $EB$ was shown to be $166;29^\circ$ in the same units.

Therefore, by subtraction, $\Theta B = 136;27^\circ$ where $\Theta A = 25;58^\circ$.

And $\Theta B^2 = 18615;16$,

$\Theta A^2 = 674;16$.

so $AB^2 = \Theta B^2 + \Theta A^2 = 19289;32$.

$\therefore AB = 138;53^\circ$ where $ED = 120^\circ$ and $AE = 39;42^\circ$.

But, where the diameter of the eccentricity is $120^\circ$, $AB = 78;31^\circ$.

since it subtends an arc of $81;44^\circ$.

Therefore where $AB = 78;31^\circ$, and the diameter of the eccentricity is $120^\circ$,

$ED = 67;50^\circ$

and $AE = 22;44^\circ$.

Therefore arc $AE$ of the eccentricity is $21;41^\circ$.

And, by addition, arc $EABG = [177;12^\circ + 21;41^\circ =] 198;53^\circ$.

Therefore the remaining arc $GE = 161;7^\circ$

and the corresponding chord $GE = 118;22^\circ$ where the diameter of the eccentricity

is $120^\circ$.

Now if $GE$ had been found equal to the diameter of the eccentricity, it is obvious

that the centre would lie on $GE$, and the ratio of the eccentricity would

immediately be apparent. But, since it is not equal [to the diameter], but makes

segment $EABG$ greater than a semi-circle, it is clear that the centre of the

eccentricity will fall within the latter. Let it be at $K$ [Fig. 10.9], and draw through

...
D and K the diameter through both centres, LKDM, and drop perpendicular KNX from K on to GE.

Then, since we showed, $EG = 118;22^\circ$ where diameter $LM = 120^\circ$, and $DE = 67;50^\circ$ in the same units.

by subtraction, $GD = 50;32^\circ$ in the same units.

Then, since $ED \cdot DG = LD \cdot DM$, $LD \cdot DM = [67;50 \times 50;32] = 3427;51.$

But $(LD \cdot DM) + DK^2$ equals the square on half the whole line $[LD + DM]$, i.e. $(LD \cdot DM) + DK^2 = LK^2$.

Now the square on the half is $3600$, and $(LD \cdot DM) = 3427;51$,

so $DK^2 = 3600 - 3427;51 = 172;9$,

and the distance between the centres,

$DK \approx 13;7^\circ$ where the radius of the eccentric, $KL = 60^\circ$.

Furthermore, since $GN = \frac{1}{2}GE = 59;11^\circ$ where diameter $LM = 120^\circ$, and, as we showed, $GD = 50;32^\circ$ in the same units,

by subtraction, $DN = 8;39^\circ$ where $DK$ was computed as $13;7^\circ$.

Therefore in the circle about right-angled triangle DKN,

$DN = 79;8^\circ$ where hypotenuse $DK = 120^\circ$,

and arc $DN = 82;30^\circ$.

$\therefore \angle DKN = \begin{cases} 82;30^\circ \text{ where 2 right angles} = 360^\circ \\ 41;15^\circ \text{ where 4 right angles} = 360^\circ \end{cases}$

And since $\angle DKN$ is an angle at the centre of the eccentric, $\text{arc} MX = 41;15^\circ$ also.

\footnote{Euclid III 35.}
\footnote{Euclid II 5.}
\footnote{Accurate computation from Ptolemy's original data gives about 13;24^\circ.}
X 7. Correction to account for equant: 1st opposition

But the whole arc GMX = \( \frac{1}{2} \) arc GXE \( [= \frac{1}{2} \cdot 161;7^\circ] = 80;34^\circ \).

Therefore, by subtraction, the arc from the third opposition to the perigee,
arc GM = 39;19^\circ.\(^{45}\)

And it is obvious that, since arc BG is given as 95;28^\circ,
by subtraction, the arc from the apogee to the second opposition,
arc LB \( [= 180^\circ - (95;28^\circ + 39;19^\circ)] = 45;13^\circ \),
and that, since arc AB is given as 81;44^\circ,
by subtraction, the arc from the first opposition to the apogee,
arc AL \( [= \text{arc AB} - \text{arc LB}] = 36;31^\circ \).

Taking the above quantities as given, let us investigate the differences which can be derived from them in the ecliptic arcs which we seek to determine at each of the oppositions [in turn]. Our investigation proceeds as follows.

[See Fig. 10.10.] From the previous figure [10.7] for the three oppositions let us draw separately the part representing the first opposition, draw the additional line AD, and drop perpendiculars DF and NQ from points D and N on to AO produced.

Then, since arc XE = 36;31^\circ,
\[ \angle EOX = \begin{cases} 36;31^\circ & \text{where 4 right angles} = 360^\circ \\ 73;2^\circ & \text{where 2 right angles} = 360^\circ \end{cases} \]

And the vertically opposite angle DOF = 73;2^\circ in the same units also.

Therefore, in the circle about right-angled triangle DOF,
arc DF = 73;2^\circ
and arc OF = 106;58^\circ (supplement).

Therefore the corresponding chords
\[ \text{DF} = 71;25^\circ \quad \text{where hypotenuse DO = 120^\circ} \]
and \[ \text{OF} = 96;27^\circ \]

Therefore where DO = 6;33^\circ and the radius of the eccentrical, DA = 60^\circ,
DF = 3;54^\circ
and FO = 5;16^\circ.

\(^{45}\) Accurate computation from Ptolemy's data gives 39;10^\circ.
X 7. Correction to account for equant: 2nd opposition

And since \( DA^2 - DF^2 = FA^2 \),
\[
AF = 59;52^\circ,
\]
and, since \( QF = F\Theta \),
by addition [of \( QF \) to \( FA \)], \( QA = 65;8^\circ \)
\[
where \( NQ = 2DF = 7;48^\circ \).
\]
Hence hypotenuse [of right-angled triangle \( NAQ \)]
\[
NA = 65;36^\circ \text{ in the same units.}
\]
Therefore, where \( NA = 120^\circ, NQ = 14;16^\circ \),
and, in the circle about right-angled triangle \( ANQ \),
\[
\text{arc } NQ = 13;40^\circ
\]
\[
\therefore \angle NAQ = 13;40^\circ \text{ where 2 right angles = 360}^\circ.
\]
Again, since \( QN \) was shown to be \( 7;48^\circ \) and \( Q\Theta \) \([= 2F\Theta] \) to be \( 10;32^\circ \),
where the radius of the eccentre. \( \Theta E = 60^\circ \),
by addition, \( Q\Theta E = 70;32^\circ \) in the same units,
and hence the hypotenuse [of right-angled triangle \( QNE \)]
\[
NE \approx 71^\circ \text{ in the same units.}
\]
Therefore, where \( NE = 120^\circ, QN = 13;10^\circ \),
and, in the circle about right-angled triangle \( ENQ \),
\[
\text{arc } QN = 12;36^\circ.
\]
\[
\therefore \angle NEQ = 12;36^\circ \text{ where 2 right angles = 360}^\circ.
\]
But we found that \( \angle NAQ = 13;40^\circ \) in the same units.
Therefore, by subtraction [of \( \angle NEQ \) from \( \angle NAQ \)],
\[
\angle ANE = \begin{cases} 1;4^\circ \text{ where 2 right angles = 360}^\circ \\ 0;32^\circ \text{ where 4 right angles = 360}^\circ \end{cases}
\]
That \( 0;32^\circ \), then, is the amount of arc \( KS \) of the ecliptic.
Next, draw a similar figure containing [the part of] the diagram for the second opposition [Fig. 10.11].

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*The roundings here are particularly crude: from the immediately preceding numbers one finds \( NE = 70;57,48^\circ \), whence \( QN = 13;11,24^\circ \). Even \( NE = 71^\circ \) leads to \( QN = 13;10,59^\circ \).*
Then, since arc XZ is given as 45;13°,
\[ \angle X\Theta Z = \begin{cases} 
45;13° & \text{where 4 right angles = 360°} \\
90;26°° & \text{where 2 right angles = 360°°},
\end{cases} \]
and the vertically opposite angle D\Theta F = 90;26°° in the same units, also.
Therefore, in the circle about right-angled triangle D\Theta F,
arc DF = 90;26°
and arc F\Theta = 89;34° (supplement).
Therefore the corresponding chords
\[ \begin{align*}
&DF = 85;10° \\
&and \ F\Theta = 84;32°
\end{align*} \]
and hypotenuse D\Theta = 120°.

Therefore where D\Theta = 6;33° and the radius of the eccentre, DB = 60°,
DF = 4;39°
and F\Theta = 4;38°.
And since DB<sup>2</sup> - DF<sup>2</sup> = BF<sup>2</sup>,
FB = 59;49°,
and since FQ = F\Theta,
by addition, QB = 64;27° where NQ (= 2DF) is computed as 9;18°.
Therefore hypotenuse [of right-angled triangle NQB]
NB = 65;6° in the same units.
Therefore, where NB = 120°, NQ = 17;9°.
and, in the circle about right-angled triangle BNQ,
arc NQ = 16;26°
\[ \bar{\angle} \ NBQ = 16;26°° \text{ where 2 right angles = 360°°}. \]
Again, since NQ was shown to be 9;18°, and Q\Theta [= 2\Theta] = 9;16°,
where the radius of the eccentre, Z\Theta = 60°.
by addition, Q\Theta Z = 69;16° in the same units.
Hence hypotenuse NZ [of right-angled triangle NZQ] = 69;52°.
Therefore, where hypotenuse NZ = 120°, NQ \cong 16°.
and, in the circle about right-angled triangle NZQ,
arc NQ = 15;20°.
\[ \bar{\angle} \ NZQ = 15;20°° \text{ where 2 right angles = 360°}. \]
But we found that \[ \bar{\angle} \ NBQ = 16;26°° \text{ in the same units}. \]
Therefore, by subtraction, \[ \bar{\angle} \ BNZ = \begin{cases}
1;6°° \text{ in the same units} \\
0;33° \text{ where 4 right angles = 360°},
\end{cases} \]
That [0;33°], then, is the amount of arc LT of the ecliptic.

Now, since we found arc KS as 0;32° for the first opposition, it is clear that the first interval, taken with respect to the eccentre, will be greater than the interval of apparent motion by the sum of both arcs. [namely] 1;5°, and [hence] will contain 68;55°.

Then let [the part of the] the diagram for the third opposition be drawn [Fig. 10.12]. Now, since arc PH is given as 39;19°,
\[ \angle P\Theta H = \begin{cases} 
39;19° \text{ where 4 right angles = 360°} \\
78;38°° \text{ where 2 right angles = 360°°},
\end{cases} \]

47 Cf. arc LB on p. 490.
48 Reading \( \xi \) (with D, Ar) for \( \xi B \) (69;6) at H335.9. The correction is assured by the preceding and subsequent computations.
49 I.e. the equant: this is made explicit in XI 1 p. 515. See n.7 there.
Therefore, in the circle about right-angled triangle $D\Theta F$,
arc $DF = 78;38^\circ$
and arc $\Theta F = 101;22^\circ$ (supplement).

Therefore the corresponding chords
\[
\begin{align*}
DF &= 76;2^\circ \\
\Theta F &= 92;50^\circ
\end{align*}
\]
where hypotenuse $D\Theta = 120^\circ$.

Therefore where the distance between the centres, $D\Theta = 6;33^{\frac{1}{2}}^\circ$, and the radius of the eccentre, $DG = 60^\circ$,
\[
\begin{align*}
DF &= 4;9^\circ \\
\Theta F &= 5;4^\circ
\end{align*}
\]
And since $GD^2 - DF^2 = GF^2$,
\[
GF = 59;51^\circ.
\]
and, since $\Theta F = FQ$,
by subtraction, $GQ = 54;47^\circ$ where $NQ = 2DF$ is computed as $8;18^\circ$.

Hence hypotenuse [of right-angled triangle $NGQ$]
\[
NG = 55;25^\circ \text{ in the same units.}
\]

Therefore, where $NG = 120^\circ$, $NQ = 17;59^\circ$,
and, in the circle about right-angled triangle $GNQ$,
arc $NQ = 17;14^\circ$
\[
\therefore \angle NGQ = 17;14^\circ \text{ where 2 right angles = } 360^\circ.
\]
Again, since $NQ$ was shown to be $8;18^\circ$, and $\Theta QF = 10;8^\circ$,
where the radius of the eccentre, $\Theta H = 60^\circ$,
by subtraction, $QH = 49;52^\circ \text{ in the same units,}$
and therefore hypotenuse $NH$ [of right-angled triangle $NHQ$] = $50;33^\circ$.

Therefore, where $NH = 120^\circ$, $NQ = 19;42^\circ$,
and, in the circle about right-angled triangle $HNQ$,
arc $NQ = 18;54^\circ$.
\[
\therefore \angle NHQ = 18;54^\circ \text{ where 2 right angles = } 360^\circ.
\]
But we showed that $\angle NGQ = 17;14^\circ \text{ in the same units.}$
Therefore by subtraction, $\angle \text{GNH} = \begin{cases} 1;40^\circ \text{ in the same units.} \\ 0;50^\circ \text{ where 4 right angles } = 360^\circ \end{cases}$

That $[0;50^\circ]$, then, is the amount of arc MY of the ecliptic.

Now since we found arc LT as $0;33^\circ$ for the second opposition, it is clear that the second interval, taken with respect to the eccentric [equant], will be less than the interval of apparent motion by the sum of both arcs, [namely] $1;23^\circ$, and will [thus] contain $92;21^\circ$.

Using the ecliptic arcs thus computed for the two intervals, and, once more, the original arcs assumed for the eccentric [equant], and following the theorem demonstrated above [pp. 486-9] for such elements, by means of which we determine [the position of] the apogee and the ratio of the eccentricity, we find (not to lengthen our account by going through the same [computations in detail again]),

\begin{align*}
\text{H339} \\
\text{the distance between the centres, } DK = 11;50^\circ \text{ where the radius of the eccentric is } 60^\circ; \\
\text{the arc of the eccentric from the third opposition to the perigee, } GM = 45;33^\circ. \\
\text{Hence } \text{arc } LB = [180^\circ - (95;28^\circ + 45;33^\circ)] = 38;59^\circ \\
\text{and } \text{arc } AL = [81;44^\circ - 38;59^\circ] = 42;45^\circ.
\end{align*}

Next, starting from these [arcs] as data, we found from our demonstration for each of the oppositions [separately] the following amounts for the true size of each of the arcs in question:

\begin{align*}
\text{arc KS} & = 0;28^\circ \\
\text{arc LT, about the same} & = 0;28^\circ \\
\text{and arc MY} & = 0;40^\circ. \text{ } \text{51}
\end{align*}

We combined the [corrections] for the first and second oppositions, added the resulting $0;56^\circ$ to the ecliptic arc of the first interval, $67;50^\circ$, and got the accurate interval with respect to the eccentric as $68;46^\circ$. Again, combining the [corrections] for the second and third oppositions, and subtracting the resulting $1;8^\circ$ from the apparent motion on the ecliptic over the second interval, $93;44^\circ$, we got the accurate interval with respect to the eccentric as $92;36^\circ$.

Next, using the same procedure [as before], we determined a more accurate value for the ratio of the eccentricity and [the position of] the apogee; we found the distance between the centres, $DK = 12^\circ$ where the radius of the eccentric, $KL = 60^\circ$,

\begin{align*}
\text{arc GM of the eccentric} & = 44;21^\circ, \text{ } \text{52} \\
\text{whence, again, } \text{arc } LB & = 40;11^\circ \\
\text{and } \text{arc AL} & = 41;33^\circ.
\end{align*}

Next, we shall show by means of the same [configurations] that the observed apparent intervals between the three oppositions are found to be in agreement with the above quantities.

\begin{footnotesize}
\text{50} \text{ From Ptolemy's elements, } \Delta \lambda_1 = 81;44^\circ, \Delta \lambda_2 = 95;28^\circ, \Delta \lambda_1 = 68;55^\circ, \Delta \lambda_2 = 92;21^\circ, \text{ I compute } 2\sigma = 11;50^\circ, \text{ } \text{GM} = 45;28^\circ. \\
\text{51} \text{ From a double eccentricity of } 11;50^\circ \text{ and Ptolemy's values for arcs GM, LB and AL, I find: arc KS = 0;27,49^\circ, \text{ arc LT = 0;26,51^\circ, arc MY = 0;39,31^\circ.} \\
\text{52} \text{ From Ptolemy's elements I find: } DK = 11;59,50^\circ \approx 12^\circ, \text{ arc GM = 44;18,45^\circ \approx 44;19^\circ. Ptolemy is quite right to terminate his calculation here, since a further iteration produces a change in the eccentricity of less than 0;0,30^\circ and in the line of the apsides of less than 5'.}
\end{footnotesize}
Let there be drawn [Fig. 10.13] the diagram for the first opposition, but with only eccentric EZ, on which the epicycle centre is always carried, drawn in. Then

\[ \angle A\Theta \epsilon E = 41;33^\circ \text{ where 4 right angles} = 360^\circ. \]

so where 2 right angles = 360°,
\[ \angle A\Theta \epsilon E = 83;6^\circ = \angle D\Theta F \text{ (vertically opposite)}. \]

Therefore, in the circle about right-angled triangle D\Theta F,
\[ \text{arc } DF = 83;6^\circ \]
and arc \( F\Theta = 96;54^\circ \) (supplement).

Therefore the corresponding chords
\[ \text{arc } DF = 79;35^\circ \]
and \( F\Theta = 89;50^\circ \).

Therefore where \( D\Theta = 6^\circ \) and hypotenuse [of right-angled triangle DAF] \( DA = 60^\circ \).
\[ \text{arc } DF = 3;58^\circ \]
and \( F\Theta = 4;30^\circ \).

And since \( DA^2 - DF^2 = FA^2 \),
\[ \text{FA} = 59;50^\circ \text{ in the same units}. \]

Furthermore, since \( F\Theta = FQ \) and \( NQ = 2DF \),
by addition, \( AQ = 64;20^\circ \) where \( NQ = 7;57^\circ \).

Hence hypotenuse [of right-angled triangle NAQ] \( NA = 64;52^\circ \) in the same units.

Therefore where \( NA = 120^\circ \), \( NQ = 14;44^\circ \),
and, in the circle about right-angled triangle ANQ,
\[ \text{arc } NQ = 14;6^\circ. \]

\[ \therefore \angle NAQ = \begin{cases} 14;6^\circ \text{ where 2 right angles} = 360^\circ \\ 7;3^\circ \text{ where 4 right angles} = 360^\circ. \end{cases} \]

But \( \angle A\Theta \epsilon E = 41;33^\circ \) in the same units.
Therefore, by subtraction, the angle of the apparent position, $\angle ANE = 34;30^\circ$.
This is the amount by which the planet was in advance of the apogee at the first opposition.

Let a similar diagram [Fig. 10.14] be drawn again for the second opposition. Then the angle of the mean position of the epicycle,

$\angle B\Theta E = 40;11^\circ$ where 4 right angles = 360°,
so where 2 right angles = 360°°,
$\angle B\Theta E = 80;22^\circ° = \angle Q\Theta N$ (vertically opposite).
Therefore, in the circle about right-angled triangle $D\Theta F$,
arc $DF = 80;22^\circ$
and arc $F\Theta = 99;38^\circ$ (supplement).

Therefore the corresponding chords
$DF = 77;26''$
and $F\Theta = 91;41''$
where hypotenuse $D\Theta = 120^\circ$.
Therefore where $D\Theta = 6^\circ$ and hypotenuse [of right-angled triangle $DBF$] $DB = 60^\circ$,
$DF = 3;52^\circ$
and $F\Theta = 4;35^\circ$.
And since $DB^2 - DF^2 = BF^2$,
$BF = 59;53^\circ$ in the same units.

And, by the same argument [as before],

since $F\Theta = FQ$, and $NQ = 2$ $DF$,
by addition, $BQ = 64;28^\circ$ where $NQ = 7;44^\circ$.
Hence hypotenuse [of right-angled triangle $BNQ$] $BN = 64;56^\circ$ in the same units.

Reading κατά ταυτά (as D. κατά τα αὐτά, Ar) for κατά ταυτά ('according to this') at H342.23.
Therefore, where hypotenuse \( BN = 120^\circ \), \( NQ = 14;19^\circ \), and, in the circle about right-angled triangle \( BNQ \),
\[
\text{arc } NQ = 13;42^\circ .
\]
\[
\therefore \angle NBQ = \begin{cases} 
13;42^\circ & \text{where 2 right angles} = 360^\circ \\
6;51^\circ & \text{where 4 right angles} = 360^\circ .
\end{cases}
\]
But \( \angle B\theta E = 40;11^\circ \) in the same units.

Therefore, by subtraction, the angle of apparent position,
\[
\angle ENB = 33;20^\circ \text{ in the same units.}
\]

That \( [33;20^\circ] \), then, is the amount by which the planet, in its apparent motion, was to the rear of the apogee at the second opposition. And we showed that at the first opposition it was \( 34;30^\circ \) in advance of the apogee. Therefore the total distance [in apparent motion] from first to second opposition comes to \( 67;50^\circ \), in agreement with what we derived from the observations [p. 485].

Let the diagram for the third opposition be drawn in the same way [Fig. 10.15]. In this case the angle of the mean position of the epicycle,

\[
\angle G\theta Z = \begin{cases} 
44;21^\circ & \text{where 4 right angles} = 360^\circ \\
88;42^\circ & \text{where 2 right angles} = 360^\circ.
\end{cases}
\]

Therefore, in the circle about right-angled triangle \( \Delta \theta F \),
\[
\text{arc } DF = 88;42^\circ \\
\text{and arc } F\theta = 91;18^\circ \text{ (supplement)}.
\]

Therefore the corresponding chords
\[
DF = 83;53^\circ \\
\text{and } F\theta = 85;49^\circ \text{ where hypotenuse } D\theta = 120^\circ .
\]

\(^{34}\) \( 7;44 \times 120 = 14;17,30 \), but if one carries out the above computations to 2 fractional sexagesimal places, one finds \( NQ = 14;18,41^\circ \). As often, Ptolemy computed with greater accuracy than the text implies.
Therefore where \( D\Theta = 6^\circ \) and the radius of the eccentre, \( DG = 60^\circ \),
\[
DF = 4;11^\circ
\]
and \( F\Theta = 4;17^\circ \).
And since \( DG^2 - DF^2 = GF^2 \),
we find that \( GF = 59;51^\circ \) in the same units.
Furthermore, since \( F\Theta = FQ \) and \( NQ = 2DF \),
we find by subtraction that \( QG = 55;34^\circ \) where \( NQ = 8;23^\circ \).
Hence we find that hypotenuse [of right-angled triangle \( GNQ \)]
\[
GN = 56;12^\circ \text{ in the same units.}
\]
Therefore, where hypotenuse \( GN = 120^\circ \), \( NQ = 17;55^\circ \),
and, in the circle about right-angled triangle \( GNQ \),
\[
\text{arc } NQ = 17;10^\circ .
\]

\[ \therefore \angle \Theta GN = \begin{cases} 17;10^\circ \text{ where 2 right angles } = 360^\circ \\ 8;35^\circ \text{ where 4 right angles } = 360^\circ. \end{cases} \]

But \( \angle G\Theta Z = 44;21^\circ \) in the same units.
That \( [52;56^\circ] \), then, is the amount by which the planet was in advance of the perigee at the third opposition. But we also showed that at the second opposition it was \( 33;20^\circ \) to the rear of the apogee. So we have found \( 93;44^\circ \) between the second and third oppositions, computed by subtraction of the sum of \( 52;56^\circ \) and \( 33;20^\circ \) from \( 180^\circ \), in agreement with the amount observed for the second interval [p. 485].
Furthermore, since the planet, when viewed at the third opposition along line \( GN \), had a longitude of \( 2;34^\circ \) according to our observation [p. 484], and angle \( GNZ \) at the centre of the ecliptic was shown to be \( 52;56^\circ \), it is clear that the perigee of the eccentre, at point \( Z \), had a longitude of \( [2;34^\circ + 52;56^\circ =] 25;30^\circ \), while the apogee was diametrically opposite in \( 25;30^\circ \).

And if [see Fig. 10.16] we draw Mars' epicycle \( KLM \) on centre \( G \) and produce line \( \Theta GM \), we will have, for the moment of the third opposition:

- mean motion of the epicycle counted from apogee of the eccentre: \( 135;39^\circ \) (for its supplement, \( \angle G\Theta Z \), was shown to be \( 44;21^\circ \));
- mean motion of the planet from the epicycle apogee \( M \) (i.e. arc \( MK \)): \( 171;25^\circ \) (for \( \angle \Theta GN \) was shown to be \( 8;35^\circ \) [above], and since it is an angle at the centre of the epicycle, the arc \( KL \) from the planet at \( K \) to the perigee at \( L \) is also \( 8;35^\circ \), hence the supplementary arc from the apogee \( M \) to the planet at \( K \) is, as already stated, \( 171;25^\circ \)).

Thus we have demonstrated, among other things, that at the moment of the third opposition, i.e. in the second year of Antoninus, Epiph 12/13 in the Egyptian calendar, 2 equinoctial hours before midnight, the mean positions of the planet Mars were:

- in longitude (so-called) from the apogee of the eccentre: \( 135;39^\circ \)
- in anomaly from the apogee of the epicycle: \( 171;25^\circ \).

Q.E.D.

\[ ^{55} \text{Reading } \Theta GM \text{ (with al-Hājījā) for } \Theta G \text{ (\( \Theta G \)) at H345,22.} \]
Our next task is to demonstrate the ratio of the size of the epicycle. For this purpose we took an observation which we obtained by sighting [with the astrolabe] about three days after the third opposition, that is, in the second year of Antoninus, Epiphi [XI] 15/16 in the Egyptian calendar [139 May 30/31], 3 equinoctial hours before midnight. [That was the time.] for the twentieth degree of Libra [i.e. \( \pm 19^\circ-20^\circ \)] was culminating according to the astrolabe, while the mean sun was in \( \Pi 5;27^\circ \) at that moment. Now when the star on the ear of wheat [Spica] was sighted in its proper position [on the instrument], Mars was seen to have a longitude of \( \varphi 1^\circ5^\circ \). At the same time it was observed to be the same distance (1.36°) to the rear of the moon’s centre. Now at that moment the moon’s position was as follows:

- mean longitude \( \varphi 4;20^\circ \)
- true longitude \( m, 29;20^\circ \)
- apparent longitude \( \varphi 0^\circ,^\text{58} \)

So from these considerations too the longitude of Mars was \( \varphi 1;36^\circ \), in agreement with the [astrolabe] sighting.

Hence, clearly, it was 53;54° in advance of the perigee.\(^{59}\)

\(^{56}\) On the method employed here see HAMA 179–80, Pedersen 283–6.

\(^{57}\) These positions are computed (accurately), not for 9 p.m., but for 8;37 p.m., i.e. Ptolemy has applied the equation of time with respect to epoch as \(-23\) minutes (it should be about \(-25\) mins.)

\(^{58}\) Literally ‘at the beginning of Sagittarius’.

\(^{59}\) Which was in \( \varphi 25;30^\circ \) (X 7 p. 498).
And the interval between the third opposition and this observation comprises
in longitude about $1^{\circ}32''$

in anomaly about $1^{\circ}21''$.

If we add the latter to the [mean] positions at the opposition in question as demonstrated above, we get, for the moment of this observation;

- distance of Mars in longitude from the apogee of the eccentre: $137;11^\circ$
- distance in anomaly from the apogee of the epicycle: $172;46^\circ$.

With these elements as data, let [Fig. 10.17] the eccentric circle carrying the centre of the epicycle be ABG on centre D and diameter ADG, on which the centre of the ecliptic is taken at E, and the point of greater eccentricity [i.e. the equant] at Z. Draw the epicycle HΘK on centre B, draw ZKBH, EΘB and DB, and drop perpendiculars EL and DM from points D and E on to ZB. Let the planet be situated at point N on the epicycle, join EN, BN, and drop perpendicular BX from B on to EN produced.

Then, since the planet’s distance from the apogee of the eccentre is $137;11^\circ$,

$$\angle BZG = [180^\circ - 137;11^\circ] = \begin{cases} 42;49^\circ & \text{where 4 right angles } = 360^\circ \\ 85;38^\circ & \text{where 2 right angles } = 360^\circ. \end{cases}$$

Therefore, in the circle about right-angled triangle DZM,

- arc DM = $85;38^\circ$
- and arc ZM = $94;22^\circ$ (supplement).

Therefore the corresponding chords

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60 These mean motions also agree better with an interval of $2^h 22^m 37''$ than with one of $2^h 23^m$ (see n.57).

61 Reading κατά τὴν ὑποκειμένην ἄκρονυκτον (with D) for κατά τὴν ὑποκειμένην γ’ ἄκρονυκτον (‘at the third opposition, which is the one in question’) at H348.9-10.
Therefore where the distance between the centres, $DZ = 6^\circ$, and the radius of the eccentre, $DB = 60^\circ$,

$$DM = 81;34^p$$

and $ZM = 88;1^p$ where hypotenuse $DZ = 120^\circ$.

And since $DB^2 - DM^2 = BM^2$,

$$BM = 59;52^p$$ in the same units.

Similarly, since $ZM = ML$, and $EL = 2DM$,

by subtraction, $BL = 55;28^p$ and $EL = 8;10^p$ in the same units.

Hence hypotenuse [of right-angled triangle EBL] $EB = 56;4^p$.

Therefore, where $EB = 120^\circ$, $EL = 17;28^\circ$, and, in the circle about right-angled triangle BEL,

arc $EL = 16;44^\circ$

$$\therefore \angle ZBE = 16;44^\circ$$ where 2 right angles = $360^\circ$.

Furthermore, the apparent distance of the planet Mars in advance of the perigee $G$,

$$\angle GEX$$ is given as $\left\{ \begin{array}{l} 53;54^p \text{ where 4 right angles = } 360^\circ \\ 107;48^\circ \text{ where 2 right angles = } 360^\circ \end{array} \right.$$

And, in the same units, $\angle ZBE = 16;44^\circ$ (shown above),

and $\angle GZB = 85;38^\circ$ (given),

so $\angle GEB = \angle ZBE + \angle GZB = 102;22^\circ$.

Therefore, by subtraction [of $\angle GEB$ from $\angle GEX$],

$\angle BEX = 5;26^\circ$ in the same units,

and, in the circle about right-angled triangle BEX

arc $BX = 5;26^\circ$.

$\therefore BX = 5;41^p$ where hypotenuse $EB = 120^\circ$.

Therefore where $EB$, as was shown, = $56;4^p$,

and the radius of the eccentre is $60^\circ$,

$BX = 2;39^p$.

Similarly, since the distance of point $N$ from the epicycle apogee $H$ was $172;46^\circ$, and [hence], from the perigee $K$, $7;14^\circ$,

$$\angle KBN$$ is given as $\left\{ \begin{array}{l} 7;14^p \text{ where 4 right angles = } 360^\circ \\ 14;28^p \text{ where 2 right angles = } 360^\circ \end{array} \right.$$

But $\angle KB\Theta$ was found as $16;44^\circ$ in the same units.

Therefore, by subtraction, $\angle NB\Theta = 2;16^\circ$,

and, by addition, [of $\angle NB\Theta$ to $\angle BEX$], $\angle XNB = 7;42^\circ$.

Therefore, in the circle about right-angled triangle $BNX$,

arc $XB = 7;42^\circ$

and $BX = 8;3^p$ where hypotenuse $BN = 120^\circ$.

Therefore where $BX = 2;39^p$ and the radius of the eccentre = $60^\circ$,

the epicycle radius $BN \approx 39;30^p$.

Therefore the ratio of the radius of the eccentre to the radius of the epicycle is $60 : 39;30$.

Q.E.D.
In order to correct the periodic mean motions we took one of the ancient observations, in which it is declared that in the 13th year of the calendar of Dionysius, Aigon 25, at dawn, Mars seemed to have occulted the northern [star in the] forehead of Scorpionius. The moment of this observation is in the 52nd year from the death of Alexander, i.e. in the 476th year from Nabonassar, Athyri [III] 20/21 in the Egyptian calendar [-271 Jan. 17/18], dawn. At this time we find the longitude of the mean sun as $23;54^\circ$; and the longitude of the star on the northern part of the forehead of Scorpionius was observed in our time as $65^\circ$. So, since the 409 years from the observation to [the beginning of] the reign of Antoninus produce about $4;5^\circ$ of shift in the position of the fixed stars, at the time of the observation in question the longitude of the star must have been $21^\circ$, and, obviously, the longitude of the planet Mars was the same. In the same way, since the longitude of the apogee of Mars in our time, that is at the beginning of the reign of Antoninus, was $25;30^\circ$, it must have been $21;25^\circ$ at the observation. Thus it is clear at that moment the apparent distance of the planet from its apogee was $100;50^\circ$, while the distance of the mean sun from the same apogee was $182;29^\circ$, and, obviously, $2;29^\circ$ from [Mars’] perigee.

With the above elements as data, let [Fig. 10.18] the eccentric circle carrying the epicycle centre be ABG on centre D and diameter ADG, on which the centre of the ecliptic is taken at E, and the point of the greater eccentricity [i.e. the equant] at Z. Draw the epicycle H0 on centre B, draw ZBH and DB, and drop perpendicular ZK from Z on to DB. Let the planet be situated at point $\Theta$ of the epicycle; join B$\Theta$ and draw EL parallel to it from E; then it is clear from our earlier demonstration [X 6, pp. 480-3] that the mean position of the sun will be seen along EL. Join E$\Theta$, and on to it drop perpendiculars DM and BN from points D and B. Also, drop perpendicular DX from D on to BN, so that the figure DMN X is a rectangular parallelogram.

Then, since the angle representing the apparent distance of the planet from the apogee,

\[ \angle A\Theta E = 100;50^\circ \text{ where } 4 \text{ right angles } = 360^\circ, \]

and the angle representing the mean motion of the sun [counted from the perigee],

\[ \angle GEL = 2;29^\circ \text{ in the same units}, \]

\[ \angle \Theta EL = \angle B\Theta E = [180^\circ - 100;50^\circ + 2;29^\circ = ] \]

\[ \begin{align*}
\text{81;39° where } 4 \text{ right angles } &= 360^\circ \\
\text{163;18°° where } 2 \text{ right angles } &= 360°°.
\end{align*} \]

\[ \text{On the method employed here see HAMA 180-2.} \]

\[ \text{Böckh (Sonnenkreise 294), in agreement with Lepsius, changed this to 'Aigon 26' on the basis of his reconstruction of Dionysius' calendar. He was followed by Manitius. The uncertainties are too many to justify emendation by a single day. It may be pertinent that the occultation [if there was one] must, according to modern calculations, have occurred two days earlier than the date Ptolemy gives.} \]

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Therefore, in the circle about right-angled triangle $B\Theta N$,  
$\text{arc } BN = 163;18^\circ$
and $BN = 118;43^\circ$ where hypotenuse $B\Theta = 120^\circ$.

Therefore where the radius of the epicycle, $B\Theta = 39;30^\circ$,
and the distance between the centres, $ED = 6^\circ$,
$BN = 39;3^\circ$.

Furthermore, since
$\angle AE\Theta = \begin{cases} 100;50^\circ & \text{where 4 right angles = 360°} \\ 201;40^\circ & \text{where 2 right angles = 360°} \end{cases}$
and therefore its supplement, $\angle DEM = 158;20^\circ$ in the same units,
in the circle about right-angled triangle $DEM$,  
$\text{arc } DM = 158;20^\circ$
and $DM = 117;52^\circ$ where hypotenuse $DE = 120^\circ$.

Therefore where $DE = 6^\circ$ and $BN$, as was shown, is $39;3^\circ$,
$DM = NX = 5;54^\circ$.

So, by subtraction, $BX = 33;9^\circ$ where the radius of the eccentre, $BD = 60^\circ$.

Therefore where hypotenuse [of right-angled triangle $BDX$] $BD = 120^\circ$,
$BX = 66;18^\circ$,
and, in the circle about right-angled triangle $BDX$,  
$\text{arc } BX = 67;4^\circ$.

$\therefore \angle BDX = 67;4^\circ$ where 2 right angles = 360°,
and, by addition [of right angle $XDM$], $\angle BDM = 247;4^\circ$.

But, since $\angle DEM$ was shown to be $158;20^\circ$,
$\angle EDM \ [= \text{a right angle minus } \angle DEM] = 21;40^\circ$ in the same units.

Therefore, by subtraction, $\angle BDE$ is computed as $225;24^\circ$,
and its supplement, $\angle BDA = 134;36^\circ$ in the same units.
Therefore, in the circle about right-angled triangle DZK,
\[ \text{arc } ZK = 134;36° \]
and \[ \text{arc } DK = 45;24° \text{ (supplement)} \].

Therefore the corresponding chords
\[ ZK = 110;42° \]
and \[ DK = 46;18° \] where hypotenuse \[ DZ = 120° \].

Therefore where \[ DZ = 6° \] and the radius of the eccentre, \[ DB = 60° \],
\[ ZK = 5;32° \] and \[ DK = 2;19° \].

And, by subtraction, \[ KB = 57;41° \].

Hence hypotenuse [of right-angled triangle BZK] \[ BZ \approx 57;57° \] in the same units.

Therefore, where \[ BZ = 120° \], \[ ZK = 11;28° \],
and, in the circle about right-angled triangle BKZ,
\[ \text{arc } ZK = 10;58° \]

\[ \angle ZBD = 10;58° \text{ where 2 right angles} = 360° \].
But \[ \angle BDA = 134;36° \text{ in the same units} \].

Therefore, by addition, \[ BZA = \left\{ \begin{array}{ll}
145;34° \text{ in the same units} \\
72;47° \text{ where 4 right angles} = 360° \end{array} \right. \]

Therefore the mean position in longitude of the planet (i.e. of B, the centre of the epicycle) at the moment of the observation in question was \[ 72;47° \] from the apogee.\(^{63}\) Hence its [mean] longitude was \[ \left[ 21;25° + 72;47° \right] \approx 4;12° \].

And \[ \angle GEL \] is given as \[ 2;29° \].

and \[ \angle GEL \] plus the two right angles of semi-circle \[ ABG \] equals the sum of the mean longitude, \[ \angle AZB \], and the [mean] anomaly (i.e. the [mean] motion of the planet on the epicycle), \[ \angle HBO \].

So, by subtraction [of \[ \angle AZB \] from \[ \angle GEL + 180° \]], we get
\[ \angle HBO = 109;42° \].

Therefore the distance of the planet in anomaly from the apogee of the epicycle at that same moment of the observation was the above \[ 109;42° \], which was what we had to determine.

Now we had [already] shown [X 7, p. 498] that at the moment of the third opposition the distance [of Mars] in anomaly from the apogee of the epicycle was \[ 171;25° \]. Therefore, in the interval between the observations, which comprises \[ 410 \text{ Egyptian years and 231} \frac{1}{2} \text{ days (approximately)} \], the planet moved \[ 61;43° \] beyond \[ 192 \text{ complete revolutions} \]. That is practically the same increment [in anomaly] which we find from the tables for Mars’ mean motion we constructed. For our [mean] daily motion was derived from these very data, by dividing the number of degrees obtained from the complete revolutions plus the increment by the number of days computed from the interval between the two observations.\(^{66}\)

\(^{63}\) Through accumulated small computational and rounding errors Ptolemy’s result is 3° too great (accurate is 72;43.50°). This would have some effect on the resulting mean motion in anomaly.

\(^{66}\) On the actual derivation of the mean motion in anomaly, which remains mysterious in the case of Mars, see Appendix C.
Furthermore, the interval from the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, until the above observation [p. 502], is 475 Egyptian years and approximately 79½ days, and that interval comprises increments of 180;40° in longitude and 142;29° in anomaly.\textsuperscript{67} If, then, we subtract the latter from the respective positions for both [longitude and anomaly] at the observation, as given above [p. 504], namely, \(\cong 4;12°\) in longitude and 109;42° in anomaly, we get the following epoch positions for the periodic motion of Mars at noon Thoth 1 in the Egyptian calendar, first year of Nabonassar:

- longitude \(\cong 3;32°\)
- anomaly 327;13° from the epicycle apogee.

Similarly, since, for the shift of the apogee in 475 years one gets by computation 41°, and the apogee of Mars was in \(\cong 21;25°\) at the observation, it is obvious that, at the above moment of epoch, longitude of the apogee was \(\cong 16;40°\).

\textsuperscript{67} The increments over 475\textsuperscript{1/2} years are (to the nearest minute) 180;39° in longitude and 142;28° in anomaly. To get Ptolemy's figures one needs about 1 hour more of motion. Perhaps he took 'dawn' as 6:30 a.m. at Dionysius' observation. But in that case the interval between Dionysius' observation and his own (p. 504) should have been less.
Book XI

1. [Demonstration of the eccentricity of Jupiter]

Now that we have established the periodic motions, anomalies and epochs of the planet Mars, we shall next deal with those of Jupiter in the same way. Once again, we first take, to demonstrate [the position of] the apogee and [the ratio of] the eccentricity, three oppositions [in which Jupiter is] directly opposite the mean sun.

[1] We observed the first of these by means of the astrolabe instrument in the seventeenth year of Hadrian, Epiphi [XI] 1/2 in the Egyptian calendar [133 May 17/18], 1 hour before midnight, in α 23:11°;

[2] the second in the twenty-first year of Hadrian, Phaophi [II] 13/14 [136 Aug. 31 - Sept. 1], 2 hours before midnight, in Χ 7:54°;


For the two intervals, that from the first to the second opposition comprises:

[in time] 3 Egyptian years 106 days 23 hours
and in apparent motion of the planet 104:43°;
while that from the second to the third opposition comprises:

[in time] 1 Egyptian year 37 days 7 hours

By computation we find the mean motion in longitude

for the first interval: 99:55°

From these intervals, following the methods expounded for Mars, we carried out the demonstration of what we proposed to determine; first of all as if there were, again, only one eccentric. The demonstration is as follows.

Let [Fig. 11.1] the eccentric be ABG, on which point A is taken as the position of the epicycle centre at the first opposition, B that of the second opposition, and G that of the third. Within the eccentric ABG take D as the centre of the ecliptic, join AD, BD and GD, produce GD to E and draw AE, EB and AB, and drop perpendiculars EZ and EH from E on to AD and BD, and perpendicular AG from A on to EB.

Then, since arc BG of the eccentric is given as subtending 36:29° of the ecliptic, the angle at the centre of the ecliptic,

\[ \angle BDG \ (= \angle EDH) = \begin{cases} 36:29° \text{ where 4 right angles = } 360°, \\ 72:58° \text{ where 2 right angles = } 360°. \end{cases} \]

1 The procedure for Jupiter and Saturn is identical to that for Mars (except that fewer iterations are required). The reader is referred to the notes on X 7-9 for elucidations of points of detail.
Therefore, in the circle about right-angled triangle EDH,

\[ \text{arc } EH = 72;58^\circ \]

and \( EH = 71;21^\circ \) where hypotenuse \( DE = 120^\circ \).

Similarly, since \( \text{arc } BG = 33;26^\circ \),

the angle [subtended by it] at the circumference,

\[ \angle BEG = 33;26^\circ \] where 2 right angles = 360°;

and, by subtraction [of \( \angle BEG \) from \( \angle EDH \)],

\[ \angle EBH = 39;32^\circ \] in the same units.

Therefore, in the circle about right-angled triangle BEH,

\[ \text{arc } EH = 39;32^\circ \]

and \( EH = 40;35^\circ \) where hypotenuse \( BE = 120^\circ \).

Therefore where \( EH \), as we showed, is 71;21°, and \( ED = 120^\circ \),

\[ BE = 210;58^\circ \].

Furthermore, since the whole arc \( ABG \) of the eccentric is given as subtending 141;12° of the ecliptic (the sum of both intervals [104;43° and 36;29°]), the angle at the centre of the ecliptic,

\[ \angle ADG = \begin{cases} 141;12^\circ & \text{where 4 right angles = 360°} \\ 282;24^\circ & \text{where 2 right angles = 360°} \end{cases} \]

and its complement, \( \angle ADE = 77;36^\circ \) in the same units.

Therefore, in the circle about right-angled triangle DEZ,

\[ \text{arc } EZ = 77;36^\circ \]

and \( EZ = 75;12^\circ \) where hypotenuse \( DE = 120^\circ \).

Similarly, since arc \( ABG \) of the eccentric is, by addition [of 99;55° + 33;26°],

133;21°, the angle [subtended by it] at the circumference,

\[ \angle AEG = 133;21^\circ \] where 2 right angles = 360°.

But \( \angle ADE \) was found to be 77;36° in the same units.

Therefore the remaining angle [in triangle EAD],

\[ \angle EAZ = 149;3^\circ \] in the same units.
Therefore, in the circle about right-angled triangle AEZ,
\[ \text{arc } EZ = 149;3^\circ \]
and \( EZ = 115;39'' \) where hypotenuse \( EA \) is \( 120'' \).
Therefore where \( EZ \), as was shown, is \( 75;12'' \), and \( ED \) is given as \( 120'' \),
\[ \text{EA} = 78;2'' \]
Furthermore, since \( \text{arc } AB \) of the eccentre is \( 99;55'' \), the angle [subtended by it] at the circumference,
\[ \angle AEB = 99;55''^\circ \text{ where 2 right angles} = 360''^\circ \]
Therefore, in the circle about right-angled triangle \( AE\Theta \),
\[ \text{arc } A\Theta = 99;55'' \]
and \( \text{arc } E\Theta = 80;5'' \) (supplement).
Therefore the corresponding chords
\[ A\Theta = 91;52'' \text{ where hypotenuse } EA = 120'' \]
and \( E\Theta = 77;12'' \).
Therefore where \( AE \), as was shown, is \( 78;2'' \), and \( DE \) is given as \( 120'' \),
\[ \text{A}0 = 59;44'' \]
and \( \text{E}0 = 50;12'' \).
But the whole line \( EB \) was shown to be \( 210;58'' \) in the same units.
So, by subtraction, \( \text{O}B = 160;46'' \) where \( A\Theta = 59;44'' \).
\[ \text{And } O\Theta^2 = 25845;55, \]
\[ \Theta A^2 = 3568;4, \]
so \( \Theta B^2 + \Theta A^2 = AB^2 = 29413;59. \)
\[ \therefore \text{AB} = 171;30'' \text{ where ED is } 120'' \text{ and EA is } 78;2''. \]
Moreover, where the diameter of the eccentre is \( 120'' \),
\[ \text{AB} = 91;52'' \text{ (for it subtends an arc of } 99;55'' \), \]
Therefore where \( AB = 91;52'' \) and the diameter of the eccentre is \( 120'' \),
\[ \text{ED} = 64;17'' \]
and \( \text{EA} = 41;47'' \).
Therefore arc \( EA \) of the eccentre equals \( 40;45'' \),
and the whole arc \( EABG \) \( = 40;45'' + 133;21'' \) \( = 174;6'' \).
Hence \( EDG = 119;50'' \text{ where the diameter of the eccentre is } 120'' \).
Now segment \( EABG \) is less than a semi-circle, so the centre of the eccentre will fall outside it. Let it, then, be at \( K \) [see Fig. 11.2], and draw through \( K \) and \( D \) the diameter through both centres. \( LKDM \), and let the perpendicular from \( K \) to \( GE \) be produced as \( KXX \).
Then, where diameter \( LM = 120'' \),
the whole line \( EG \) was shown to be \( 119;50'' \), and \( ED \) to be \( 64;17'' \);
so, by subtraction, \( GD = 55;33'' \) in the same units.
So, since \( ED.DG = LD.DM. \)
\[ LD.DM = 3570;56'' \text{ where diameter } LM = 120''. \]
But \( LD.DM + DK^2 = LK^2 \) (i.e. the square on half the diameter).
Therefore, if we subtract \( (LD.DM) \), i.e. \( 3570;56 \), from the square on half the diameter, i.e. \( 3600 \), the remainder will be the square on \( DK \),
i.e. \( DK^2 = 29;4. \)
Therefore the distance between the centres, \( DK \approx 5;23'' \)
where the radius of the eccentre, \( KL = 60'' \).
\[ ^3 \text{Because of an accumulation of rounding errors this should be } 5;20''. \]
Furthermore, since

\[ GN = \frac{1}{4} GE = 59;55^\circ \text{ where diameter } LM = 120^\circ, \]

and GD was shown to be 55;33^\circ in the same units.

by subtraction, \( DN = 4;22^\circ \text{ where } DK = 5;23^\circ. \)

Therefore where hypotenuse [of right-angled triangle DKN] \( DK = 120^\circ, \)

\[ DN = 97;20^\circ, \]

and, in the circle about right-angled triangle DKN,

\[ \text{arc } DN = 108;24^\circ. \]

\[ \therefore \angle DKN = \begin{cases} 108;24^\circ & \text{where } 2 \text{ right angles } = 360^\circ \\ 54;12^\circ & \text{where } 4 \text{ right angles } = 360^\circ. \end{cases} \]

And since DKN is an angle at the centre of the eccentre,

\[ \text{arc } MX = 54;12^\circ \text{ also.} \]

But the whole arc GMX, which is \( \frac{1}{4} \) arc GXE, equals 87;3^\circ.

Therefore, by subtraction, the arc from the perigee to the third opposition,

\[ \text{arc } MG = 32;51^\circ. \]

And clearly, since the interval BG is given as 33;26^\circ,

by subtraction, we find the arc from the second opposition to the perigee,

\[ \text{arc } BM = 0;35^\circ; \]

and since the interval AB is given as 99;55^\circ,

by subtraction [of (arc AB + arc BM) from 180^\circ], we find the arc from the apogee to the first opposition,

\[ \text{arc } LA = 79;30^\circ. \]

\(^3\) Ptolemy's accumulation of rounding errors has led to the considerable discrepancy of 1° from the accurate result, 32;21°.

\(^4\) The smallness of the corrections for this and the next opposition shows that these oppositions have been badly chosen. To display the greatest difference between the simple eccentric and equant models, all three oppositions should be near the octants (as they are for Mars).
Now if it were this eccentre on which the epicycle centre is carried, the above quantities would be sufficiently accurate to use. However, since, according to our hypothesis, [the epicycle centre] moves on a different circle, namely the circle described with centre at the point bisecting DK and with radius KL, we must once again, as we did for Mars, first calculate the differences which result in the apparent intervals [i.e. the arcs of the ecliptic between the oppositions]: we must show what the sizes of these differences would be (taking the above ratio for the eccentricity as approximately correct), if the epicycle centre were carried, not on the second eccentre, but on the first eccentre [i.e. the equant], which produces the ecliptic anomaly, i.e. the one drawn on centre K.

Then [see Fig. 11.3] let the eccentre carrying the epicycle centre be LM on centre D, and the eccentre of the planet’s mean motion be NX on centre Z.

Fig. 11.3

equal to LM. Draw the diameter through the centres, NLM, and take on it the centre of the ecliptic E. Let the epicycle centre be situated, first, at A, for the first opposition. Draw DA, EA, ZAX and EX, and drop perpendiculas DH and EΘ from D and E on to AZ produced.

Then, since the angle of mean motion in longitude, $\angle NZX$, was shown to be $79;30^\circ$ where 4 right angles = $360^\circ$, the angle vertically opposite to it,

$$\angle DZH = \begin{cases} 
79;30^\circ \text{ where 2 right angles} = 360^\circ \\
159^\circ \text{ where 2 right angles} = 360^\circ 
\end{cases}$$

Therefore, in the circle about right-angled triangle DZH,

arc $DH = 159^\circ$

and arc $ZH = 21^\circ$ (supplement).
XI 1. Correction to account for equant: 2nd opposition

Therefore the corresponding chords

\[
\begin{align*}
\text{DH} &= 117;59'' \quad \text{where hypotenuse } DZ = 120°, \\
\text{and } ZH &= 21;52''
\end{align*}
\]

Therefore where \( DZ (= \frac{1}{2}EZ) \approx 2;42° \) and the radius of the eccentric, \( DA = 60°, \)
\[
\begin{align*}
\text{DH} &= 2;39'' \\
\text{and } ZH &= 0;30''.
\end{align*}
\]

And since \( DA^2 - DH^2 = AH^2, \)
\[
\text{AH} = 59;56'' \text{ in the same units.}
\]

Similarly, since \( ZH = H\Theta, \) and \( \Theta = 2DH, \)
by addition, \( \Theta = 60;26° \text{ where } Z\Theta = 5;18°, \)
and hence hypotenuse \([\text{of right-angled triangle } \triangle AE\Theta] \)
\[
\text{AE} = 60;40'' \text{ in the same units.}
\]

Therefore, where \( AE = 120°, \Theta = 10;29°, \)
and, in the circle about right-angled triangle \( \triangle AE\Theta, \)
\[
\text{arc } \Theta = 10;1°.
\]

\( \therefore \angle E\Theta = 10;1° \) where 2 right angles = 360°°.

Furthermore, where \( E\Theta = 5;18°, \)
the radius of the eccentric, \( ZX = 60° \) and \( Z\Theta (= 2ZH) = 1°, \)
(hence, obviously, by addition, \( X\Theta = 61°). \)

So we find hypotenuse \([\text{of right-angled triangle } \triangle E\Theta X]\) \( EX \) as 61;14° in the same units.

Therefore, where \( EX = 120°, \Theta = 10;23°, \)
and, in the circle about right-angled triangle \( \triangle E\Theta X, \)
\[
\text{arc } \Theta = 9;55°.
\]

\( \therefore \angle E\Theta = 9;55° \) where 2 right angles = 360°°.

But we showed that \( \angle E\Theta = 10;1° \) in the same units.

Therefore, by subtraction, the angle of the difference in question.
\[
\angle AEX = \begin{cases} 
0;6° \text{ where } 2 \text{ right angles } = 360°° \\
0;3° \text{ where } 4 \text{ right angles } = 360°
\end{cases}
\]

But at the first opposition the planet, viewed along the line \( EA, \) had an apparent longitude of \( 23;11°. \) Thus it is clear that, if the epicycle centre were carried, not on eccentric \( LM, \) but on \( [\text{eccentric}] NX, \) it would have been at point \( X \) on that eccentric, and the planet would have appeared along line \( EX, \)
differing by \( 0;3° \) [from the actual position], and thus would have had a longitude of \( 23;14°. \)

Let the diagram for the second opposition be drawn, again with a similar figure \([\text{Fig. 11.4}],\)[ with the epicycle centre] depicted as a little in advance of the perigee.

Then, since \( \text{arc } XN \text{ of the eccentric was shown } [\text{p. 510, arc } BM] \) to be \( 0;35°, \)
\[
\angle XZN = \begin{cases} 
0;35° \text{ where } 4 \text{ right angles } = 360° \\
1;10° \text{ where } 2 \text{ right angles } = 360°°
\end{cases}
\]

Therefore, in the circle about right-angled triangle \( \triangle DZH, \)
\[
\text{arc } DH = 1;10°
\]
and arc \( ZH = 178;50° \) (supplement).

\footnote{Heiberg's figure (p. 371) is wrong: \( \Delta X \) has been connected instead of \( \Delta B, \) and \( \Lambda \) is misprinted as \( A. \) Corrected by Manitius.}
XI 1. Correction to account for equant: 1st and 2nd oppositions

Therefore the corresponding chords

\[ DH = 1;13^\circ \]
and \[ ZH \approx 120^\circ \]

Therefore where \[ DZ = 2;42^\circ \] and the radius of the eccentre, \( DB = 60^\circ \),
\[ DH = 0;2^\circ \] and \[ ZH = 2;42^\circ \].

And \( HB = 60^\circ \) in the same units (for it is negligibly smaller than hypotenuse [of right-angled triangle \( HBD \)] \( BD \)).

Furthermore, since \( 0H = HZ \), and \( E0 = 2DH \),
by subtraction, \( 0B = 57;18^\circ \) where \( E0 = 0;4^\circ \).
Hence hypotenuse [of right-angled triangle \( E0B \)] \( EB = 57;18^\circ \) in the same units.

Therefore, where \( EB = 120^\circ \), \( E\Theta \approx 0;8^\circ \),
and, in the circle about right-angled triangle \( BE\Theta \),
\[ arc \ E\Theta = 0;8^\circ \] also.
\[ \therefore \angle E\Theta = 0;8^\circ \] where 2 right angles = 360\(^\circ\).

In the same way, since we showed that the whole line \( Z\Theta = 2ZH = 5;24^\circ \)
where the radius of the eccentre, \( ZX = 60^\circ \),
by subtraction, \( \Theta X = 54;36^\circ \) where \( E\Theta = 0;4^\circ \).
Hence hypotenuse [of right-angled triangle \( E\Theta X \)] \( EX = 54;36^\circ \) in the same units.

Therefore, where \( EX = 120^\circ \), \( E\Theta \approx 0;10^\circ \),
and, in the circle about right-angled triangle \( E\Theta X \),
\[ arc \ E\Theta = 0;10^\circ \).
\[ \therefore \angle E\Theta = 0;10^\circ \] where 2 right angles = 360\(^\circ\),
and, by subtraction [of \( \angle E\Theta \)], \( \angle BEX = \begin{cases} 0;2^\circ \text{ in the same units} \\ 0;1^\circ \text{ where 4 right angles = 360}^\circ \end{cases} \).

Here, then, it is clear that the planet, since its apparent longitude at the
XI 1. Correction to account for equant: 3rd opposition

second opposition, when it was viewed along line EB, was $7;54^\circ$, would, if it
had been viewed along line EX, have had a longitude of only $7;53^\circ$.

So let the diagram for the third opposition be drawn, to the rear of the perigee
[Fig. 11.5].

Then, since arc NX of the eccentre is given as $32;51^\circ$,

$$\angle NZX = \begin{cases} 32;51^\circ & \text{where 4 right angles = 360}^\circ \\ 65;42^\circ & \text{where 2 right angles = 360}^\circ \end{cases}$$

Therefore, in the circle about right-angled triangle DZH,

arc $DH = 65;42^\circ$
and arc $ZH = 114;18^\circ$ (supplement).

Therefore the corresponding chords

$$DH = 65;6^\prime$$
and $ZH = 100;49^\prime$.

Therefore where $DZ = 2;42^\circ$ and the radius of the eccentre, $DG = 60^\circ$,

$$DH = 1;28^\circ$$
and $ZH = 2;16^\circ$.

And since $GD^2 - DH^2 = GH^2$,

GH $\approx 59;59^\circ$.

Similarly, since $\Theta H = HZ$, and $E\Theta = 2DH$,

by subtraction, $G\Theta = 57;43^\circ$ where $E\Theta = 2;56^\circ$.

Hence hypotenuse [of right-angled triangle $E\Theta G$] $EG = 57;47^\circ$ in the same units.

Therefore, where $EG = 120^\circ$, $E\Theta = 6;5^\circ$.

---

Heiberg's figure (p. 373) is wrong: $\Delta \Xi$ has been connected instead of $\Delta \Gamma$, and $\Lambda$ is in the wrong
place and misprinted as $A$. Corrected by Manitius.
XI 1. Iteration for Jupiter

and, in the circle about right-angled triangle $\text{GEO}$, arc $\text{E} \Theta \approx 5;48^\circ$.
\[ \therefore \angle \text{EG} \Theta = 5;48^\circ \text{ where 2 right angles = } 360^\circ. \]
In the same way, since the whole line $Z \Theta (= 2ZH)$ comes to $4;32^\circ$
where the radius of the eccentre, $ZX = 60^\circ$,
by subtraction, $X \Theta = 55;28^\circ$ where $E \Theta$ was found to be $2;56^\circ$.
Hence hypotenuse of right-angled triangle $E \Theta X$.
Therefore, where $EX = 120^\circ$, $E \Theta = 6;20^\circ$,
and, in the circle about right-angled triangle $E \Theta X$,
arc $E \Theta = 6;2^\circ$.
\[ \therefore \angle \text{EX} \Theta = 6;2^\circ \text{ where 2 right angles = } 360^\circ, \]
and, by subtraction $\angle \text{EG} \Theta$, $\angle \text{GEX} = 0;14^\circ$ in the same units
and, by subtraction $\angle \text{EG} \Theta$, $\angle \text{GEX} = 0;7^\circ$ where 4 right angles = $360^\circ$.
Hence, since the planet at the 3rd opposition, when viewed along line $EG$, had a longitude of $\Psi 14;23^\circ$, it is clear that, if it had been on line $EX$, it would have had a longitude of $\Psi 14;30^\circ$.
And we showed that its [corrected] longitudes [would have been]
at the first opposition $\Psi 23;14^\circ$
at the second opposition $\Psi 7;53^\circ$.

Hence we calculate the apparent intervals [in longitude] of the planet, taken, not with respect to the eccentre carrying the epicycle centre, but with respect to the eccentre producing the mean motion [i.e. the equant],\(^7\) as
from first to second oppositions $104;39^\circ$
from second to third oppositions $36;37^\circ$.

Starting from these data, by means of the previously demonstrated theorem we find the distance between the centres of the ecliptic and the eccentre producing the mean motion of the epicycle as about $5;30^\circ$ where the diameter of the eccentre is $120^\circ$;
and, for the arcs of the eccentre,
from the apogee to the first opposition: $77;15^\circ$
from the second opposition to the perigee $2;50^\circ$
from the perigee to the third opposition $30;36^\circ$.

The above quantities have been accurately determined by this method, for the differences in the intervals [as measured along deferent and equant], when calculated from these data, are very nearly the same as the previous set.\(^8\) That is [also] clear from the fact that the apparent intervals [in longitude] of the planet derived from the ratios we have thus found turn out to be the same as those observed; we can show this as follows.

Once again, let the diagram for the first opposition be drawn [Fig. 11.6], but containing only the eccentre carrying the epicycle centre. Then, since $\angle \text{LZA}$ was shown to be $77;15^\circ$ where 4 right angles = $360^\circ$,
\[ \angle \text{LZA} = \angle \text{DZH} \text{ (vertically opposite)} = 154;30^\circ \text{ where 2 right angles = } 360^\circ. \]

\(^7\)I.e. the apparent intervals which would result if the epicycle were carried, not on the actual deferent, but on the equant. Cf. XI 5 p. 529, where this is stated explicitly. Cf. also p. 492.

\(^8\)Indeed, a further iteration produces a change of much less than $0;1^\circ$ in the eccentricity, and about $0;10^\circ$ in the line of the apsides.
Therefore, in the circle about right-angled triangle DZH,
arc $DH = 154;30^\circ$
and arc $ZH = 25;30^\circ$ (supplement).
Therefore the corresponding chords
$$DH = 117;2''$$
and $ZH = 26;29''$ where hypotenuse $DZ = 120^\circ$.

Therefore where $ZD = 2;45^\circ$ and the radius of the eccentric $DA = 60^\circ$,
$$DH = 2;41^\circ$$
and $ZH = 0;36''$.

Then, by the same argument as in the previous proof,
$$AH = \sqrt{AD^2 - DH^2} = 59;56^\circ$$
in the same units,
and, by addition [of $H\Theta = ZH$], $A\Theta = 60;32^\circ$ where $E\Theta (= 2DH) = 5;22^\circ$.
Therefore hypotenuse [of right-angled triangle $AE\Theta$] $AE$ comes to $60;46^\circ$ in the same units.

Therefore, where $AE = 120^\circ$, $E\Theta = 10;36^\circ$,
and, in the circle about right-angled triangle $AE\Theta$,
$$E\Theta = 10;8^\circ$$
and, by subtraction [of $E\Theta$ from $LZA$],
$$\angle LEA = \begin{cases} 144;22^\circ & \text{in the same units} \\ 72;11^\circ & \text{where 4 right angles = 360°} \end{cases}$$
That $[72;11^\circ]$, then, was the distance in the ecliptic\(^9\) of the planet from its apogee at the first opposition.

\(^9\)So we must translate τού ἐσακτάκο (i.e. take it closely with μοιράζ) at H377.16, to make any sense at all. But its position in the sentence, and redundance, make me suspect it as an interpolation, although it is in all branches of the ms. tradition.
Again, let the [corresponding] diagram for the second opposition be drawn [Fig. 11.7]. [Then,] since

\[ \angle BZM \text{ is given as } \begin{cases} 2;50^\circ & \text{where 4 right angles } = 360^\circ \\ 5;40^\circ & \text{where 2 right angles } = 360^\circ \end{cases} \]

in the circle about right-angled triangle DZH,

\[ \text{arc } DH = 5;40^\circ \]

and \( \text{arc } ZH = 174;20^\circ \) (supplement).

Therefore the corresponding chords

\[ \text{DH} = 5;55^\circ \]

and \( \text{ZH} = 119;51^\circ \) where hypotenuse \( DZ = 120^\circ \).

Therefore where \( DZ = 2;45^\circ \) and the radius of the eccentric, \( DB = 60^\circ \),

\[ \text{DH} = 0;8^\circ \]

and \( \text{ZH} = 2;45^\circ \).

And, by the same [argument as previously],

\[ BH \approx 60^\circ \] in the same units,

and, by subtraction [of \( H\Theta = ZH \)], \( B\Theta = 57;15^\circ \) where \( E\Theta = 0;16^\circ \).

Hence hypotenuse [of right-angled triangle \( E\Theta \)] \( EB \) comes to \( 57;15^\circ \) in the same units.

Therefore, where \( EB = 120^\circ \), \( E\Theta = 0;33^\circ \),

and, in the circle about right-angled triangle \( BE\Theta \),

\[ \text{arc } E\Theta = 0;32^\circ \].

\[ \therefore \angle E\Theta = 0;32^\circ \] where 2 right angles = \( 360^\circ \).

And, by addition [of \( \angle BZM \)], \( \angle BEM = \begin{cases} 6;12^\circ \text{ in the same units} \\ 3;6^\circ \text{ where 4 right angles } = 360^\circ \end{cases} \).

Therefore the distance of the planet in advance of the perigee at the second opposition was \( 3;6^\circ \). And we showed [p. 516] that at the first opposition it was
XI I. Verification: Jupiter’s 3rd opposition

72;11° to the rear of the apogee.\(^\text{10}\) Thus the computed apparent interval from first to second oppositions is the supplement [of 3;6° + 72;11°], 104;43°, in agreement with the interval derived from the observations [p. 507].

So let the [corresponding] diagram for the third opposition be drawn [Fig. 11.8]. [Then,] since

\[ \angle MZG \text{ was shown to be } \begin{cases} 30;36° \text{ where 4 right angles } = 360° \\ 61;12°° \text{ where 2 right angles } = 360°° \end{cases}, \]

in the circle about right-angled triangle DZH,

\[
\begin{align*}
\text{arc } DH & = 61;12° \\
\text{and arc } ZH & = 118;48° \text{ (supplement)}.
\end{align*}
\]

![Fig. 11.8](image_url)

Therefore the corresponding chords

\[
\begin{align*}
\text{DH} & = 61;6° \\
\text{and } ZH & = 103;17°
\end{align*}
\]

Therefore where DZ = 2;45° and the radius of the eccentric, GD = 60°,

\[
\begin{align*}
\text{DH} & = 1;24° \\
\text{and } ZH & = 2;22°.
\end{align*}
\]

And, by the same [argument as previously],

\[
\text{GH} = 59;59°,
\]

and, by subtraction [of HΘ = ZH], GΘ = 57;37° where EΘ = 2;48°.

Therefore hypotenuse [of right-angled triangle EΘ] EG = 57;41° in the same units;

and hence, where EG = 120°, EΘ = 5;50°,

and, in the circle about right-angled triangle GEΘ,

\[
\begin{align*}
\text{arc } EΘ & = 5;34° \\
\therefore \angle EGΘ & = 5;34°° \text{ where 2 right angles } = 360°°.
\end{align*}
\]

\(^\text{10}\) Reading eίς τά ἐπόμενα τοῦ ἀπογείου (with D, Ar) at H379,3 for eίς τά ἐπόμενα (‘to the rear’).

Corrected by Manutius.
And, by addition \([\angle MZG]\),
\[
\angle MEG = \begin{cases} 
66;46^\circ \text{ in the same units} \\
33;23^\circ \text{ where 4 right angles } = 360^\circ.
\end{cases}
\]

That \([33;23^\circ]\), then, was the distance of the planet to the rear of the perigee at the third opposition. And we showed that at the second opposition its distance in advance of the same perigee was \(3;6^\circ\). Therefore the apparent interval [in longitude] from the second to the third oppositions is computed as the sum [of the above], \(36;29^\circ\), once again in agreement with the observed interval [p. 507].

It is immediately clear, since the planet at the third opposition had an observed longitude of \(\Omega 14;23^\circ\) and, as we showed, was \(33;23^\circ\) to the rear of the perigee, that at that moment the perigee of its eccentric had a longitude of \(\aleph 11^\circ\), while its apogee was diametrically opposite at \(\wp 11^\circ\).

And if [see Fig. 11.9] we draw the epicycle \(H\Theta K\) about centre \(G\), we will immediately have:

\[\text{Fig. 11.9}\]

the mean position in longitude [counted] from the apogee of the eccentric, \(L\), as \(210;36^\circ\) (for we have shown that \(\angle MZG = 30;36^\circ\));

and the arc \(\Theta K\) of the epicycle from the perigee \(\Theta\) to the planet \(K\) as \(2;47^\circ\) (for we showed that
\[
\angle EGZ = \begin{cases} 
5;34^\circ \text{ where 2 right angles } = 360^\circ \\
2;47^\circ \text{ where 4 right angles } = 360^\circ.
\end{cases}
\]

Therefore at the moment of the third opposition, namely in the first year of Antoninus, Athyr \([\text{III}] 20/21\) in the Egyptian calendar, 5 hours after midnight, the planet Jupiter had the following mean positions:

- in longitude \(210;36^\circ\) from the apogee of the eccentric (i.e. its mean longitude was \(\Omega 11;36^\circ\))

- in anomaly \(182;47^\circ\) from the apogee of the epicycle, \(H\).

\(^{11}\) Heiberg's figure on p. 38 is wrong: he has connected \(\Delta \Gamma\) instead of \(EG\). Corrected by Manitius.
2. \{Demonstration of the size of Jupiter's epicycle\}

Next, to demonstrate the size of the epicycle, we again took an observation, which we obtained by sighting [with the astrolabe], in the second year of Antoninus, Mesore [XII] 26/27 in the Egyptian calendar [139 July 10/11], before sunrise, i.e. about 5 equinoctial hours after midnight (for the mean longitude of the sun was $16;11^\circ$, and the second degree of Aries [i.e. $10^\circ-2^\circ$] was culminating according to the astrolabe). At that moment Jupiter, when sighted with respect to the bright star in the Hyades, was seen to have a longitude of $15\frac{1}{2}^\circ$, and also had the same apparent longitude as the centre of the moon, which lay to the south of it. For that moment\textsuperscript{12} we find, by means of the [kind of] calculations [previously] explained:

- moon's mean longitude $9;0^\circ$
- moon's [mean] anomaly counted from the epicycle apogee $272;5^\circ$
- hence its true position $14:50^\circ$
- and its apparent position at Alexandria $15;45^\circ$

Thus from these considerations too Jupiter's longitude was $15\frac{1}{2}^\circ$.

Furthermore, the time interval from the third opposition to the above observation comprises
- 1 Egyptian year and 276 days,
and this interval produces
- in longitude: $53;17^\circ$
- and in anomaly: $218;31^\circ$

(for it will make no sensible difference even if this kind of calculation is carried out rather crudely);\textsuperscript{13} so, if we add the latter to the [mean] positions derived for the third opposition, we will get, for the moment of the present observation, [the mean positions]:

- in longitude $263;53^\circ$ from the apogee (which is in approximately the same position [as at the third opposition])\textsuperscript{14}
- in anomaly $41;18^\circ$ from the apogee of the epicycle.

With the above as data, let the diagram for the similar demonstration in the case of Mars [Fig. 10.17] be repeated [Fig. 11.10], [but] with the epicycle in a position to the rear of the perigee of the eccentric, and with the planet past the apogee of the epicycle, in accordance with the mean positions in longitude and anomaly set out here.

Then, since the mean position in longitude from the apogee of the eccentric is

\[
\angle BZG = \begin{cases} 
83;53^\circ & \text{where 4 right angles = 360}^\circ \\
167;46^\circ & \text{where 2 right angles = 360}^\circ 
\end{cases}
\]

\textsuperscript{12} These positions were (correctly) computed, not for 5 a.m., but for 4:42 a.m., i.e. the correct equation of time with respect to epoch of era Nabonassar has been applied. Cf. p. 499 n.57.

\textsuperscript{13} These intervals are correct to the nearest minute if one computes for exactly 1\textsuperscript{st} 276\textsuperscript{nd}. However, for 18 mins. less (cf. n.12) one finds 218;30\textsuperscript{th} for the motion in anomaly. Is it this neglect of the equation of time to which Ptolemy refers by 'rather crudely'?

\textsuperscript{14} I.e. in less than 2 years the precessional motion of the apogee is negligible.
XI 2. Geometrical determination of size of Jupiter's epicycle

Therefore, in the circle about right-angled triangle DZM,

arc DM = 167;46°
and arc ZM = 12;14° (supplement).

Therefore the corresponding chords

\[
\begin{align*}
DM &= 119;19' \quad \text{where hypotenuse DZ = 120°,} \\
ZM &= 12;47' \\
\end{align*}
\]

Therefore where DZ = 2;45° and the radius of the eccentre, DB = 60°,

\[
\begin{align*}
DM &\approx 2;44' \\
ZM &= 0;18' \\
\end{align*}
\]

And since \(DB^2 - DM^2 = MB^2\),

\[
MB = 59;56' \text{ in the same units.}
\]

Similarly, since ZM = ML and EL = 2DM,

by subtraction, \(LB = 59;38'\) where EL is computed as 5;28'.

Hence hypotenuse [of right-angled triangle LBE] \(EB = 59;52'\) in the same units.

Therefore, where \(EB = 120°\), \(EL \approx 10;58°\),

and in the circle about right-angled triangle BEL,

\[
\begin{align*}
\angle EBZ &= 10;30'\quad \text{where 2 right angles = 360°} \\
\angle BZG &= 167;46° \text{ in the same units.}
\end{align*}
\]

But \(\angle KEG = 178;16°\) in the same units.

Furthermore, since the approximate longitude of the perigee G is \(\geq 11°\), and the apparent longitude of the planet, as viewed along line EK, was \(\Pi 15;45°\),

\[
\angle KEG = \begin{cases} 94;45° \quad \text{where 4 right angles = 360°} \\
189;30° \quad \text{where 2 right angles = 360°.}
\end{cases}
\]

And, by subtraction [of \(\angle BEG\)], \(\angle BEK = 11;14°\) in the same units.

Therefore, in the circle about right-angled triangle BEN,

\[
\begin{align*}
\text{arc BN} &= 11;14° \\
\text{and BN} &= 11;44° \text{ where hypotenuse } EB = 120°.
\end{align*}
\]
XI 3. Ancient observation of Jupiter

Therefore, where \( EB = 59;52^\circ \), and the radius of the eccentre is \( 60^\circ \),
\[ \text{BN} = 5;50^\circ. \]

Similarly, since arc \( HK = 41;18^\circ \),
\[ \angle HBK = \begin{cases} 41;18^\circ \text{ where 4 right angles } = 360^\circ \\ 82;36^\circ \text{ where 2 right angles } = 360^\circ. \end{cases} \]

But \( \angle EBZ (= \angle HB\Theta) = 10;30^\circ \) in the same units.

Therefore, by subtraction, \( \angle ZBK = 72;6^\circ \).

And we showed that \( \angle KE\Theta = 11;14^\circ \) in the same units.

Therefore, by subtraction, \( \angle BKN = 60;52^\circ \) in the same units.

Therefore, in the circle about right-angled triangle \( BKN \),
\[ \text{arc } BN = 60;52^\circ \]
and \( BN = 60;47'' \) where hypotenuse \( BK = 120'' \).

Therefore where \( BN = 5;50'' \) and the radius of the eccentre is \( 60^\circ \),
the radius of the epicycle, \( BK = 11;30^\circ. \)

Q.E.D.

3. [On the correction of the periodic motions of Jupiter]

Next, to determine the periodic motions, we again took one of the precisely recorded ancient observations. In this it is declared that in the 45th year of the calendar of Dionysius, on Parthenon 10, the planet Jupiter occulted\(^{16}\) the southernmost of the 2 Aselli at dawn. Now the moment of the observation is in the 83rd year from the death of Alexander, Epiphi [XI] 17/18 in the Egyptian calendar [-240 Sept. 3/4], dawn. For that time we find the longitude of the mean sun as \( \mu 9;56^\circ \). But the star called 'the southern Asellus' among those surrounding the nebula in Cancer had a longitude, at the time of our observation [of it], of \( \equiv 11\frac{1}{2}^\circ \) [catalogue XXV 5]. Hence, obviously, its longitude at the observation in question was \( \equiv 7;33^\circ \), since to the 378 years between the observations\(^{17}\) corresponds [a precessional motion of] \( 3;47^\circ \). Therefore the longitude of Jupiter at that moment (since it had occulted the star) was also \( \equiv 7;33^\circ \). Similarly, since the apogee was in \( \mu 11^\circ \) in our times, it must have had a longitude of \( \equiv 7;13^\circ \) at the observation. Hence it is clear that the distance of the apparent planet from the then apogee of the eccentre was \( 300;20^\circ \), while the distance of the mean sun from that same apogee was \( 2;43^\circ \).

With the above elements as data, let there again be drawn [Fig. 11.11] a diagram similar to that for the [corresponding] demonstration for Mars [Fig. 10.18], but in this case in accordance with the positions given for the observation: [i.e.,] have the epicycle, on centre B, positioned before the apogee A, and the point L, representing the mean position of the sun, a little after that

---

\(^{15}\) There are a series of small miscalculations and rounding errors, which result in a not negligible final error (one finds \( 11;38^\circ \) to the nearest minute). No doubt Ptolemy was aiming at a convenient round number.

\(^{16}\) Literally 'covered' (ἐπεκάλυψεν). Modern calculations show that Jupiter in fact passed ca. \(^{16}\) to the north of \( \delta \) Cnc (cf. p. 658), but Ptolemy's wording is unambiguous here (cf. p. 477 n.17).

\(^{17}\) The epoch of the star catalogue is Antoninus I = Nabonassar 885. And 885—507 = 378. But since the observation took place in the 11th month of the Egyptian year, 377 would have been more accurate.
same apogee, and hence the point $\Theta$, representing the planet, after $H$, the apogee of the epicycle. And, as we always do in similar situations, we join $ZBH$, $DB$, $B\Theta$ and $E\Theta$, and drop perpendiculars $ZK$ on to $DB$, $DM$ and $BN$ on to $E\Theta$, and $DX$ on to $NB$ (produced in this case), which forms the rectangular parallelogram $DMNX$.

Then $\angle AE\Theta$ contains one revolution in the ecliptic less $300;20^\circ$, or $59;40^\circ$.

And $\angle AEL = 2;43^\circ$.

Therefore, by addition,

$$\angle LE\Theta = \angle B\Theta E = \begin{cases} 62;23^\circ & \text{where 4 right angles} = 360^\circ \\ 124;46^\circ & \text{where 2 right angles} = 360^\circ \end{cases}.$$

Therefore, in the circle about right-angled triangle $B\Theta N$,

arc $BN = 124;46^\circ$

and $BN = 106;20^\circ$ where hypotenuse $B\Theta = 120^\circ$.

Therefore where the radius of the epicycle, $B\Theta^{18} = 11;30^\circ$,

BN = 10;12^\circ$.

Again, since $\angle DEM$ is given as

$$\begin{cases} 59;40^\circ & \text{where 4 right angles} = 360^\circ \\ 119;20^\circ & \text{where 2 right angles} = 360^\circ \end{cases},$$

and $\angle MDE = 60;40^\circ$ in the same units (complement),

in the circle about right-angled triangle $DEM$

arc $DM = 119;20^\circ$

and $DM = 103;34^\circ$ where hypotenuse $ED = 120^\circ$.

Therefore where $ED = 2;45^\circ$ and the radius of the eccentre, $DB = 60^\circ$,

$DM = 2;23^\circ$.

and, by addition, $BNX = 12;35^\circ$.

$^{18}$ Reading ἡ $B\Theta$ ἐκ τοῦ κέντρου (with D, Ar) for ἡ ἐκ τοῦ κέντρου ('the radius of the epicycle') at H389,2-3.
Therefore where hypotenuse \([\text{of right-angled triangle } BDX]\) \(BD = 120\), 
\[BX = 25;\,10°,\]
and, in the circle about right-angled triangle \(BDX\),
\[\text{arc } BX = 24;\,14°\]
\[\therefore \angle BDX = 24;\,14° \text{ where } 2 \text{ right angles } = 360°,\]
and, by subtraction \([\text{from a right angle}]\), \(\angle BDM = 155;\,46° \text{ in the same units};\)
and, by addition \([\text{of } \angle MDE]\), \(\angle BDE = 216;\,26° \text{ in the same units};\)
and, again by subtraction \([\text{from } 2 \text{ right angles}]\), \(\angle BDZ = 143;\,34° \text{ in the same units}.\)
Therefore, in the circle about right-angled triangle \(ZDK\),
\[\text{arc } ZK = 143;\,34°\]
\[\text{and arc } DK = 36;\,26° \text{ (supplement)}.\]

Hence hypotenuse \([\text{of right-angled triangle } ZBK]\) \(ZB = 59;\,12° \text{ in the same units}.\)
Therefore where \(ZB = 120°\), \(ZK = 5;\,18°,\)
\[\text{and, in the circle about right-angled triangle } BZK,\]
\[\text{arc } ZK = 5;\,4°,\]
\[\therefore \angle ZBD = 5;\,4° \text{ where } 2 \text{ right angles } = 360°,\]
\[\angle AZB \text{ (which comprises the mean motion in longitude) } = \begin{cases} 148;\,38° & \text{in the same units} \\ 74;\,19° & \text{where } 4 \text{ right angles } = 360°. \end{cases}\]
And since \(\angle HBΩ + \angle BZG + 180° \text{ (i.e. here } \angle HBΩ - \angle AZB) = \angle AEL = 2;\,43°,\)
we find that \(\angle HBΩ \text{ (which comprises the planet's position [in anomaly] from the apogee of the epicycle) is } 77;\,2°.\)

Therefore we have shown that at the moment of the observation in question the planet Jupiter had the following mean positions:
in longitude, from the apogee of the eccentre. \(285;\,41°\)
\[\text{(i.e. its mean longitude was } Π \text{ } 22;\,54°\).

in anomaly, from the apogee of the epicycle. \(77;\,2°.\)

And we had [already] shown that at the moment of the third opposition its distance from the apogee of the epicycle was \(182;\,47°.\) Thus in the interval between the two observations, which comprises
377 Egyptian years and 128 days less approximately 1 hour,
its motion in anomaly was
\(105;\,45° \text{ beyond } 345 \text{ complete revolutions.}\)
That is, again, very nearly the same increment in anomaly as one derives from the [tables for] mean motions which we constructed. For it was from these very same elements that we derived the daily [mean motion in anomaly], by dividing

\[19\text{ There are numerous small inaccuracies and rounding errors in the preceding calculations, which to some extent cancel each other. Accurate computation gives } 77;\,0° \text{ to the nearest minute.}\]
the number of degrees contained in the complete revolutions plus the increment by the number of days contained in the time-interval.

4. {On the epoch of Jupiter's periodic motions}

Here too again, then, since the interval from the first year of Nabonassar, Thoth I in the Egyptian calendar, noon, to the above-mentioned ancient observation is

506 Egyptian years and approximately \( 316\frac{1}{2} \) days,
and this interval comprises increments of

- 258;13° in longitude
- 290:58° in anomaly,

if we subtract the latter from the respective [mean] positions listed above for the observation, we get, for the same moment of epoch as for the other [heavenly bodies], for Jupiter:

- mean longitude \( \cong 4;41° \)
- mean anomaly \( 146;4° \) from the epicyclic apogee.

And, by the same [kind of computation as before], the apogee of its eccentric will be in \( \nu 2;9° \).

5. {Demonstration of Saturn's eccentricity and [the position of] its apogee}

To complete this topic, it remains to demonstrate the anomalies and epochs for the theory of the planet Saturn. Once again, as for the other planets, we took, first, for our investigation of [the position of] the apogee and the eccentricity, three opposition situations of the planet, in which it was diametrically opposite the sun's mean position.

1. The first of these was observed by us, using the astrolabe instruments, in the eleventh year of Hadrian, Pachon [IX] 7.8 in the Egyptian calendar [127 Mar. 26/27], in the evening, in \( 1;13° \);
2. the second, in the seventeenth year of Hadrian, Epiphi [XI] 18 in the Egyptian calendar [133 June 3]. We computed the time and place of exact opposition from nearby observations as 4 hours after noon on the 18th, in \( 9;40° \);
3. we observed the third opposition in the twentieth year of Hadrian, Mesore [XII] 24 in the Egyptian calendar [136 July 8]. As before, we computed the time of exact opposition as having occurred precisely at noon on the 24th, and computed the place as \( \nu 14;14° \).

Of these two intervals, then, that from the first to the second opposition comprises

\[ \begin{align*}
\text{H392} & \text{On the actual derivation of the mean motion in anomaly for Jupiter, which remains obscure, see Appendix C.}
\text{H393} & \text{These intervals are precise (to the nearest minute) for an increment of exactly \( \frac{1}{2} \) day.}
\text{H392} & \text{The apogee was in \( \nu 7;13° \) at the observation (p. 522). In \( 507° \) (at the rate of 1° in 100 years) it moves about \( 5;4° \). Hence at epoch it was in \( \nu 2;9° \).}
\end{align*} \]
XI 5. Preliminary determination of Saturn’s apogee and eccentricity

[In time] 6 Egyptian years 70 days 22 hours in apparent motion of the planet 68°27′; while that from the second to the third opposition comprises [in time] 3 Egyptian years 35 days 20 hours [in apparent motion] 34°34′.

And we compute for the mean motion in longitude, using rough figures, for the first interval: 75°43′ and for the second interval: 37°52′.

These intervals [in mean and true longitude] being given, we again demonstrate the required [parameters] by means of the same theorem [as before] (as if there were only one eccentric), as follows.

To avoid repetition, let there be drawn a diagram [Fig. 11.12] like those used for the same proof [previously, Figs. 10.8, 11.1]. Then since arc BG of the eccentric is given as subtending 34°34′ of the ecliptic, the [corresponding] angle at the centre of the ecliptic.

\[ \angle BDG = \angle EDH = \begin{cases} 34°34′ & \text{where 4 right angles = 360°} \\ 69°8′′ & \text{where 2 right angles = 360°} \end{cases} \]

Therefore, in the circle about right-angled triangle DEH,

arc EH = 69°8′′

and EH = 68°5′ where hypotenuse DE = 120°.

Similarly, since arc BG = 37°52′, the angle at the circumference,

\[ \angle BEG = 37°52′′ \]

and, by subtraction [from \( \angle BDG \)], \( \angle EBH = 31°16′′ \) in the same units.

Therefore, in the circle about right-angled triangle EBH,

arc EH = 31°16′′

and EH = 32°20′ where hypotenuse BE = 120°.

\[ \text{Despite Ptolemy’s phrase here, the intervals in mean longitude are accurate to the nearest minute according to his own tables. Nor would the equation of time make any difference.} \]
XI 5. Preliminary determination of Saturn’s apogee and eccentricity 527

Therefore where EH, as we showed, is 68;5', and ED = 120°,

BE = 252;41°.

Furthermore, since the whole arc ABG subtends 103;1° of the ecliptic (the sum of both intervals [in true longitude]), the [corresponding] angle at the centre of the ecliptic,

\[ \angle ADG = 103;1° \text{ where 4 right angles = 360°.} \]

Hence the supplementary \[ \angle ADE = 153;58° \text{ where 2 right angles = 360°.} \]

Therefore, in the circle about right-angled triangle DEZ,

\[ \text{arc } EZ = 153;58° \]

and \[ EZ = 116;55'' \text{ where hypotenuse } DE = 120°. \]

Similarly, since arc ABG of the eccentric is found by addition [of 75;43° and 37;52°] as 113;35°, the angle at the circumference,

\[ \angle AEG = 113;35° \text{ where 2 right angles = 360°.} \]

But we found that \[ \angle ADE = 153;58° \text{ in the same units.} \]

Therefore the remaining angle [in triangle ADE],

\[ \angle ZAE = 92;27° \text{ in the same units.} \]

Therefore, in the circle about right-angled triangle AEZ,

\[ \text{arc } EZ = 92;27° \]

and \[ EZ = 86;39'' \text{ where hypotenuse } AE = 120°. \]

Therefore where EZ, as we showed, is 116;55'', and ED = 120°

EA = 161;55°.

Furthermore, since arc AB of the eccentric is 75;43°, the angle at the circumference

\[ \angle AEB = 75;43° \text{ where 2 right angles = 360°.} \]

Therefore, in the circle about right-angled triangle AE\(\Theta\),

\[ \text{arc } \Theta = 75;43°. \]

and arc \( \Theta = 104;17° \) (supplement).

Therefore the corresponding chords

\[ \Theta = 73;39° \text{ where hypotenuse } EA = 120°. \]

Therefore where AE, as we showed, is 161;55'', and DE = 120°,

\[ \Theta = 99;23° \text{ and } \Theta = 127;51°. \]

But we showed that the whole line EB = 252;41° in the same units.

Therefore, by subtraction, \( \Theta B = 124;50° \text{ where } \Theta = 99;23°. \)

And \( \Theta B = 15583;22 \)

and \( \Theta = 9877;3 \)

and \( \Theta^2 + \Theta = AB = 25460;25. \)

\[ \therefore AB = 159;34° \text{ where ED = 120° and EA = 161;55°.} \]

And, where the diameter of the eccentric is 120°, \( \Theta B = 73;39° \)

(for it subtends an arc of 75;43°).

Therefore where \( AB = 73;39° \) and the diameter of the eccentric is 120°,

\[ ED = 55;23° \text{ and } EA = 74;43°. \]

24 Reading \( \Theta \) for \( \Theta \) (99;43°) at H396,10 and 13. '23', which is guaranteed by the rest of Ptolemy’s working, is found in Ger.
Therefore arc EA of the eccentric = 77;1°
and, by addition [of arc ABG], arc EABG = 190;36°,
and hence, by subtraction [from the circle], arc GE = 169;24°.
Therefore GDE ≈ 119;28° where the diameter of the eccentric is 120°.

So [see Fig. 11.13] let the centre of the eccentric be taken inside segment EAG
(since it is greater than a semi-circle) as point K. Draw through K and D the
diameter from K on to GE be produced [to meet the circumference] as KNX.

Then, where the diameter, LM = 120°,
the whole line EG was shown to be 119;28° and ED to be 55;23°;
so, by subtraction, DG = 64;5° in the same units.

So, since ED·DG = LD·DM,
LD·DM = 3549;9° where diameter LM is 120°.
But LD·DM + DK² = LK² (the square on half the diameter).

Therefore, if from the square on half the diameter, 3600, we subtract 3549;9, we
are left with DK² as 50;51° in the same units.

Therefore the distance between the centres, DK ≈ 7;8°
where the diameter of the eccentric is 120°. 25

Furthermore, since EN (= \frac{1}{4}GE) = 59;44° where diameter LM = 120°,
and we showed that ED = 55;23° in the same units,
by subtraction, DN = 4;21° where DK, as we showed, = 7;8°.
Therefore where hypotenuse [of right-angled triangle DKN] DK = 120°,
DN = 73;11°.

25 DG and ED have been computed with only small inaccuracies (I find 64;5.21 and 55;23.39 for
Ptolemy’s 64;5 and 55;23), but the resulting value for the eccentricity, 7;3.33°, differs significantly
from Ptolemy’s 7;8°.
XI 5. Preliminary determination of Saturn’s apogee and eccentricity

and, in the circle about right-angled triangle DKN

\[
\text{arc } DN = 75;10^\circ.
\]

\[
\therefore \angle DKN = \begin{cases} 
75;10^\circ & \text{where 2 right angles} = 360^\circ \\
37;35^\circ & \text{where 4 right angles} = 360^\circ.
\end{cases}
\]

And since \( \angle DKN \) is an angle at the centre of the eccentric,

\[
\text{arc } XM = 37;35^\circ. \tag{26}
\]

But arc GX = \( \frac{1}{4} \) arc GXE = 84;42°.

Therefore, by subtraction (of (arc GX + arc XM) from 180°), the arc from the apogee to the third opposition,

\[
\text{arc } GL = 57;43^\circ.
\]

But arc BG is given as 37;52°.

Therefore, by subtraction, the arc from the apogee to the second opposition,

\[
\text{arc } LB = 19;51^\circ.
\]

Similarly, since arc AB is given as 75;43°,

by subtraction, the arc from the first opposition to the apogee,

\[
\text{arc } AL = 55;52^\circ.
\]

Now again, since the epicycle centre is carried, not on this eccentric, but on that drawn with centre the point bisecting DK and with radius KL, we computed in due order, as we did for the other [planets], the differences in the apparent intervals [in true longitude] on the ecliptic which result from the above ratios (taking them to be approximately correct), if we transfer the epicycle’s path to the eccentric in question, which produces the ecliptic anomaly [i.e. the equant].

Thus, let there be drawn [Fig. 11.14] the diagram for the first opposition, [similar to the previous one in the same demonstration, but drawn in advance of the apogee L. Then, since the angle of the mean position in longitude,

\[
\angle NZX (= \angle DZH) = \begin{cases} 
55;52^\circ & \text{where 4 right angles} = 360^\circ \\
111;44^\circ & \text{where 2 right angles} = 360^\circ,
\end{cases}
\]

in the circle about right-angled triangle DZH,

\[
\text{arc } DH = 111;44^\circ
\]

and arc ZH = 68;16° (supplement).

Therefore the corresponding chords

\[
\text{DH} = 99;20^\circ \text{ where hypotenuse } DZ = 120^\circ
\]

and ZH = 67;20°.

Therefore where the distance between the centres, DZ = 3;34°, and the radius of the eccentric, DA = 60°,

\[
\text{DH} = 2;57^\circ
\]

and ZH = 2;0°.

And since \( DA^2 - DH^2 = AH^2 \),

\[
AH = 59;56^\circ \text{ in the same units.}
\]

Similarly, since ZH = \( \Theta H \),

and \( \Theta E = 2DH \),

by addition, \( A\Theta = 61;56^\circ \) where \( E\Theta = 5;54^\circ \).

\( \tag{26} \)

The accumulation of small errors again leads to a significant difference between Ptolemy’s result and the accurately computed value. 38;1°.
Hence hypotenuse [of right-angled triangle $\Theta AE$]

$$AE = 62:13^\circ \text{ in the same units.}$$

Therefore, where hypotenuse $AE = 120^\circ$, $E\Theta = 11:21^\circ$, and in the circle about right-angled triangle $AE\Theta$.

$$\text{arc } E\Theta \approx 10:51^\circ$$

$$\therefore \angle E\Theta A = 10:51^\circ \text{ where 2 right angles = } 360^\circ.$$

Furthermore, where $E\Theta = 5:54^\circ$,

the radius of the eccentric, $ZX = 60^\circ$, and $Z\Theta = 4^\circ$;

hence, by addition, $E\Theta X$, obviously, = $64^\circ$,

and we get hypotenuse [of right-angled triangle $E\Theta X$]

$$EX \text{ as } 64:16^\circ \text{ in the same units.}$$

Therefore, where hypotenuse $EX = 120^\circ$, $\Theta E = 11:2^\circ$.

and, in the circle about right-angled triangle $E\Theta X$.

$$\text{arc } \Theta E = 10:33^\circ.$$

$$\therefore \angle EX\Theta = 10:33^\circ \text{ where 2 right angles = } 360^\circ.$$

But we showed that $\angle EA\Theta = 10:51^\circ \text{ in the same units.}$

Therefore, by subtraction, the angle of the required difference,

$$\angle AEX = \begin{cases} 
0:18^\circ \text{ where 2 right angles = } 360^\circ \\
0:9^\circ \text{ where 4 right angles = } 360^\circ.
\end{cases}$$

But the planet at the first opposition, when viewed along line $AE$, had an apparent longitude of $\equiv 1:13^\circ$. Thus it is clear that if the epicycle centre were carried, not on $AL$, but on $NX$, it would have been at point $X$ [at the first

---

$^{27}$ I find $11:23^\circ$, leading to $\text{arc } E\Theta = 10:53^\circ$. 

---
opposition], and the planet would have been seen along line EX, 9° in advance of its [actual] position at A, with a longitude of ≈ 1;4°.

Again, let there be drawn [Fig. 11.15] the diagram for the second opposition, [like that] in the same demonstration [previously], but drawn to the rear of the apogee. [Then,] since arc NX of the eccentre was shown to be 19;51°,

\[ \angle NZX = \angle DZH \ (\text{vertically opposite}) = \begin{cases} 19;51° & \text{where 4 right angles = 360°} \\ 39;42° & \text{where 2 right angles = 360°} \end{cases} \]

Therefore, in the circle about right-angled triangle DZH,

arc DH = 39;42°

and arc ZH = 140;18° (supplement).

Therefore the corresponding chords

\[ DH = 40;45° \text{ where hypotenuse } DZ = 120°, \]

\[ \text{and } ZH = 112;52° \]

Therefore, where DZ = 3;34° and the radius of the eccentre, DB = 60°,

\[ DH = 1;13° \]

and \[ ZH = 3;21° \]

And, since \[ DB^2 - DH^2 = BH^2, \]

\[ BH \approx 59;59° \text{ in the same units.} \]

Similarly, since \[ ZH = H\Theta, \text{ and } E\Theta = 2DH, \]

by addition, \[ B\Theta = 63;20° \text{ where } E\Theta = 2;26°. \]

Hence hypotenuse [of right-angled triangle BE\Theta]

\[ EB = 63;23° \text{ in the same units.} \]

Therefore where hypotenuse \[ BE = 120°, E\Theta = 4;36°, \]
and, in the circle about right-angled triangle BEΘ,
  \[ \text{arc } EΘ = 4;24° \]
  \[ \therefore \angle EBTheta = 4;24^{\circ}\text{ where 2 right angles = 360}^{\circ}. \]
Likewise, where the radius of the eccentre, \( XZ = 60^\circ \),
  \[ ZΘ \text{ is computed as } 6;42^\circ; \]
  so, by addition, \( XΘ = 66;42^\circ \) where \( EΘ \) is given as \( 2;26^\circ \).
Hence we find hypotenuse [of right-angled triangle \( EΘX \)] \( EX \) as \( 66;45^\circ \) in the
same units.

Therefore, where hypotenuse \( EX = 120^\circ \), \( EΘ = 4;23^\circ \),
and, in the circle about right-angled triangle \( EΘX \),
  \[ \text{arc } EΘ = 4;12^\circ. \]
  \[ \therefore \angle EXΘ = 4;12^{\circ}\text{ where 2 right angles = 360}^{\circ}. \]
But \( \angle EBTheta \) was shown to be \( 4;24^{\circ}\text{ in the same units. } \)

Therefore, by subtraction, \( \angle BEX = \begin{cases} 0;12^\circ \text{ in the same units} \\ 0;6^\circ \text{ where 4 right angles = 360}^{\circ}. \end{cases} \)
Here too, then, it is clear, since the planet at the second opposition, when
viewed along \( EB \), had a longitude of \( \approx 9;40^\circ \), that if, instead, it were viewed
along \( EX \), it would have a longitude of \( \approx 9;46^\circ \). And we showed that at the first
opposition it would, on the same hypothesis, have had a longitude of \( \approx 1;4^\circ \).
Hence it is clear that the interval in apparent [longitude] from the first to the
second opposition, if it were taken with respect to the eccentre \( NX \), would be
\( 68;42^\circ \) of the ecliptic.

Let the diagram for the third opposition be drawn [Fig. 11.16], with the same
layout as that set out above for the second. [Then.] since we showed [p. 529] that
arc \( NX = 57;43^\circ \),

![Fig. 11.16](image-url)
\[ \angle NZX (= \angle DZH) = \begin{cases} 57;43^\circ & \text{where 4 right angles} = 360^\circ \\ 115;26^\circ & \text{where 2 right angles} = 360^\circ. \end{cases} \]

Therefore, in the circle about right-angled triangle DZH,

\[
\begin{align*}
\text{arc } DH &= 115;26^\circ \\
\text{and arc } ZH &= 64;34^\circ \text{ (supplement).}
\end{align*}
\]

Therefore the corresponding chords

\[
\begin{align*}
DH &= 101;27'' \\
ZH &= 64;6''
\end{align*}
\]

where hypotenuse \( DZ = 120^\circ \).

Therefore where \( DZ = 3;34^p \) and the radius of the eccentre, \( DG = 60^p, \)

\[
\begin{align*}
DH &= 3;1^p \\
\text{and } ZH &= 1;54^p.
\end{align*}
\]

Again, since \( DG^2 - DH^2 = GH^2 \).

Similarly, since \( ZH = \Theta H \), and \( E\Theta = 2DH \),

by addition, \( G\Theta = 61;59^p \) where \( E\Theta \) is computed as \( 6;2^p \);

Hence hypotenuse [of right-angled triangle \( G\Theta E \)]

\[
\begin{align*}
EG &= 62;5^p \text{ in the same units.}
\end{align*}
\]

Therefore, where hypotenuse \( GE = 120^p, E\Theta = 11;39^p \),

and, in the circle about right-angled triangle \( G\Theta E \),

\[
\begin{align*}
\text{arc } E\Theta &= 11;9^p \\
\therefore \angle G\Theta E &= 11;9^\circ \text{ where 2 right angles} = 360^\circ.
\end{align*}
\]

Similarly, where the radius of the eccentre, \( XZ = 60^p, \)

\( Z\Theta \) is computed as \( 3;48^p \):

so, by addition, \( X\Theta = 63;48^p \) where \( E\Theta \) was found to be \( 6;2^p \).

Hence hypotenuse [of right-angled triangle \( E\Theta X \)]

\[
\begin{align*}
EX &= 64;5^p \text{ in the same units.}
\end{align*}
\]

Therefore, where hypotenuse \( EX = 120^p, E\Theta = 11;18^p \),

and, in the circle about right-angled triangle \( E\Theta X \),

\[
\begin{align*}
\text{arc } E\Theta &= 10;49^p \\
\therefore \angle E\Theta X &= 10;49^\circ \text{ where 2 right angles} = 360^\circ.
\end{align*}
\]

But we showed that \( \angle G\Theta E = 11;9^\circ \) in the same units.

Therefore, by subtraction, \( \angle GEX = \begin{cases} 0;20^\circ \text{ in the same units} \\ 0;10^\circ \text{ where 4 right angles} = 360^\circ. \end{cases} \)

Hence, since the planet at the third opposition, when viewed along line \( EG \), had a longitude of \( 1^\circ 14;14^p \), it is clear that, if it had been on line \( EX \), it would have had a longitude of \( 1^\circ 14;24^p \), and the interval from the second opposition to the third in apparent [longitude], taken with respect to eccentre NX, would have been \( [1^\circ 14;24^p - 9;46^\circ] = 34;38^p \).

Starting from these intervals, then, we follow through the same theorem, and find the distance between the centres of the ecliptic and the eccentre which produces the uniform motion of the epicycle (i.e. the distance equal to \( EZ \) [in Fig. 11.16]) as about \( 6;50^p \) where the diameter of the eccentre is \( 120^p \), and [the following values] for the arcs of that same eccentre:

- from the first opposition to the apogee \( 57;5^p \)
- from the apogee to the second opposition \( 18;38^p \)
- from the apogee to the third opposition \( 56;30^p \).

Here again, the above quantities have been accurately derived by this
XI.5. Verification of Saturn’s apogee and eccentricity

method; for the differences in the ecliptic arcs computed from these arcs are very nearly the same as the previous set, and the apparent intervals [in longitude] of the planet are found to be in agreement with those observed, as we shall show by a procedure similar [to the preceding ones for Jupiter and Mars].

Let the diagram for the first opposition be drawn [Fig. 11.17], with only the eccentric carrying the epicycle centre. Then since the angle subtending 57;5° of the eccentric [i.e. equant],

\[ \angle AZL = 57;5° \] where 4 right angles = 360°,

and \[ \angle AZL = \angle DZH \] (vertically opposite) = 114;10°° where 2 right angles = 360°°,

in the circle about right-angled triangle DZH.

\[ \text{arc } DH = 114;10° \]

and \[ \text{arc } ZH = 65;50° \] (supplement).

Therefore the corresponding chords

\[ DH = 100;44'' \]

and \[ ZH = 65;13'' \] where hypotenuse \( DZ = 120' \).

H408 Therefore where the distance between the centres, \( DZ = 3;25' \),

and the radius of the eccentric, \( DA = 60° \),

\[ DH = 2;52' \]

and \[ ZH = 1;51' \].

Furthermore, since \( AD^2 - DH^2 = AH^2 \),

\[ AH = 59;56° \] in the same units.

Similarly, since \( ZH = H\Theta \), and \( E\Theta = 2DH \),

Indeed, with one more iteration, one finds corrections of 0;9,28°, 0;5,36° and 0;9,40° (compare Ptolemy's 9', 6' and 10'), and a result for the eccentricity and apogee agreeing very closely with that adopted by Ptolemy.
XI 5. Verification: Saturn’s 1st and 2nd oppositions

by addition, \( A\Theta = 61;47^o \) where \( E\Theta \) is computed as \( 5;44^o \).

Hence hypotenuse [of right-angled triangle \( AE\Theta \)]
\[ AE = 62;3^o \] in the same units.

Therefore, where hypotenuse \( AE = 120^o \), \( E\Theta = 11;5^o \),
and, in the circle about right-angled triangle \( AE\Theta \),
\[ \text{arc } E\Theta = 10;36^o. \]
\[ \therefore \angle EAZ = 10;36^o \text{ where } 2 \text{ right angles} = 360^o. \]

But \( \angle AZL \) was given as \( 114;10^o \) in the same units.
Therefore, by subtraction, \( \angle AEL = \begin{cases} 103;34^o \text{ in the same units} \\ 51;47^o \text{ where } 4 \text{ right angles} = 360^o. \end{cases} \)
That \( [51;47^o] \), then, was the amount by which the planet was in advance of the apogee at the first opposition.

Again, let the diagram for the second opposition be drawn in the same manner [Fig. 11.18]. Then, since

\[ \angle BZL \text{ was shown to be } 18;38^o \text{ where } 4 \text{ right angles} = 360^o, \]
and \( \angle BZL = \angle DZH \) (vertically opposite) = \( 37;16^o \) where \( 2 \text{ right angles} = 360^o \),
in the circle about right-angled triangle \( DZH \),
\[ \text{arc } DH = 37;16^o \]
and \( \text{arc } ZH = 142;44^o \) (supplement).
Therefore the corresponding chords
\[ DH = 38;20^o \text{ where hypotenuse } DZ = 120^o, \]
and \( ZH = 113;43^o \).
So where \( DZ = 3;25^o \) and the radius of the ecentre, \( DB = 60^o \),
\[ DH = 1;5^o \]
and \( ZH = 3;14^o. \)
And since $DB^2 - DH^2 = BH^2$,  
$BH = 59;59^o$ in the same units.

Similarly, since $ZH = H\Theta$, and $E\Theta = 2DH$,  
by addition, $B\Theta = 63;13^o$ where $E\Theta$ is computed as $2;10^o$.

Hence hypotenuse [of right-angled triangle $BE\Theta$]  
$EB = 63;15^o$ in the same units.

Therefore, where hypotenuse $EB = 120^o$, $\Theta E = 4;7^o$,  
and, in the circle about right-angled triangle $BE\Theta$,  
$\Theta E = 3;56^o$.

$\therefore \angle EBL = 3;56^o$ where 2 right angles = $360^o$.

But $\angle BZL$ was given as $37;16^o$ in the same units.

Therefore, by subtraction, $\angle BEL = \begin{cases} 33;20^o \\ 16;40^o \end{cases}$ where 4 right angles = $360^o$.

Therefore at the second opposition the apparent position of the planet was $16;40^o$ to the rear of the apogee. And we showed that at the first opposition it was $51;47^o$ in advance of the same apogee. Therefore the interval in apparent [longitude] from the first opposition to the second is computed as the sum of the above amounts, $68;27^o$, in agreement with the distance found from the observations [p. 526].

Now let the diagram for the third opposition be drawn [Fig. 11.19]. [Then.] since  
$\angle GZL$ was shown to be $56;30^o$ where 4 right angles = $360^o$,  
and $\angle GZL = \angle DZH$ (vertically opposite) = $113;0^o$ where 2 right angles = $360^o$,

in the circle about right-angled triangle $DZH$,  
$\text{arc } DH = 113^o$  
and $\text{arc } ZH = 67^o$ (supplement).

Fig. 11.19
XI 5. Agreement of computation with observations for Saturn

Therefore the corresponding chords

\[ DH = 100;4^p \] \text{where hypotenuse } DZ = 120^p. \\
and \[ ZH = 66;14^p \]

Therefore, where \[ DZ = 3;25^p, \] and the radius of the eccentre, \[ DG = 60^p, \]

\[ DH = 2;51^p \]

and \[ ZH = 1;53^p. \]

Again, since \[ DG^2 - DH^2 = GH^2, \]

\[ GH = 59;56^p \] in the same units.

Similarly, since \[ ZH = H\Theta, \] and \[ E\Theta = 2DH, \]

by addition, \[ G\Theta = 61;49^p \] where \[ E\Theta \] is computed as \[ 5;42^p; \]

hence hypotenuse [of right-angled triangle \( GE\Theta \)]

\[ EG = 62;5^p \] in the same units.

Therefore, where hypotenuse \( GE = 120^p, \) \( E\Theta = 11;1^p, \)

and, in the circle about right-angled triangle \( GE\Theta, \)

\[ \text{arc } E\Theta = 10;32^p \]

\[ : \angle EG\Theta = 10;32^\circ \text{ where } 2 \text{ right angles } = 360^\circ. \]

But \( \angle GZL \) was given\(^9\) as \( 113^\circ \) in the same units.

That \( [51;14^p], \) then, is the amount by which the planet was to the rear of the apogee at the third opposition. And we showed that at the second opposition it was \( 16;40^p \) to the rear of the same apogee. So the distance in apparent [longitude] from the second opposition to the third is computed as the difference [between \( 51;14^p \) and \( 16;40^p \)], \( 34;34^p, \) which is, again, in agreement with that derived from the observations [p. 526].

It is immediately clear, since the planet at the third opposition had a longitude of \( \Theta \) \( 14;14^p, \) and was shown to be \( 51;14^p \) to the rear of the apogee, that the apogee of its eccentre had at that moment a longitude of \( m \) \( 2^p, \) while its perigee was diametrically opposite at \( B \) \( 23^p. \)

In the same way [as before], if we draw [Fig. 11.20] the epicycle \( H\Theta \) about centre \( G, \) we immediately get the mean position of the epicycle in longitude from the apogee of the eccentre as \( 56;30^p \) [as demonstrated [p. 533]], and arc \( \Theta K \) of the epicycle as \( 5;16^p \) [for \( \angle EGZ \) was shown [above] to be \( 10;32^\circ \text{ where } 2 \text{ right angles } = 360^\circ. \)] Therefore, by subtraction [from \( 180^p\)],

\[ \text{arc } H\Theta, \] the arc from the apogee of the epicycle to the planet, is \( 174;44^p. \)

Therefore at the moment of the third opposition, namely in the twentieth year of Hadrian, Mesore 24 in the Egyptian calendar, at noon, the planet Saturn had the following mean positions:

in longitude: \[ 56;30^p \text{ from the apogee of the eccentre} \]

(i.e. its [mean] longitude was \( \psi \) \( 19;30^p); \)

in anomaly: \[ 174;44^p \text{ from the apogee of the epicycle}. \]

Q.E.D.

\(^9\) Reading \( \text{ά} \) (with Ar) for \( \text{i} \) (11;10^p) at H411.22. The reading is confirmed by the surrounding computations.

\(^{10}\) Reading \( \text{υπόκεισται} \) with D, for \( \text{υπόκειθαι} \) ('is given') at H412.1.
Next, once again, in order to demonstrate the size of the epicycle, we took an observation which we made in the second year of Antoninus, Mechir [VI] 6/7 in the Egyptian calendar [138 Dec. 22/23]. It was 4 equinoctial hours before midnight, for according to the astrolabe the last degree of Aries was culminating, while the longitude of the mean sun was $\varpi 28;41^\circ$. At that moment the planet Saturn, sighted with respect to the bright star in the Hyades [catalogue XXIII 14], was seen to have a longitude of $\approx 9^\circ$ and was about $\frac{1}{10}$ to the rear of the centre of the moon (for that was its distance from the moon's northern horn). Now at that moment the moon's positions were as follows:

- mean longitude $\approx 8;55^\circ$
- anomaly $174;15^\circ$ from the apogee of the epicycle
- hence its true longitude must have been $\approx 9;40^\circ$
- and its apparent longitude at Alexandria $\approx 8;34^\circ$.

Thus from these considerations too the planet Saturn must have had a longitude of $\approx 9^\circ$ (since it was about $\frac{1}{10}$ to the rear of the moon's centre).

It is far from clear for what moment these amounts are computed. The equation of time with respect to epoch is about $-131$ minutes, and indeed the mean positions seem to be computed for 7:50 p.m. rather than 8 p.m.; but then Ptolemy's true longitude is much too big. I find:

<table>
<thead>
<tr>
<th></th>
<th>for 7:50 p.m.</th>
<th>for 8 p.m.</th>
<th>Ptolemy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>308;52°</td>
<td>308;58°</td>
<td>308;55°</td>
</tr>
<tr>
<td>$\delta$</td>
<td>174;15°</td>
<td>174;20°</td>
<td>174;15°</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>309;29°</td>
<td>309;35°</td>
<td>309;40°</td>
</tr>
</tbody>
</table>

Since the moon was almost on the horizon, the parallax was large: from Ptolemy's tables I find a longitudinal parallax of about $-11^\circ$ ($-1;6^\circ$ text), leading to a discrepancy of about $1^\circ$ in the final result.
And its distance from the apogee of the eccentric (which was [in] the same position as at the third opposition, since its shift over so short an interval is negligible), was 76;4°.

Now the interval from the third opposition to this observation is 2 Egyptian years 167 days 8 hours.

And the [mean] motions of Saturn over this interval, calculated roughly, are

in longitude: 30;3°
in anomaly: 134;24°.

If we add the latter to the positions at the third opposition as found above [p. 537], we get, for the moment of the observation in question:

in [mean] longitude 86;33° from the apogee of the eccentric
in anomaly 309;8° from the apogee of the epicycle.

With the above as data, let us again draw the diagram [Fig. 11.21] as in the similar proof [for Mars and Jupiter. Figs. 10.17 and 11.10], but with the epicycle situated to the rear of the apogee of the eccentric, and the planet in advance of the apogee of the epicycle, in accordance with their given positions.

Then, since

\[ \angle AZB \left(= \angle DZM\right) = \begin{cases} 
86;33^\circ & \text{where 4 right angles = } 360^\circ \text{ (given)} \\
173;6^\circ & \text{where 2 right angles = } 360^\circ 
\end{cases} \]

in the circle about right-angled triangle DZM,

arc DM = 173;6°
and arc ZM = 6;54° (supplement).

Therefore the corresponding chords

\[ DM = 119;47^\circ \]
and ZM = \( 7;13^\circ \) where hypotenuse DZ = 120°.

32 These agree, to the nearest minute, with those found from the tables. Cf. p. 526 n.23.
Therefore, where the distance between the centres, $DZ = 3;25^\circ$,
and the radius of the eccentre, $DB = 60^\circ$,
\begin{align*}
DM & = 3;25^\circ \\
and ZM & = 0;12^\circ.
\end{align*}

And since $DB^2 - DM^2 = BM^2$,
$$BM = 59;54^\circ$$ in the same units.

Similarly, since $ZM = ML$, and $EL = 2DM$,
by addition, $BL = 60;6^\circ$ where $EL$ is computed as $6;50^\circ$.

Hence hypotenuse [of right-angled triangle $BEL$]
$$EB = 60;29^\circ$$ in the same units.

Therefore, where hypotenuse $EB = 120^\circ$, $EL = 13;33^\circ$,
and, in the circle about right-angled triangle $BEL$,
$$\text{arc } EL = 12;58^\circ$$
\[\therefore \angle EBZ = 12;58^\circ \text{ where } 2 \text{ right angles} = 360^\circ.\]

But $\angle AZB$ was given as $173;6^\circ$ in the same units.

Therefore, by subtraction, $\angle AEB = 160;8^\circ$ in the same units.

But the angle representing the apparent distance of the planet from the apogee,
$$\angle AEK$$ was given as \[\begin{cases} 76;4^\circ \text{ where } 4 \text{ right angles} = 360^\circ \\
152;8^\circ \text{ where } 2 \text{ right angles} = 360^\circ.\end{cases}\]

Therefore, by subtraction, $\angle KEB = 8;0^\circ$ in the same units.

Therefore, in the circle about right-angled triangle $BEN$,
$$\text{arc } BN = 8^\circ$$
and $BN = 8;22^\circ$ where hypotenuse $EB = 120^\circ$.

Furthermore, since the distance of the planet from H, the apogee of the epicycle, was $309;8^\circ$,
by subtraction [from $360^\circ$], $\text{arc } HK = 50;52^\circ$.
\[\therefore \angle HBK = \begin{cases} 50;52^\circ \text{ where } 4 \text{ right angles} = 360^\circ \\
101;44^\circ \text{ where } 2 \text{ right angles} = 360^\circ.\end{cases}\]

But we found that $\angle EBZ (= \angle HB\Theta) = 12;58^\circ$.

Therefore, by subtraction, $\angle \Theta BK = 88;46^\circ$ where $\angle KEB$ was shown to be $8^\circ$.

Therefore, by subtraction, $\angle BKN = 80;46^\circ$ in the same units.

Therefore, in the circle about right-angled triangle $BKN$,
$$\text{arc } BN = 80;46^\circ$$
and $BN = 77;45^\circ$ where hypotenuse $BK = 120^\circ$.

Therefore, where $BN$ was found as $4;13^\circ$, and the radius of the eccentre is $60^\circ$,
the radius of the epicycle, $BK \approx 61^\circ$.

Thus we have computed the following:
round about the beginning of the reign of Antoninus the longitude of Saturn's
apogee was $ \mp 23^\circ$;
where the radius of the eccentre carrying the epicycle is $60^\circ$,
the distance between the centres of the ecliptic and the eccentre which produces
the uniform motion is $6;50^\circ$,
and the radius of the epicycle is $6;30^\circ$.

Q.E.D.

$^{13}$ Reading ὑπέκειτο (with D) for ὑπόκειται ('is given') at H417.13.
It remains to demonstrate the correction of the periodic motions. For this purpose we again selected one of the accurately recorded ancient observations. In this it is declared that in the 82nd year in the Chaldaean calendar, Xanthikos 5, in the evening, the planet Saturn was 2 digits [i.e. 10 minutes] below [the star on] the southern shoulder of Virgo. Now that moment is in the 519th year from Nabonassar, Tybi [V] 14 in the Egyptian calendar [-228 Mar. 1], evening, at which time we find the longitude of the mean sun as $\pi 6;10^\circ$. But the fixed star on the southern shoulder of Virgo had a longitude at the time of our observation of $\nu 13;10^\circ$, thus at the moment of the observation in question, since to the intervening 366 years corresponds a motion of the fixed stars of about $3;3^\circ$, its longitude was, obviously, $\nu 9;1^\circ$. And the planet Saturn had the same longitude, since it was 2 digits to the south of the fixed star. By the same argument, since we showed that in our time its apogee was at $\nu 23^\circ$, at the observation in question it must have had a longitude of $\nu 19;1^\circ$. From this we conclude that at the above moment the apparent distance of the planet from the then apogee was $290;10^\circ$ of the ecliptic, while the mean sun was $106;50^\circ$ from the same apogee.

With the above as data, let there be drawn [Fig. 11.22] the diagram as for the same demonstration [for Mars and Jupiter, Figs. 10.18 and 11.11], [but] with the epicycle located in advance of the apogee of the eccentric, and the [mean] sun in advance of the perigee, with the radius from the epicycle centre to the

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34 This is clearly a Babylonian observation: see Introduction p. 13. On the 'digit' see p. 322 n.5. The star in question, $\gamma$ Vir, is one of the Babylonian 'normal stars' (cf. p. 453 n.70).

35 Catalogue no. XXVII 7.
planet drawn parallel to [the line indicating] the sun's position. Then, since the apparent position of Saturn was in advance of the apogee by 69;50° (the difference of 290;10° from one revolution), the angle at the centre of the ecliptic,

\[ \angle AE\Theta = \begin{cases} 69;50° & \text{where 4 right angles} = 360° \\ 139;40° & \text{where 2 right angles} = 360°. \end{cases} \]

And the angle of the sun's distance [from the apogee],

\[ \angle AEL \text{ is given as} \begin{cases} 106;50° & \text{where 4 right angles} = 360° \\ 213;40° & \text{where 2 right angles} = 360°. \end{cases} \]

Therefore, by addition, \( \angle OEL = \angle B\Theta E \), since \( B\Theta \) is parallel to \( EL \), is 353;20° where 2 right angles = 360°,

and, by subtraction [of \( \angle B\Theta E \) from 2 right angles]

\( \angle B\Theta N = 6;40° \) in the same units.

Therefore, in the circle about right-angled triangle \( B\Theta N \),

arc \( BN = 6;40° \)

and \( BN = 6;58° \) where hypotenuse \( B\Theta = 120° \).

H422 Therefore where the radius of the epicycle, \( B\Theta = 6;30° \),

\( BN = 0;23° \).

Similarly, since \( \angle AE\Theta = 139;40° \) where 2 right angles = 360°

and \( \angle EDM = 40;20° \) in the same units [complement],

in the circle about right-angled triangle \( DEM \),

arc \( DM = 139;40° \)

and \( DM = 112;39° \) where hypotenuse \( ED = 120° \).

Therefore, where the distance between the centres, \( ED = 3;25° \),

and the radius of the eccenter, \( DB = 60° \),

\( DM (= XN) = 3;12° \).

and, by addition, \( BNX = 3;35° \) where hypotenuse [of right-angled triangle \( BDX \)] \( DB = 60° \).

Therefore, where \( DB = 120° \), \( BX = 7;10° \),

and, in the circle about right-angled triangle \( BDX \),

\( \angle BDX = 6;52° \)

\( \therefore \angle BDX = 6;52° \) where 2 right angles = 360°

and, by subtraction [from a right angle],

\( \angle BDM = 173;8° \) in the same units.

And, by addition [of \( \angle EDM \)], \( \angle BDE = 213;28° \) in the same units,

and, by subtraction [from 2 right angles],

\( \angle BDA = 146;32° \) in the same units.

Therefore, in the circle about right-angled triangle \( DZK \),

arc \( ZK = 146;32° \)

and arc \( DK = 33;28° \) (supplement).

Therefore the corresponding chords

\( ZK = 114;55° \) where hypotenuse \( DZ = 120° \).

H423 Therefore, where the distance between the centres, \( DZ = 3;25° \),

and the radius of the eccenter, \( DB = 60° \),

\( ZK = 3;17° \)

and \( DK = 0;59° \),
XI 7. Derivation of Saturn's mean motion from observations 543

and, by subtraction [from DB], KB = 59;1° where ZK = 3;17°.
Hence hypotenuse [of right-angled triangle BZK]

ZB = 59;6° in the same units.

Therefore, where hypotenuse ZB = 120°, ZK = 6;40°,
and, in the circle about right-angled triangle BZK,
arc ZK = 6;22°.
∴ ∠ ZBK = 6;22° where 2 right angles = 360°.

But we found that ∠ ADB = 146;32° in the same units.
Therefore, by addition, the angle representing the mean position in longitude,

∠ AZB = \left\{152;54°, 76;27°\right\} where 4 right angles = 360°.

Therefore at the moment of the above observation Saturn's distance from the apogee in mean longitudinal motion was 283;33°, i.e. its [mean] longitude was [m 19;20° + 283;33° =] m 2;53°.

And since the sun's mean position is given as 106;50°, if we add the 360° of one revolution to the latter and from the resulting 466;50° subtract the 283;33° of the longitude [from apogee], we get, for the anomaly at that moment, 183;17° from the apogee of the epicycle.46

So, since we have shown that at the moment of the above observation, which is in the 519th year from Nabonassar, Tybi [V] 14,37 in the evening, [Saturn] was 183;17° [in anomaly] from the apogee of the epicycle, and at the moment of the third opposition, which was in the 883rd year from Nabonassar, Mesore [XII] 24. noon, it was 174;44°, it is clear that in the interval between the observations, which comprises 364 Egyptian years and 219\frac{1}{2} days,
the planet Saturn has moved 351;27° (beyond 351 complete revolutions in anomaly).
That is again almost the same increment as one derives from the [tables for] mean motions which we constructed. For it was from these very same elements that we derived the daily mean motion [in anomaly], by dividing the total in degrees computed from the number of complete revolutions plus the increment by the total in days computed from the time [interval].38

8. {On the epoch of Saturn's periodic motions}

Now since the time interval from the first year of Nabonassar, Thoth 1, noon, to the above ancient observation is 518 Egyptian years 133\frac{1}{2} days,
and this interval comprises increments of 216;10° in longitude39

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36 Accurate computation gives 183;16° to the nearest minute.
37 Reading i° for δ° (4) at H424.6. The latter is found as the reading of the first hand in D, but is probably a misprint in Heiberg's text. Corrected by Manitius.
38 On the actual derivation of Saturn's mean motion in anomaly see Appendix C.
39 Reading σ 2 (with GD, Ar) for σ 3 (216;9°), which is Heiberg's correction (most Greek mss. have 216° or 216;9°). Heiberg was no doubt influenced by the fact that the mean motion, according
XI 8. Epoch positions of Saturn in mean motion

544

and $149;15^\circ$ in anomaly, if we subtract the latter from the [respective] positions at the observation, we get, for the same moment of epoch, the mean position of the planet Saturn as in longitude: $26;43^\circ$
in anomaly: $34;2^\circ$ from the apogee of the epicycle.

By the same computation [as before], we find the apogee of its eccentre in $\mu_e 14;10^\circ$. Q.E.D.

9. {How the true positions can be found geometrically from the periodic motions}

Furthermore, conversely, given the arcs of the periodic [motions] on the eccentre which produces the uniform motion [i.e. the equant] and on the epicycle, one can readily obtain the apparent positions of the planets geometrically, as will become clear to us through the same [diagrams as above, e.g. Fig. 11.21].

For [see Fig. 11.23], in the simplified diagram containing [only] the eccentre and epicycle, we join $ZB\Theta$ and $EBH$. Then, if we are given the mean position in longitude, i.e. $\angle AZB$, from what we proved previously, $\angle AEB$ will be given according to both hypotheses,\(^{11}\) and so will $\angle EBZ$, (which is the same as

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Fig. 11.23

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\(^{10}\) To Ptolemy's table, is only 216;8,27°. But 216,10° is confirmed by the reading 26;43° below (in which all mss. agree here and in IX 4: Heiberg's correction to 26;44° must be rejected), and we must admit that Ptolemy made a small computing error. Cf. HAMA 182 n.15.

\(^{11}\) I presume that by 'both hypotheses' Ptolemy means the simple eccentric model and the full, equant model. A possible alternative would be eccentric and epicyclic models, but since these are not discussed (for the planets) until Bk. XII, this seems unlikely.
And if we also suppose that the planet is located on the epicycle, e.g. at point K, and, when EK and BK are joined, arc ΘK from the planet K on to EB, then \( \angle HBK \) will be given by addition [of the given angles \( \angle \Theta BK, \angle HB\Theta \)], and hence the ratio of KL and LB to BK and also, obviously, [their ratio] to EB. Accordingly, the ratio of the whole line EBL to LK will be given. Hence \( \angle LEK \) will be given, and we will have computed the angle AEK which comprises the apparent distance of the planet from the apogee.

10. [Method of constructing tables for the anomalies]44

However, to avoid always computing the apparent positions geometrically (for although that method is the only one which provides a fully accurate solution to the problem, it is too cumbersome to be convenient for [astronomical] investigations), we have constructed for each of the five planets a table which is as easy to use as we could devise, while at the same time being very close to full accuracy. [Each table] contains the individually determined anomalies of the planets, so that we can use them as a ready means of computing any particular apparent position, once we are given the periodic motions from the respective apogees.

We have again arranged each of the tables in 45 lines for the sake of symmetry, and we have arranged each in 8 columns. The first 2 columns will contain the numbers of the mean positions arranged as for the sun and moon [III 6 and V 8]: in the first column the 180 degrees beginning from the apogee, from the top down, and in the second the remaining 180 degrees of the [other] semi-circle, from the bottom up, in such a way that the number '180' is in the last line in both columns, and the increment in the numbers is 6° in the top 15 lines, but 3° in the 30 lines remaining below (for the differences between [successive] values for the anomalies remain almost constant for longer stretches near the apogee, whereas they change faster near the perigee). As for the next two columns, the third will contain the equations corresponding to the mean position in longitude (each to the arguments on the same line), computed for the greater eccentricity, but under the simplifying assumption that the centre of the epicycle is carried on the equant. The fourth column will contain the corrections to the equations due to the fact that the epicycle centre is carried, not on the above circle, but on another. The method by which each of these quantities [the equation and its correction], both in combination and separately, can be found geometrically has

42 Euclid, Data Props. 40 and 8.
43 Euclid, Data Props. 6 and 8.
44 See HAMA 183–6, Pedersen 291–4.
45 Reading δυοθεσι (with D.1s) for δυοθεσι πρώτων (‘first top’) at H428.18.
46 I.e. the equations of center computed for the double eccentricity (ZE in Fig. 11.23, where the equation is \( \angle ZBE \)).
already been made plain by numerous preceding theorems. In this place, since this is a [scientific] treatise, it was appropriate to display this way of separating the zodiacal anomaly, and hence to tabulate it in two columns. However, for actual use, a single column formed by combining these two will suffice.

Each of the next three columns will contain the equations due to the epicycle. These, again, are computed under a simplifying assumption, [namely] that the apogee or perigee of the epicycle is viewed along the line from the observer [to the epicycle centre]. The way in which this kind of demonstration is performed has also been made plain by the previous theorems. The midmost of these three columns (which is the sixth from the beginning) will contain the equations computed for the ratio [of epicycle radius to distance of epicycle centre] at mean distance; the fifth will contain, [for each argument], the difference between the equation at greatest distance [of the epicycle] and the equation for the same argument at mean distance; the seventh will contain the differences between the equations at least distance and the [corresponding] equations at mean distance. For we have shown that for the following epicycle sizes (from now on it would be best to list [the planets] in order from the outermost):

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Venus</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
<td>6;30''</td>
<td>11;30''</td>
<td>39;30''</td>
<td>43;10''</td>
<td>22;30''</td>
</tr>
</tbody>
</table>

the mean distance, i.e. the distance [equivalent] to the radius of the eccentric which carries the epicycle, is 60'' in all cases; and the greatest distances (with respect to the centre of the ecliptic), are:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Venus</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
<td>63;25''</td>
<td>62;45''</td>
<td>66''</td>
<td>61;15''</td>
<td>69''</td>
</tr>
</tbody>
</table>

The least distances (defined similarly) are:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Venus</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
<td>56;35''</td>
<td>57;15''</td>
<td>54''</td>
<td>58;45''</td>
<td>55;34''</td>
</tr>
</tbody>
</table>

As for the remaining, eighth column, we provided it in order that one may find the applicable fraction of the above differences [in cols. 5 and 7] when the planet’s epicycle is not exactly at mean, greatest or least distance, but in an intermediate position. The computation of this correction is based only on the maximum equation ([i.e.] that formed by the tangent from the observer to the epicycle) at each intermediate distance; for the [fraction] of the difference to be applied for any particular position [of the planet] on the epicycle is not significantly different from that for the greatest equation.

But in order to make our meaning clearer, and to explain the actual method of computing the [fractions] to be applied, let us draw [see Fig. 11.24] the line

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47 E.g. XI 5 pp. 529-37 and XI 9.
48 The didactic purpose of the Almagest is made explicit here. [scientific] treatise is my translation of σπουταϊς. For this meaning, which is typical of Hellenistic prose, but seems not to be classical, see LSJ s.v. 3. In the Handy Tables Ptolemy does indeed combine the two columns into one, and that is the pattern of all subsequent ancient and mediaeval astronomical tables.
49 I.e. the equation of anomaly is computed as a function, not of the mean anomaly, but of the true, that is as counted from the true apogee of the epicycle.
50 For this value for the least distance of the centre of the epicycle for Mercury see IX 9 p. 460 with n.89.
through both centres (the centre of the ecliptic and the centre of the equant point) at B. Produce line BEZ, describe the epicycle ZH about centre E, and draw the tangent to it from G, line GH. Join GE and perpendicular EH, and let us suppose, *exempli gratia*, that for each of the five planets the epicycle centre is 30° from the apogee of the eccentric in mean motion.

Then (to avoid lengthening the computation by demonstrating the same thing over and over again), we have demonstrated at length in what preceded, both in the hypothesis for Mercury and in that for the other planets, that if \( \angle ABE \) is given, the ratio of GE to the radius of the epicycle (HE) is also given. Hence, by means of the computations for each particular planet, with \( \angle ABE \) taken as 30°, this ratio comes to:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>63:2:6:30</td>
</tr>
<tr>
<td>Jupiter</td>
<td>62:26:11:30</td>
</tr>
<tr>
<td>Mars</td>
<td>65:24:39:30</td>
</tr>
<tr>
<td>Venus</td>
<td>61:6:43:10</td>
</tr>
<tr>
<td>Mercury</td>
<td>66:35:22:30</td>
</tr>
</tbody>
</table>

Thus we will get for \( \angle EGH \), which comprises the maximum epicyclic equation at that point,

<table>
<thead>
<tr>
<th>Planet</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>5:55°</td>
</tr>
<tr>
<td>Jupiter</td>
<td>10:36°</td>
</tr>
<tr>
<td>Mars</td>
<td>37:9°</td>
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<td>Venus</td>
<td>44:56°</td>
</tr>
<tr>
<td>Mercury</td>
<td>19:45°</td>
</tr>
</tbody>
</table>

And we compute the greatest equations at the mean distance, according to the ratios set out just above, as (to avoid repetition, we [simply list them] in an order corresponding to the above order of the planets):

<table>
<thead>
<tr>
<th>Planet</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>6:13°</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11:30°</td>
</tr>
<tr>
<td>Mars</td>
<td>41:10°</td>
</tr>
<tr>
<td>Venus</td>
<td>46:00°</td>
</tr>
<tr>
<td>Mercury</td>
<td>22:20°</td>
</tr>
</tbody>
</table>

31 Mercury, IX 9 pp. 457-60; other planets, X 2, X 8, XI 2, XI 6.
32 Reading \( \xi \alpha \) \( \zeta \) (with AD,Ar) for \( \xi \alpha \ \kappa \zeta \) (61:26) at H433,4. At H503,5 all mss. have 61:6. Corrected by Manitius.
those at the greatest distances as
5;53° 10;34° 36;45° 44;48° 19;2°;
and those at the least distances as
6;36° 11;35° 47;1° 47;17° 23;53°.

Thus the differences between the equations at mean distance and those at
least distance are
0;23° 0;32° 5;51° 1;17° 1;51°.
while the differences between [those at mean distance and] those at least
distance are
0;20° 0;29° 4;25° 1;12° 3;0°.

Now the equations of the distances in question [for a mean longitude of 30°
from the apogee] are less than those for mean distance, and differ from the latter
by the following amounts:
0;17° 0;26° 4;1° 1;31° 2;17°,
and the latter (expressed as sixtieths of the above total differences between [the
equations for] mean and greatest distance) are
for Saturn Jupiter Mars Venus Mercury
52;30 54;50 54;34 52;55 45;40.

So those are the values, in sixtieths, which we put in the 8th column of the
appropriate table, on the line containing the number '30' for the mean motion
in longitude.

For those distances which have equations greater than those at mean
distance, we again reduced the [resulting] differences to sixtieths, but in this
case expressed as fractions, not of the [corresponding] equations at greatest
distance, but of those at least distance. In the same way [as above], we
performed the computation for all other positions [of the epicycle] at 6°
intervals of mean longitude, and tabulated the resulting fractions, expressed
in sixtieths, opposite the appropriate arguments. As we said, the fraction of the
difference to be applied is sensibly the same even when the position of a planet is
not at the greatest epicyclic equation, but at some other point on the epicycle.

The layout of the five tables is as follows.

11. {Planetary equation tables}
### XI 11. Table for anomaly of Saturn

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>Common Numbers</td>
<td>Equation in Longitude</td>
<td>Equation</td>
<td>Difference</td>
<td>Equation Difference</td>
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<td>0 0</td>
<td>0 0</td>
<td>+39 28</td>
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So when we want to determine the apparent position of any one of the planets from the periodic motions in longitude and anomaly, by employing the above [tables], we carry out the numerical computation (which is one and the same for all five planets) in the following way.

From the tables for mean motion we compute the mean positions in longitude and anomaly for the moment required (by addition, and casting out complete revolutions). Then, taking as argument the distance from the apogee of the eccentre at that moment to the mean position in longitude, we enter the anomaly table belonging to the planet in question, and take the value for the longitudinal correction corresponding to that argument in the third column, together with the value (in minutes) in the fourth column (which has to be added or subtracted). We subtract the result from the [mean] longitude and add it to the anomaly if the above-mentioned argument for the longitude [i.e. the mean centrum] falls in the first column, but if it falls in the second column, we add the result to the longitude and subtract it from the anomaly, to get both positions corrected.

Then we enter with the corrected anomaly [counted] from the [epicyclic] apogee into [one of] the first two columns, take the corresponding amount in the sixth column (the equation for mean distance), and write it down separately. Similarly, we enter with the amount for the mean longitude [i.e. mean centrum] (which we used as argument at the beginning) into the same argument [columns]; then, if [that argument] falls in the upper lines, which are closer to the apogee than that for mean distance (this will be clear from the entries in the eighth column), we take the corresponding number of sixtieths in the eighth column, take, from the fifth column (for the [difference at] greatest distance), the entry on the same line as that for the equation at mean distance which was written down separately, form the fraction of that [entry for the] difference corresponding to the above number of sixtieths, and subtract the result from the amount which we wrote down separately. But if the argument of the above longitude [i.e. the mean centrum] falls in the lower lines, which are closer to the perigee than that for mean distance, we take the corresponding number of sixtieths in the eighth column, as before, take, from the seventh column (for the [difference at] least distance), the entry corresponding to the equation for mean [distance] which was written down separately, form the fraction of that difference corresponding to the above number of sixtieths, and add the result to the number we wrote down separately. The result will be the corrected equation [of anomaly]. If the corrected anomaly is in the first column, we add that corrected equation to the amount for the corrected longitude, but we subtract it if the corrected anomaly is in the second column. Using the result to count from the apogee of the planet at that moment, we reach its apparent position.

56 See HAMA 186-7 and Appendix A, Example 14.
57 I.e. if the entry in the eighth column is subtractive, the epicycle centre is closer to apogee than to mean distance; if additive, closer to perigee (for Mercury, to least distance) than to mean distance.
1. \{On the preliminaries for the retrogradations\}\(^1\)

Now that we have demonstrated the above, the appropriate sequel would be to examine the greatest and least retrogradations associated with each of the 5 planets, and to show that the sizes of these, [as computed] from the above hypotheses, are in as close agreement as possible with those found from observations.

In the definition of this kind of problem, there is a preliminary lemma demonstrated (for a single anomaly, that related to the sun) by a number of mathematicians, notably Apollonius of Perge, to the following effect.

[1] If [the synodic anomaly] is represented by the epicyclic hypothesis, in which the epicycle performs the [mean] motion in longitude on the circle concentric with the ecliptic towards the rear [i.e. in the order] of the signs, and the planet performs the motion in anomaly on the epicycle [uniformly] with respect to its centre, towards the rear along the arc near the apogee, and if a line is drawn from our point of view intersecting the epicycle in such a way that the ratio of half that segment of the line intercepted within the epicycle to that segment intercepted between the observer and the point where the line intersects the epicycle nearer its perigee is equal to the ratio of the speed of the epicycle to the speed of the planet, then the point on the arc of the epicycle nearer the perigee determined by the line so drawn is the boundary between forward motion and retrogradation, so that when the planet reaches that point it creates the appearance of station.

[2] If the anomaly related to the sun is represented by the eccentric hypothesis (which is a viable hypothesis only for the three [outer] planets which can reach any elongation from the sun),\(^2\) in which the centre of the eccentre moves [uniformly] about the centre of the ecliptic with the speed of the [mean] sun towards the rear [i.e. in the order] of the signs, while the planet moves on the eccentre in advance [i.e. in the reverse order] of the signs with a speed [uniform] with respect to the centre of the eccentre and equal to the [mean] motion in anomaly, and if a line is drawn in the eccentre through the centre of the ecliptic (i.e. the observer) in such a way that the ratio of half the whole line to the smaller of the two segments of the line formed by [the position of] the observer is

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\(^1\) On chs. 1–6 see HAMA 190–201, Pedersen 331–49.

\(^2\) This type of eccentric model is in fact applicable to the inner planets as well, provided that, for the speed of the centre of the eccentre, one uses, not the speed of the mean sun, but the sum of the speeds of the mean sun and the planet’s anomaly (which sum is the same as the modern heliocentric mean motion). I do not understand why Ptolemy does not recognise this.
equal to the ratio of the speed of the eccentre to the speed of the planet, then when the planet arrives at the point in which the above line cuts the arc of the eccentre near the perigee, it will produce the appearance of station.

H452 We too shall achieve the required result by a method which, though summary, is none the less more convenient: we employ a proof which contains both hypotheses combined in a common [figure], to demonstrate their agreement and similarity in these ratios of theirs too.

Let [Fig. 12.1] the epicycle be ABGD on centre E and diameter AEG, which is produced to Z, the centre of the ecliptic (i.e. our point of view). Cut off equal arcs, GH, GΘ, on either side of the perigee G, and draw ZHB and ZΘD from Z through points H and Θ. Join DH and ΘD to intersect each other at point K, which will, obviously, lie on diameter AG.

We say, first, that

\[ AZ:ZG = AK:KG. \]

[Proof:] Join AD, DG, and draw LGM through G parallel to AD. Then LGM will, obviously, be perpendicular to DG (for \( \angle ADG \) is right).

Then, since \( \angle GDH = \angle GDG \) [on equal arcs, Euclid III 27],

\[ GL = GM \] [triangles LDG, MDG congruent].

Fig. 12.1

3 'in these . . . too' refers to the earlier demonstrations of the equivalence of the hypotheses in III 3 and IV 5. Note that Ptolemy opposes his proof (ἡμεῖς δὲ) to that of the earlier mathematicians, notably Apollonius (προαποδεικνύοντι μὲν, H450.9). This counts against Neugebauer's supposition (H.A.M.1 264) that Ptolemy has taken this elegant equivalence theorem from Apollonius, despite its relationship to Conics III 37-40 and to Plane Loci II 8 ("Circle of Apollonius").
\[ \therefore AD:GL = AD:GM. \]

But \[ AD:GM = AZ:ZG \] \[ \text{triangle ADZ} \parallel \text{triangle GMZ} \]
and \[ AD:LG = AK:KG \] \[ \text{triangle ADK} \parallel \text{triangle GLK}. \]
\[ \therefore AZ:ZG = AK:KG. \]

So, if we imagine epicycle ABGD to be the actual eccentre in the eccentric hypothesis, the point K will be the centre of the ecliptic, and diameter AG will be divided by it in the same ratio as [the corresponding amounts] in the epicyclic hypothesis. For we have shown that the ratio of the greatest distance in the epicyclic hypothesis, AZ, to the least distance, ZG, is the same as the greatest distance in the eccentric hypothesis, AK, to the least distance, KG.

We also say, [secondly], that
\[ DZ:Z\Theta = BK:K\Theta. \]

[Proof:] In the similar diagram [Fig. 12.2] join the line BND (obviously, this will be perpendicular to diameter AG), and draw \( \Theta X \) parallel to it from \( \Theta \). Then, since

\[ \begin{align*}
BN &= ND, \\
BN: \Theta X &= ND: \Theta X.
\end{align*} \]

But \[ ND: \Theta X = DZ: Z\Theta \] \[ \text{triangle ZND} \parallel \text{triangle ZX}\Theta \]
and \[ BN: \Theta X = BK: K\Theta \] \[ \text{triangle BNK} \parallel \text{triangle } \Theta XK. \]
\[ \therefore DZ: Z\Theta = BK: K\Theta. \]

So, \textit{componendo},
\[ (DZ + Z\Theta): Z\Theta = K\Theta: K \Theta. \]
And, dropping perpendiculars $EO$ and $EP$, and dividendo, [we get],
$$OZ:Z\Theta = P\Theta:K\Theta.$$  

And, dividendo once again,
$$O\Theta:Z\Theta = PK:K\Theta.$$

Therefore, if, in the epicyclic hypothesis, $DZ$ is drawn in such a way that the ratio of $O\Theta$ to $Z\Theta$ equals the ratio of the speed of the epicycle to the speed of the planet, in the eccentric hypothesis $PK:K\Theta$ will have that same ratio.

The reason that in this case [i.e. in the eccentric hypothesis] we do not use this ratio obtained dividendo (namely $PK:K\Theta$) to get the stations, but rather the undivided ratio (namely $P\Theta:K\Theta$), is that the epicycle's speed is in the same ratio to the planet's as the [mean] motion in longitude (alone) to the [mean] motion in anomaly, whereas the ratio of the eccentric's speed to the planet's is the same as that of the sun's mean motion (i.e. the sum of the planet's [mean] motions in longitude and anomaly) to the motion in anomaly. Thus, e.g. for Mars.

\[
\text{speed of epicycle : speed of planet} \approx 42:37
\]

(or that, approximately, is the ratio which, as we demonstrated, holds between the [mean] motions in longitude and anomaly).^5

Hence that is also the ratio of $O\Theta:Z\Theta$.

But speed of eccentric : speed of planet $\approx [42 + 37 =] 79:37$.

i.e. this is the same as the ratio obtained componendo, $P\Theta:K\Theta$.

since we found that the divided ratio, $PK:K\Theta$, is equal to $O\Theta:Z\Theta$ (i.e. 42:37).

Let the above suffice us as preliminary theorems. It remains to prove that when one takes lines [corresponding to $ZD, B\Theta$] divided in the ratio described, then in both hypotheses $H$ and $\Theta$ represent the points in which station appears to take place, and [thus] arc $HG\Theta$ must be retrograde, and the remainder [of the circle] possessing forward motion. [For this purpose] Apollonius proposes the following preliminary lemma.

[See Fig. 12.3.] In triangle $ABG$, in which

\[BG > AG,\]

if we cut off [from $GB$] $GD > AG,^6$ then

\[GD:BD > \angle ABG: \angle BGA.\]

His proof is as follows.

Complete the parallelogram $ADGE$ (he says), and let $BA$ and $GE$ be produced to meet at $Z$. Then, since

\[AE (= GD) > AG,\]

the circle drawn on centre $A$ with radius $AE$ will either pass through $G$ or beyond $G$. Let it be drawn to pass through $G$, as $HEG$. Then, since

triangle $AEZ > \text{sector } AEH$

and triangle $AEG < \text{sector } AEG$,

triangle $AEZ : \text{triangle } AEG > \text{sector } AEH : \text{sector } AEG$.

But sector $AEH : \text{sector } AEG = \angle EAZ : \angle EAG$

^4 For $DZ + Z\Theta = 2OZ$, and $\Theta = 2P\Theta$ (Euclid III 3). \[2OZ:Z\Theta = 2P\Theta:K\Theta. \]

\[OZ:Z\Theta = P\Theta:K\Theta.\]  

It is this last step which is described as dividendo (διδυότα). See Introduction pp. 17-18 for the two senses of this term.

^5 IX 3 p. 424. 37 returns in anomaly correspond to about 42 revolutions in longitude and 79 years.

^6 Literally 'not less than $AG$'.
and triangle $\triangle AEZ : \triangle AEG = ZE:EG$ (bases).\footnote{Euclid VI 1: triangles with the same height are in proportion to their bases.}
\[ \therefore ZE:EG > \angle ZAE: \angle EAG. \]
And $\angle ZAE = \angle ABG$  
and $\angle EAG = \angle BGA$.  
\[ \therefore GD:DB > \angle ABG: \angle AGB. \]

And it is obvious that if $GD (= AE)$ is supposed, not equal to $AG$, but greater, \footnote{The situation where $EG:GZ = speed of epicycle : speed of planet$ is the limiting situation for retrogradation to occur: see p. 561.} the difference in the ratios will be even greater.

Now that we have established this preliminary lemma, let [Fig. 12.4] the epicycle be $ABGD$ on centre $E$ and diameter $AEG$. Produce $AEG$ to $Z$, [representing] our point of view, so that
\[ EG:GZ > speed of epicycle : speed of planet. \]
Thus it will be possible to draw a line $ZHB\footnote{Because of Euclid III 8, which proves that of all lines drawn to a circle from a point outside it, that through the centre is the least.}$ in such a way that
\[ BH:HZ = speed of epicycle : speed of planet. \]
Then, by what we proved previously, if we cut off arc $AD$ equal to arc $AB$, and join $D\Theta H$, point $\Theta$ will represent our point of view in the eccentric hypothesis, and
\[ DH:\Theta H = speed of eccentric : speed of planet. \]

We say, then, that in either hypothesis, when the planet reaches point $H$, it will produce the appearance of station, and if we cut off arcs, however small, on either side of $H$, we will find that the arc intercepted towards the apogee will be an arc of forward motion, and the arc towards the perigee will be retrograde. \footnote{Euclid III 15.}

[Proof:] First, cut off an arbitrary arc towards the apogee, $KH$, draw $ZKL$ and $K\Theta M$, and join $BK$, $DK$ and also $EK$ and $EH$.

Then since, in triangle $BKZ$,
\[ BH > BK, \]
\[ BH:HZ > \angle HZK: \angle HBK \] [cf. above].
Fig. 12.4

\[ \frac{1}{2} BH:HZ > \angle HZK : 2 \angle KBH = \angle HZK : \angle KEH \]
But \( \frac{1}{2} BH:HZ = \text{speed of epicycle : speed of planet} \).
\[ \therefore \angle HZK : \angle KEH < \text{speed of epicycle : speed of planet.} \]

Therefore the angle which has the same ratio to \( \angle KEH \) as the ratio (speed of epicycle : speed of planet) is greater than \( \angle HZK \). Let that angle be \( \angle HZN \).

Then, in the time that the planet takes to travel arc \( KH \) of the epicycle, the epicycle centre has moved in the opposite direction by an amount equal to the [angular] distance from \( ZH \) to \( ZN \). So it is clear that arc \( KH \) of the epicycle has moved the planet in advance through an angle at our eye (\( \angle HZK \)) which is less than the angle (\( \angle HZN \)) through which [the motion of] the epicycle itself has moved it towards the rear during the same space of time. Thus the planet has undergone a forward motion [of the amount] of \( \angle KZN \).

Similarly, to carry out the reasoning as if the circle [ABGD] were an eccentric:

11 Reading ως ἐπὶ ἐκκέντρου τοῦ κύκλου (with C²D) for ως ἐπὶ τοῦ ἐκκέντρου κύκλου ('as on the eccentric circle') at H460,13.
since $BH:HZ > \angle HZK:\angle HBK$,

*componendo*, $BZ:ZH > [\angle HZK + \angle HBK =] \angle BKL:HBK$.

But $BZ:ZH = D\Theta:\Theta H$.\(^{12}\)

And $\angle BKL = \angle DMK$.\(^{13}\)

and $\angle HBK = \angle HDK$.

\[\therefore D\Theta:\Theta H > \angle DMK:HDK.\]

So, *componendo*, $DH:H\Theta > [\angle DMK + \angle HDK =] \angle H\Theta K:\angle HDK$.

Therefore, *dividendo*, $\frac{1}{2}DH:H\Theta > \angle H\Theta K:2\angle HDK = \angle H\Theta K:HEK$.

But $\frac{1}{2}DH:H\Theta = \text{speed of eccentric} : \text{speed of planet}$.

\[\therefore \angle H\Theta K:HEK < \text{speed of eccentric} : \text{speed of planet}.\]

Therefore the angle which bears the same ratio to $\angle HEK$ as the speed of the eccentric bears to the speed of the planet is greater than $\angle H\Theta K$. Let it, again, be $\angle H\Theta N$. So, since the planet, in its own motion along $KH$, has travelled in advance through $\angle KEH$, and in the same space of time has been carried by the motion of the eccentric towards the rear through $\angle H\Theta N$, which is greater than $\angle K\Theta H$, it is clear that, by this [hypothesis] too, the planet will appear to have undergone a forward motion [of the amount] of $\angle K\Theta N$.

It is easy to see that the same method can be used to prove the opposite case,\(^{14}\)

if in the same figure [Fig. 12.5] we suppose that

\[
\frac{1}{2}LK:KZ = \text{speed of epicycle} : \text{speed of planet}
\]

and hence $\frac{1}{2}MK:K\Theta = \text{speed of eccentric} : \text{speed of planet}$;

and imagine arc $KH$ cut off towards the perigee side of line $LZ$.

For, if we join $LH$ to produce the triangle $LZH$, in which there is cut off $LK > ZH$, then

\[
LK:KZ < \angle HZK:HLK.
\]

\[\therefore \frac{1}{2}LK:KZ < \angle HZK:2\angle HLK = \angle HZK:KEH,
\]

which is the opposite of what was proved above.\(^{15}\)

And, by the same reasoning, one will come to a conclusion opposite [to the above, namely] that

\[
\angle KEH:\angle HZK < \text{speed of planet} : \text{speed of epicycle}
\]

and $\angle KEH:\angle H\Theta K < \text{speed of planet} : \text{speed of eccentric}$.

So the angle which has the same ratio [to $\angle HZK$ or $\angle H\Theta K$ as the speed of the planet has to the speed of the epicycle or eccentric] turns out to be greater than $\angle KEH$, and the resulting retrograde [component of] motion is greater than the forward.

Furthermore, it is clear that for distances at which

$EG:GZ \leq \text{speed of epicycle} : \text{speed of planet}$

it will be impossible to draw another line [to the circle which will be cut] in a ratio equal to that [of the speeds of epicycle and planet], and the planet will not appear stationary or retrograde.

\(^{12}\)This was proven p. 557 (in Fig. 12.2 $DZ:Z\Theta = BK:K\Theta$).

\(^{13}\)Euclid III 27: angles standing on equal arcs are equal. I.e. Ptolemy assumes that arc $BL = arc DM$. This follows from the fact that $\Theta $ is a fixed point for given $Z$ (cf. HAMA 264–5). Cf. p. 556, where it is shown that $AZ:ZG = AK:KG$, hence $K$ (corresponding to $\Theta$ here) is a fixed point.

\(^{14}\)I.e. that the planet will be retrograde on the other side of the point defined by the ratio of the speeds.

\(^{15}\)p. 560, where $\frac{1}{2}BH:HZ > \angle HZK:KEH$. 
For since, in triangle $EKZ$, $EG$ has been cut off and is [equal to, i.e.] not less than $EK$,

$$\angle GZK: \angle GEK < EG:GZ.$$  

But $EG:GZ \leq$ speed of epicycle : speed of planet.

$$\therefore \angle GZK: \angle GEK < \text{speed of epicycle : speed of planet.}$$

Hence, since we have shown [p. 560] that, where this occurs, the planet has undergone a forward motion, we shall find no arc either on epicycle or on eccentric on which it will appear retrograde.

2. \textit{Demonstration of the retrogradations of Saturn}

That being established, we shall next set out the calculations of the retrogradations for each of the planets, in accordance with the hypotheses [previously] demonstrated, beginning with Saturn. The method is as follows.
Fig. Q

[See Fig. 12.6.]² Let the circle carrying the epicycle centre be AB on diameter AGB, on which G represents the centre of the ecliptic, i.e. our point of view. Describe the epicycle DEZH on centre A, and draw line GZE in such a way that, when perpendicular AΘ is dropped on to it, the ratio of half EZ (i.e. ΘZ) to ZG is that of the speed of the epicycle to the speed of the planet. Let us suppose, first, that the epicycle is situated at mean distance: thus the mean motions in longitude and anomaly are very nearly the same as the motions [in longitude and anomaly] taken with respect to the centre of the ecliptic.¹⁷

Now for Saturn, as we demonstrated [XI 6], where the mean distance GA is 60°, the epicycle radius AD = 62°.

Thus, by addition, DG = 66;30°,
and, by subtraction, GH = 53;30° in the same units.

¹⁶ Ptolemy uses an identical simplified figure (Figs. 12.6 - 12.12), in which the observer, G, is represented as the centre of the circle, for all situations. The actual situation is depicted in Fig. Q (copied from Manitius), where the subscripts 1, 2 and 3 represent the situations at mean, greatest and least distances respectively.

¹⁷ I.e. because the epicycle centre is the same distance from the observer as it would be in the simple model treated in ch. 1, one can assimilate the situation to that, and use the mean motions unmodified. As Ptolemy says, this involves an approximation, since the centre of motion is not the observer, but the equant point. However, for small eccentricities this is negligible.
Thus their product\(^{18}\) is 3557:45\(^{p}\).

But DG.GH = EG.GZ.

so \(EG.GZ = 3557:45\)\(^{p}\) in the same units.

Furthermore (in accordance with the mean motions), where the speed of the epicycle (i.e. \(\Theta Z\)) is 1\(^{p}\), the speed of the planet (i.e. \(ZG\)) is about 28:25,46\(^{p}\).\(^{19}\)

Therefore, by addition, \(EG = ZG + 2\Theta Z\) = 30:25,46\(^{p}\),

and \(EG.GZ = 865:5.32\)\(^{p}\) in the same units.

So if we divide\(^{20}\) 3557:45 by 865:5.32, which gives a quotient of 4:6,45, take the square root of the latter, 2:1,40, and multiply this factor into \(\Theta Z\) (\(= 1\)\(^{p}\)) and \(ZG\) (\(= 28:25,46\)\(^{p}\)) separately, we get

\[
\Theta Z = \frac{2:1.40}{2:1.40}\]

and \(ZG = 57:38,55\)\(^{p}\) where \((EG.GZ) = 3557:45\)\(^{p}\).

Then if we join \(AZ\), where \(AZ = 6:30\)\(^{p}\),

\(Z\Theta = 2:1,40\)\(^{p}\).

so where \(AZ = 120\)\(^{p}\), \(Z\Theta = 37:26,9\)\(^{p}\).

Therefore, in the circle about right-angled triangle \(AZ\Theta\),

\[
\text{arc } \Theta Z = 36:21,15\)\(^{p}\),\(^{21}\)

so \(Z\Theta = 36:21,15\)\(^{o}\) where 2 right angles = 360\(^{o}\).

\(\approx 18:10,38\)\(^{o}\) where 4 right angles = 360\(^{o}\).

---

\(^{18}\) Literally 'the rectangle contained by them'.

\(^{19}\) Taking the mean daily motions tabulated in IX 4 one finds the ratio of longitude to anomaly as \(1 : 28:25,55\) ... Ptolemy may have taken the rounded numbers \(0:57,7,43\)\(^{p}\)/\(^{o}\) and \(0:2,0.34\)\(^{p}\)/\(^{o}\), which lead to 28:25,46.

\(^{20}\)\(\piαραβάλωμεν παρά\). literally 'measure it by laying alongside'.

\(^{21}\) Accurately, 36:21,20\(^{p}\).
Furthermore, where hypotenuse [of right-angled triangle $AG\Theta$] $GHA = 60^\circ$, by addition, $GZ\Theta [= 57;38,55^\circ + 2;1,40^\circ] = 59;40,35^\circ$.

So, in the circle about right-angled triangle $AG\Theta$,

$$\text{arc } G\Theta = 168;5,39^\circ.$$  

So where $GHA = 120^\circ$, $GZ\Theta = 119;21,10^\circ$.

So, in the circle about right-angled triangle $AG\Theta$,

$$\angle G\Theta \begin{cases} = 168;5,39^\circ \text{ where } 2 \text{ right angles } = 360^\circ, \\ \approx 84;2,50^\circ \text{ where } 4 \text{ right angles } = 360^\circ. \end{cases}$$

Hence we get $\angle AG\Theta = 5;57,10^\circ$ (complement),

and $\angle ZAH = \angle GA\Theta - \angle ZA\Theta = 65;52,12^\circ$.

So, since the planet is seen along line $GZ$ at first station, and along $GH$ at [mean] opposition, it is clear that, if the epicycle centre had no motion towards the rear [during this interval], arc $ZH$ of the epicycle, comprising $65;52,12^\circ$, would produce a retrograde motion of the amount of $\angle AGZ$, $5;57,10^\circ$. But since, according to the above ratio of the speed of the epicycle to the speed of the planet, to this anomaly of $65;52,12^\circ$ correspond approximately $2;19^\circ$ in longitude, we get a retrograde motion of:

- from either station to opposition $3;38,10^\circ$ and $69^\circ$, (the latter is approximately the time the planet takes to move $2;19^\circ$ in mean longitude),

and a total retrogradation of $7;16,20^\circ$ and $138^\circ$.

Next we will investigate the [corresponding] quantities near the greatest distance under the same conditions, namely when the opposition halfway between the [two] stations brings the epicycle centre precisely to the apogee of the eccentre, and, obviously, brings each of the two stations to a distance in corrected longitude from the opposition (i.e. from the apogee) which is close to the $2;19^\circ$ which was derived [above] from the ratio between the mean [motions]. In this situation $AG$, which represents the distance at that moment, is negligibly different from the greatest distance, and hence is obtained via the theorems previously developed, and to $1^\circ$ of longitude corresponds an equation of about $6;30^\prime$. Therefore the ratio of the corrected [motion in] longitude to the corrected [motion in] anomaly, i.e. of the apparent speed of the epicycle at that moment to the apparent speed of the planet, is $0;53,30 : 28;32,16$. Then, repeating the same figure [Fig. 12.7], where the radius of the epicycle $DA$ is $6;30^\circ$, $GA$ (which is negligibly different from the greatest distance) is $63;25^\circ$.

Hence, by addition, $DG$ is computed as $69;55^\circ$,

and, by subtraction, $GH = 56;55^\circ$.

And $DG.GH = 6979;25,25^\circ$.

---

22 $65;52,12/28;25,46 = 2;19,1.$

23 $5;57,10^\circ - 2;19^\circ = 3;38,10^\circ$. In 69 days the planet moves $2;18,39^\circ$ in longitude, i.e. here (and throughout) Ptolemy rounds to the nearest day or convenient fraction of a day.

24 Since this must be the meaning, one has to correct Heiberg's punctuation at H468,3, deleting the comma after $\mu\kappa\chi\omega\zeta$, and inserting a comma after $\alpha\pi\omega\gamma\iota\omicron\omicron\omicron\omicron$.

25 Since the epicycle centre is in the apogee of the eccentric halfway between the stations, at the actual stations the epicycle is a little before or after apogee: hence 'negligibly different'.

26 In the anomaly table for Saturn (XI 11), to $6^\circ$ corresponds an equation of centre of $39^\prime$: hence to $1^\prime$ corresponds exactly $6^\prime$.

27 I.e. $1^\circ - 0;6,30^\circ$ and $28;25,46^\circ + 0;6,30^\circ$ (cf. p. 564 n.19). On the rationale for this procedure see $\text{HAMA}$ 193-4.
And, by hypothesis, where $Z\Theta$ (representing the speed of the epicycle) is $0;53,30^\circ$, $GZ$ (representing the speed of the planet) is $28;32,16^\circ$;
so, by addition, $EG [= GZ + 2Z\Theta] = 30;19,16^\circ$,
and $EG.GZ = 865;17,50^\circ$.

So, again, dividing $3979;25,25$ by $865;17,50$, which gives $4;35,56$, taking the square root of the latter, $2;8,40$, and multiplying this factor into $\Theta Z (= 0;53,30^\circ)$
and $ZG (= 28;32,16^\circ)$ separately, we get
\[
\Theta Z = 1;54,44^\circ,
\]
and $GZ = 61;11,52^\circ$.

And, by addition, $G\Theta = 63;6,36^\circ$ in the same units.
Therefore where hypotenuse $AZ$ [of right-angled triangle $AZ\Theta] = 120^\circ$,
$\Theta Z = 35;18,3^\circ$,
and where hypotenuse $GA$ [of right-angled triangle $AG\Theta] = 120^\circ$,
$G\Theta = 119;25,11^\circ$.
Therefore, in the circle about right-angled triangle $AZ\Theta$,
arc $\Theta Z = 34;13,4^\circ$,
and, in the circle about right-angled triangle $AG\Theta$,
arc $G\Theta = 168;43,38^\circ$.
\[
\therefore \angle ZA\Theta = 34;13,4^\circ
\]
and $\angle GA\Theta = 168;43,38^\circ$
\[
\therefore \angle ZA\Theta = 17;6,32^\circ
\]
and $\angle GA\Theta = 84;21,49^\circ$
where 2 right angles = $360^\circ$.

Hence, by subtraction [from $90^\circ$], $\angle AG\Theta$ (which represents the amount of
retrogradation\(^{28}\) which there would be between either of the stations and opposition, if the epicycle had no\(^{29}\) forward motion) is 5;38,11°,
and, by subtraction [of \(\angle ZA\Theta\) from \(\angle GA\Theta\)], \(\angle ZAH\) (which represents the apparent motion on the epicycle\(^{30}\) at the same [unchanging] distance) is 67;15,17°.

Now, according to the ratio of the speeds at the apogee, to the latter amount correspond 2;6,6° in corrected longitude;\(^{31}\) so we get, for half of the total retrogradation,

\[
[5;38,11° - 2;6,6° = ] 3;32,5° and 70^\circ\]

(the latter is approximately the time the planet takes to travel 2;21,25° in mean longitude, which is the amount corresponding to the above 2;6,6° in corrected longitude);

and, for the total retrogradation,

7;4,10° and 140^\circ\(^{32}\).

Again, we will investigate the [corresponding] quantities near the least distance, using the same figure [Fig. 12.8] and under similar conditions, i.e.

![Fig. 12.8](image-url)

\(^{28}\) Reading τῆς (with C\(^{2}\)D) for τῶ at H470.6. Cf. H473.1. Corrected by Manitius.

\(^{29}\) Reading μηδὲν at H470.8 for μηδὲνως. There is no ms. authority for my correction, but it is necessary for the sense. As a consequence of the corruption of the τῆς to τῶ just above, it was assumed that προηγησεως was connected with ὑπελείπετο, hence μηδὲν was changed to μηδὲνος to agree with it.

\(^{30}\) By ‘apparent motion’ Ptolemy means ‘as counted from the true [and not the mean] epicyclic perige’. 

\(^{31}\) One might suppose from what he says here that Ptolemy computes 67;15,17° × 0;53,30/28;32,16. This leads to 2;6,5°. The actual method of computation is explained at the end of XII6 (p. 582). It is as follows: 67;15,17° × 1/28;32,16 = 2;21,24°. To the latter corresponds an equation of 0;15,19°, which, subtracted from 67;15,17°, gives about 67°. Then 67° × 1/28;25,46 = 2;21,25°. 2;21,25° – 0;15,19° = 2;6,6°.
when the opposition halfway between the [two] stations is precisely at the perigee of the eccentric, and both stations are the above [ca. 2;19°] distance in longitude from the opposition (i.e. from the perigee).

In this situation the distance at that moment, AG, is found in the same way [as at greatest distance], since it is negligibly different from the least distance. And to 1° of longitude corresponds an equation of about 7;20 minutes. So here apparent speed of epicycle : apparent speed of planet = 1;7,20 : 28;18,26. Hence, where \( \Theta Z = 1;7,20^p \), \( GZ = 28;18,26^p \),
and, by addition, \( EG = 30;33,6^p \)
and \( EG.GZ = 864;49,58^p \).

But where the epicycle radius, \( DA = 6;30^p \),
AG (which is negligibly different from the least distance) is 56;35°;
hence, by addition, \( DG = 63;5^p \),
and, by subtraction, \( GH = 50;5^p \),
and \( DG.GH (= EG.GZ) = 3159;25,25'' \).

Therefore if, as before, we divide 3159;25,25 by 864;49,58, which gives 3:39,12, take the square root of that, 1;54,41, and multiply the latter factor into \( \Theta Z \) (= 1;7,20°) and \( ZG \) (= 28;18,26°) separately, we get
\[ \Theta Z = 2;8,43^p \]
where the epicycle radius, \( AZ = 6;30^p \), and the distance at that moment, \( AG = 56;35^p \);
and \( GZ = 54;6,22^p \) in the same units.

Hence, by addition, \( G\Theta = 56;15,5^p \) in the same units. Therefore, where hypotenuse \( AZ = 120^p \), \( \Theta Z = 39;36,18^p \),
and, where hypotenuse \( GA = 120^p \), \( G\Theta = 119;17,46^p \).

Hence, in the circle about right-angled triangle \( AZ\Theta \),
arc \( Z\Theta = 38;32,34^o \),
and, in the circle about right-angled triangle \( AG\Theta \),
arc \( G\Theta = 167;34,54^o \).

\[ \therefore \angle ZA\Theta = 38;32,34^o \] where 2 right angles = 360°,
and \( \angle GA\Theta = 167;34,54^o \) where 4 right angles = 360°.

Therefore, by subtraction [from 90°], we get \( \angle AG\Theta \), which represents the retrogradation (due to the planet's speed) between either of the stations and opposition, as 6;12,33°.

---

^2 To an argument of 177° (= 180° - 3°) corresponds (Table XI 11) an equation of centre of 0;22°. Hence to 1° near perigee corresponds 0;7,20°.
^3 I.e. 1 + 0;7,20 and 28;25,46 = 0;7,20.
^4 Deleting \( \alpha \) at H471,18-19 (with D,Ar).
^5 Reading \( \alpha \) for \( \eta \) (misprint in Heiberg) at H471,20.
^6 Reading \( \mu \) at H472,5 for \( \mu \) (1;54,42). The latter has no ms. authority, but is Heiberg's correction for the \( \mu \) (45) or \( \mu \) (49) of the Greek mss. '41' is the reading of Ger (all other Arabic mss. I have seen have '49'), and is shown to be correct not only because it is the square root of 3;39,12 (accurate to two sexagesimal places), but because (below) 1;54,41 x 28;18,26 = 54;6,22 (in agreement with the text), whereas 1;54,42 x 28;18,26 = 54;6,50.
^7 119;17,45° would be a more accurate result, and corresponds better to the arc 167;34,54° given below. But in the absence of any ms. authority I hesitate to change it.
and, again by subtraction [of $\angle ZA\Theta$ from $\angle GA\Theta$], $\angle ZAH$, which represents the apparent motion on the epicycle at the same [unchanging] distance, as $64;31,10^\circ$.

According to the ratio of the speeds at the perigee, to the latter amount correspond $2;33,28^\circ$ in corrected longitude.\(^{38}\) Hence we get for half the total retrogradation,

\[ [6;12,33^\circ - 2;33,28^\circ = ] 3;39,5^\circ \text{ and } 68^d \]

(\textit{the latter is approximately the time taken by the planet to travel, at mean speed, }2;16,45^\circ, \textit{which is the amount in mean longitude corresponding to the above }2;33,28^\circ \textit{of corrected longitude}).

[Thus] the total retrogradation is $7;18,10^\circ$ and $136^d$.

3. \textit{Demonstration of the retrogradations of Jupiter}

For Jupiter [see Fig. 12.9], according to our calculations for mean distance,

$\Theta Z:GZ = 1:10;51,29,^{39}$

and $EG:ZG = 12;51,29:10;51,29$,

\[ \text{Fig. 12.9} \]

\(^{38}\) Cf. p. 567 n.31. Computation: $64;31,10^\circ \times 1/28;18,26 = 2;16,45^\circ$. Equation for $180^\circ - 2;16,45^\circ$ is $0;16,43^\circ, 64;31,10^\circ + 0;16,43^\circ = 64;47,53^\circ$. The latter multiplied by $1/28;25,46$ gives $2;16,45^\circ$, and $2;16,45^\circ + 3;16,43^\circ = 2;33,28^\circ$.

\(^{39}\) Taking the first three places (rounded) of the mean daily motions from IX 4 (cf. p. 564 n.19), one gets $0;54,9,3 : 0;4,59,14 = 10;51,28,29 \ldots$
so \( \text{EG:ZG} = 139;37,39. \)

Furthermore \( \text{GA:AD} = 60 : 11;30 \)
and \( \text{GD:GH} = 71;30 : 48;30, \)
so \( \text{GD:GH} = 3467;45. \)

Dividing \([3467;45 \text{ by } 139;37,39]\) we get \( 24;50,9 \), the square root of which, \( 4;59,1^\circ \), we multiply into the above ratio of \( \Theta Z:GZ \), and get, in terms of the given sizes of \( \text{GA and AZ} \) [i.e. \( 60 \) and \( 11;30 \)],

\[
\Theta Z = 4;59,1^\circ
\]
and \( GZ = 54;6,44^\circ \) in the same units.

Hence, expressed in units where hypotenuses \( \text{AZ and AG} \) [respectively] are \( 120^\circ \),

\[
\Theta Z = 52;0,10^\circ
\]
and \( G\Theta = 118;11,30^\circ, \)
and the corresponding\(^{41}\) arcs are:

\[
\text{arc } Z\Theta = 51;21,41^\circ
\]
and \( \text{arc } G\Theta = 160;4,55^\circ. \)

Accordingly we compute \( \angle ZA\Theta \approx 25;40,50^\circ \)
and \( \angle G\Theta \approx 80;2,28^\circ, \)
and, by subtraction [of \( \angle G\Theta \) from \( 90^\circ \)], \( \angle ZGA \), which represents the retrogradation due to the planet's speed, is \( 9;57,32^\circ \), and \( \angle ZAH \), which represents the apparent [motion in] anomaly, is \( [\angle G\Theta - \angle ZA\Theta = ] 54;21,38^\circ. \) To the latter correspond \( 5;1,24^\circ \) in longitudinal motion, according to the above ratio [of \( 1 : 10;51,29 \)].\(^{42}\) Thus half the retrogradation is \( 4;56,8^\circ \) and about \( 60^\text{d}. \)

and the total retrogradation is \( 9;52,16^\circ \) and \( 121^\text{d}. \)

The distance at an elongation of about \( 5^\circ \) from apogee or perigee is [respectively] negligibly smaller than the greatest distance and negligibly larger than the least distance.

According to our calculations for greatest distance, the equation [corresponding to \( 1^\circ \)] for correcting [the speeds] is \( 5^\circ \) minutes.\(^{43}\) Hence

\[
\Theta Z:GZ = 0;54,50 : 10;56,39
\]
and \( \text{EG:GZ} = 12;46,19 : 10;56,39, \)
and \( \text{EG:GZ} = 139;46,42. \)

\(^{40}\) Ptolemy has made a computing error: correct is \( 139;36,48 \), and this is indeed found in Ger, derived no doubt from the kind of marginal correction found in \( D^2 \) (139;36,48,32). That the error is Ptolemy's is shown by the subsequent calculations (at H474.5 Ger reads \( 24;50,17 \), again in agreement with \( D^2 \) and the above amount, but the square root should be \( 4;59,2 \), whereas the whole tradition agrees on \( 4;59,1 \), which is confirmed by the following computations).

\(^{41}\) Reading \( \kappa\varepsilon' \alpha\upsilon\tau\alpha\tau\varsigma \) at H474.16 (with all mss.) for Heiberg's correction \( \kappa\varepsilon' \omega\upsilon\tau\omega\nu. \) Although the genitive is normal in the Almagest in expressions of the type \( \eta \varepsilon\varepsilon' \tau\nu\varsigma \Theta \nu\pi\rho\nu\varepsilon\varepsilon\varsigma \), the dative after \( \kappa\varepsilon' \) is perfectly good Greek, and is explicable here as avoiding the ambiguity of two genitive plurals referring to different things. I have restored the mss.' reading in the similar passages H476.9 and H477.18.

\(^{42}\) In fact \( 54;21,38/10;51,29 = 5;0,23^\circ \). But the number in the text is confirmed by the following computations.

\(^{43}\) Reading \( \xi' \) (with L, Ger) at H475.14 for \( \xi \) (5;6). The correction was made by Manitius, who notes that, in the table of anomaly, to an argument of \( 6^\circ \) corresponds an equation of centre of \( 0;31^\circ \), hence, to \( 1^\circ, 0;5,10^\circ \).
Furthermore, GA:AD = 62;45 : 11;30,
DG:GH = 74;15 : 51;15,
and DG:GH = 3805;18,45.
Dividing [3805;18,45 by 139;46,42], we get 27;13,26, the square root of which, 5;13,4, when multiplied into the above ratio of ΘZ:GZ, gives, in terms of the given sizes of GA and AZ [i.e. 62;45 and 11;30]

\[
\begin{align*}
\Theta Z &= 4;46,6^\circ, \\
GZ &= 57;6,19^\circ,
\end{align*}
\]

and, by addition, GΘ = 61;52,25^\circ.

Hence, expressed in units where hypotenuses AZ and AG [respectively] are 120^\circ,

\[
\begin{align*}
\Theta Z &= 49;45,23^p, \\
GZ &= 57;6,19^p, \\
\end{align*}
\]

and the corresponding arcs are:

\[
\begin{align*}
\text{arc } \Theta Z &= 48;59,34^\circ, \\
\text{arc } G\Theta &= 160;49,36^\circ.
\end{align*}
\]

Accordingly, \( \angle ZA\Theta = 24;29,47^\circ \)

and \( \angle GA\Theta = 80;24,48^\circ \).

And, by subtraction, \( \angle ZGA \), which represents the retrogradation due to the planet's speed, is \( [90^\circ - \angle GA\Theta = ] 93;5,12^\circ \), and \( \angle ZAH \), which represents the apparent [motion in] anomaly, is \( [\angle GA\Theta - \angle ZA\Theta = ] 55;55,1^\circ \). To the latter correspond 4;40,35^\circ in corrected longitudinal motion, and 5;6,35^\circ in mean [longitudinal] motion, according to the ratio [of speeds] at the apogee. Thus half the retrogradation is

\[
[93;5,12^\circ - 4;40,35^\circ = ] 4;54,37^\circ \]

and about 61\;1^d,

and the total retrogradation

9:49,14^\circ and 123\;1^d.

According to our calculations for least distance, the equation [corresponding to 1°] for correcting [the speeds] is found to be 5\;1^m. Hence

\[
\begin{align*}
\Theta Z:ZG &= 1:45 : 10:45,49, \\
EG:ZG &= 12:57,9 : 10:45,49, \\
\text{and } EG:ZG &= 139;24,56.
\end{align*}
\]

Furthermore, GA:AD = 57;15 : 11;30,
DG:GH = 68;45 : 45,45,
and DG:GH = 3145;18,45.

Dividing [the latter by 139;24,56], we get 22;33,39, the square root of which, 4;45, multiplied into the above ratio of ΘZ:GZ, gives, in terms of the above sizes of GA and AZ [i.e. 57;15 and 11;30],

\[
\begin{align*}
\Theta Z &= 5;11,55^p, \\
ZG &= 51;7,38^p,
\end{align*}
\]

and, by addition, GΘ = 56;19,33^p.

\[\text{Hence more accurate would be 57;6,15, which is the reading of D and is given as an alternative in ABC. But the text is guaranteed by the following computations.}\]

\[\text{Cf. p. 567 n.31. Computation: 55;55,1^p \times 1/10:56,39 = 5;6,33^p, to which corresponds an equation of } 0;26,24^\circ \approx 26^\circ. 55;55,1^p - 0;26^\circ = 55;29,1^p. \text{ This multiplied by } 1/10:51,29 = 5;6,35^p \text{ so text; accurately 5;6,36}. 5;6,35^p - 0;26^\circ = 4,40,35^p.\]

\[\text{In the table of anomaly, to an argument of } [180^\circ - 3^\circ = ] 177^\circ \text{ corresponds an equation of } 0;17^\circ, \text{ hence to } 1^\circ \text{ near perigee corresponds } 51^\circ.\]
**XII 3. Jupiter’s retrogradation at least distance**

Hence, expressed in units where hypotenuses ZA and AG [respectively] are 120°.

\[
\begin{align*}
Z\Theta &= 54:14,47^o \\
G\Theta &= 118:3,46^o,
\end{align*}
\]

and the corresponding arcs

\[
\begin{align*}
\text{arc } Z\Theta &= 53:45,4^o \\
\text{arc } G\Theta &= 159:22,4^o.
\end{align*}
\]

Accordingly \( \angle ZA\Theta = 26:52,3^o \)

and \( \angle GA\Theta = 79:41,2^o \).

And, by subtraction, \( \angle ZGA \), which represents the retrogradation due to the planet’s speed, is \([90^o - \angle GA\Theta =] 10:18,40^o \), and \( \angle ZAH \), which represents the apparent [motion in] anomaly, is \([\angle GA\Theta - \angle ZA\Theta =] 52:48,48^o \). To the latter correspond 5:21,20° in corrected longitudinal motion,\(^{47}\) and 4:54,20° in mean [longitudinal] motion, according to the ratio [of speeds] at the perigece. Thus half the retrogradation is

\[10:18,40^o - 5:21,20^o = 4:57,20^o \]

and the total retrogradation is

\[9:54,40^o \text{ and } 118^o.\]

4. *Demonstration of the retrogradations of Mars*

Again, in the case of Mars [see Fig. 12.10], according to our calculations for near mean distance,

\[
\begin{align*}
\Theta Z:ZG &= 1 : 0;52,51,\text{,}^{18} \\
\text{and } EG:GZ &= 2;52,51 : 0;52,51, \\
\text{so } EG:GZ &= 2;32,15.
\end{align*}
\]

Furthermore, \( GA:AH = 60 : 39:30, \)

and \( DG:GH = 99:30 : 20:30, \)

so \( DG:GH = 2039:45. \)

Dividing \([2039:45 \text{ by } 2;32,15\] ), we get 803:50,50 the square root of which, 28:21,8, multiplied into the above ratio of \( \Theta Z:ZG \), gives, in terms of the above sizes of \( GA \) and \( AZ \) [i.e. 60 and 39:30],

\[
\begin{align*}
\Theta Z &= 28:21,8^o, \\
GZ &= 24:58,25^o \text{ in the same units,}
\end{align*}
\]

and, by addition, \( G\Theta = 53;19,33^o. \)

Hence, in units where hypotenuses \( AZ \) and \( AG \) are each [respectively] 120°,

\[
\begin{align*}
Z\Theta &= 86:8,0^o \\
\text{and } G\Theta &= 106:39,6^o.
\end{align*}
\]

\(^{47}\) Cf. p. 567 n.31. Computation: 52:48,48° × 1/10;45,49 = 4;54,24°, to which corresponds an equation of 27° [so text: accurate would be 29°]. 52:48,48° + 0.27° = 53:15,48°, which multiplied by 1/10;51.29 gives 4;54,20° [accurately 4;54,19°]. 4;54,20° + 0.27° = 5:21,20°.

\(^{18}\) From the mean daily motions (IX 4) : 0:27,41,40/0:31,26,36 = 0:52,50,47 . . .

\(^{49}\) Accurate would be 803,50,33, which is found as the reading of the second hand in D. Ger has 803,50,32, T 803,50,30. The variation has no further consequences, since the square root of all (to the nearest second) is 28:21,8.
The corresponding arcs are
\[
\text{arc } Z\Theta = 91;44.34^\circ
\]
and \( \text{arc } G\Theta = 125;26.10^\circ. \)

Accordingly \( \angle ZA\Theta = 45;52.17^\circ \)
and \( \angle GA\Theta = 62;43.5^\circ. \)

And, by subtraction, \( \angle ZGA, \) which represents the retrogradation due to the planet's speed, is \( [90^\circ - \angle GA\Theta =] 27;16.55^\circ, \) and \( \angle ZAH, \) which represents the [motion in] anomaly, is \( [\angle GA\Theta - \angle ZA\Theta =] 16;50.48^\circ. \) To the latter amount correspond \( 19;7,33^\circ \) in [mean] longitudinal motion, according to the above ratio [of speeds, of 1 : 0.52,51]. Thus half the retrogradation is
\[
[27;16.55^\circ - 19;7,33^\circ =] 8;9.22^\circ \text{ and about } 36^\circ.
\]

And the total retrogradation is
\[
16;18.44^\circ \text{ and } 73^\circ.
\]

[Hence] the distance at the elongation of the stations from apogee and perigee is [respectively] about \( 0.20^\circ \) of the mean distance [i.e. \( 60^\circ \)] less than the greatest distance, and about the same amount greater than the least distance.\(^{50}\)

According to our calculations for near greatest distance, the equation corresponding to an argument of \( 1^\circ \) for correcting [the speeds] is found to be \( 10\frac{1}{4}^\circ. \)\(^{51}\) Hence

\(^{50}\) For a true centrum \((\kappa)\) of \( 19;7,33^\circ, \) the distance of the centre of the epicycle, \( \rho = 65;38.12^\circ = 66^\circ - 22^\circ. \) For \( \kappa = 160;52.27^\circ, \rho = 54;17.56^\circ = 54^\circ + 18^\circ, \) i.e. \( 20^\circ \) is a reasonable mean.

\(^{51}\) In the anomaly table for Mars (XI 11), to an argument of \( 18^\circ \) corresponds an equation of \( 3;13^\circ \) and to \( 24^\circ, 4;16^\circ; \) hence, as Manitius notes, the correct amount corresponding to \( 1^\circ \) should be \( (4;16 - 3;13)/6 = 10\frac{1}{4}. \)
XII 4. Mars’ retrogradation at greatest distance

\[ \Theta Z : ZG = 0;49,40 : 1;3,11, \]
\[ EG : GZ = 2;42,31 : 1;3,11, \]
\[ and \ EG : GZ = 2;51,8. \]

Furthermore, \( GA : AH = 65;40 : 39;30, \)
\[ DG : GH = 105;10 : 26;10, \]
\[ and \ DG : GH = 2751;51,40. \]

And, when we divide \([2751;51,40 \div 2;51,8], \) we get \( 964;48,47, \) the square root of which, \( 3I ;3,41, \) multiplied into the above ratio of \( \Theta Z : ZG, \) gives, in terms of the above sizes of \( GA \) and \( AZ \) [i.e. \( 65;40 \) and \( 39;30), \]

\[ \Theta Z = 25;42,43, \]
\[ GZ = 32;42,34, \]

and, by addition, \( G\Theta = 58;25,17. \)

Hence, expressed in units where hypotenuses \( AZ \) and \( AG \) are each [respectively] \( 120^\circ, \)

\[ Z\Theta = 78;6,44^p, \]
\[ and \ G\Theta = 106;45,36^p. \]

The corresponding arcs are

\[ \text{arc} \ Z\Theta = 81;13,8^52 \]
\[ \text{and arc} \ G\Theta = 125;39,46^p. \]

Accordingly \( \angle ZA\Theta = 40;36,34^p \)
\[ \text{and} \ \angle GA\Theta = 62;49,53^p. \]

And, by subtraction, \( \angle ZGA, \) which represents the retrogradation due to the planet’s speed, is \([90^\circ - \angle GA\Theta = 27;10,7^p, \) while \( \angle ZAH, \) which represents the [motion in] apparent anomaly, is \([\angle GA\Theta - \angle ZA\Theta = 22;13,19^p. \) To the latter correspond [motions in] corrected longitude of \( 17;13,21^p, \) and in mean [longitude] of \( 20;58,21^p. \) according to the ratios [of the speeds] at the apogee.

Thus half the retrogradation is

\[ [27;10,7^p - 17;13,21^p = ] 9;56,46^p \text{ and about } 40^d, \]

and the total retrogradation is

\[ 19;53,32^p \text{ and } 80^d. \]

According to our calculations for near least distance, the equation [corresponding to an argument of \( 1^\circ \)] for correcting [the speeds] is found to be \( 125^\circ. \)

Hence

\[ \Theta Z : ZG = 1;12,40 : 0;40,11, \]
\[ EG : GZ = 3;5,31 : 0;40,11, \]
\[ and \ EG : GZ = 2;4,14. \]

Furthermore, \( GA : AH = 54;20 : 39;30, \)
\[ DG : GH = 93;50 : 14;50, \]
\[ and \ DG : GH = 1391;51,40. \]

Dividing \([1391;51,40 \div 2;4,14], \) we get \( 672;13, \) the square root of which,

\[ ^{52} \text{Correct would be } 81;13,28^p, \text{ and this is the reading of BCL.Ger. However, all mss. agree in the reading for the half of this, } 40;36,34^p, \text{ which would seem to confirm Heiberg’s reading here. It is possible, however, that Ptolemy made an error in halving, and that the reading ‘8’ in AD is due to scribal correction.} \]

\[ ^{54} \text{Ptolemy gives the computation for this at XII 6 p. 582.} \]

\[ ^{54} \text{In the anomaly table for Mars (XI 11), to an argument of } 162^\circ \text{ corresponds an equation of } 3;55^p, \text{ and to } 159^\circ, 4;33^p. \text{ Therefore to } 1^\circ, \text{ at about } 20^\circ \text{ from perigee, corresponds } (4;33 - 3;55) : 3 = 121^\circ. \]
25;55,38, multiplied into the above ratio of \( \Theta Z : ZG \), gives, in terms of the above sizes of \( GA \) and \( AZ \) [i.e. 54;20 and 39;30],

\[
\Theta Z = 31;24,3^\circ,
\]

\[
GZ = 17;21,51^\circ
\]

in the same units,

and, by addition, \( G\Theta = 48;45,54^\circ \).

Hence, where the hypotenuses \( AZ \) and \( AG \) are each \( 120^\circ \),

\[
Z\Theta = 95;23,42^\circ
\]

\[
G\Theta = 107;42,7^\circ
\]

The corresponding arcs are

\[
\text{arc } Z\Theta = 105;18,10^\circ
\]

\[
\text{arc } G\Theta = 127;40,22^\circ
\]

Accordingly \( \angle Z\Theta \Theta = 52;39,5^\circ \)

\[
\text{and } \angle G\Theta \Theta = 63;50,11^\circ
\]

And, by subtraction, \( \angle Z\Theta \Theta \), which represents the [amount of] retrogradation due to the planet's speed, is \( [90^\circ - \angle G\Theta \Theta] = 26;9,49^\circ \), while \( \angle ZA\Theta \), which represents the [motion in] apparent anomaly, is \( [\angle G\Theta \Theta - \angle Z\Theta \Theta] = 11;11,6^\circ \).

To the latter correspond [motions in] corrected longitude of 20;33,42\(^\circ \), and in mean longitude of 16;52,52\(^\circ \), according to the ratios [of the speeds] at the perige.

So half the retrogradation comes out as

\[
[26;9,49\(^\circ \) - 20;33,42\(^\circ \) = 5;36,7\(^\circ \) and about 32\(^d\).

and the total retrogradation is

\[
11;12,14\(^d\) and 64\(^d\).
\]

5. \{Demonstration of the retrogradations of Venus\}

Again, in the case of the planet Venus [see Fig. 12.11], according to our calculations for mean distance,

\[
\Theta Z : ZG = 1 : 0;37,31,57
\]

\[
EG : GZ = 2;37,31 : 0;37,31
\]

\[
\text{and } EG : GZ = 1;38,30
\]

Furthermore, \( GA : AH = 60 : 43;10 \)

\[
DG : GH = 103;10 : 16;50\]

\[
\text{and } DG : GH = 1736;38,20
\]

Dividing [1736;38,20 by 1;38,30], we get 1057;51,58 the square root of which,
H484  32;31,29, multiplied into the above ratio of $\Theta Z:ZG$, gives, in terms of the above
sizes of $GA$ and $AZ$ [i.e. 60 and 43;10],

$$\Theta Z = 32;31,29^p,$$
$$GZ = 20;20,11^p$$ in the same units,

and, by addition, $G\Theta = 52;51,40^p$.

Hence, where hypotenuses $AZ$ and $AG$ are each [respectively] $120^p$,

$$Z\Theta = 90;24,58^p$$
$$and \ G\Theta = 105;43,20^p.$$  

The [corresponding] arcs are:

$$\text{arc } Z\Theta = 97;47,0^o$$
$$\text{and arc } G\Theta = 123;31,49^o.$$  

Accordingly $\angle ZA\Theta = 48;53,30^o$
$$\text{and } \angle GAO \approx 61;45,54^o.$$  

And, by subtraction, $\angle ZGA$, which represents the [amount of] retrogradation due to the planet's speed, is $[90^o - \angle GAO = ] 28;14,6^o$, while $\angle ZAH$, which represents the [motion in mean] anomaly, is $[\angle GAO - \angle ZA\Theta = ] 12;52,24^o$.

To the latter corresponds a motion in [mean] longitude of $20;35,19^o$, according to the above mean ratio [of the speeds], and half the retrogradation is computed to be

$$[28;14,6^o - 20;35,19^o = ] 7;38,47^o$$ and about $20^d$.  

The total retrogradation is

$$15;17,34^o$$ and $41^d$.

\[\text{Footnote: } 12;52,24/0;37,31 \text{ is, accurately, } 20;35,17.\]
Hence the distance at the elongation of the stations from apogee and perigee is respectively about 0.5° of the mean distance (i.e., 60°) less than the greatest distance, and about the same amount greater than the least distance.

According to our calculations for near greatest distance, the equation [corresponding to 1°] for correcting [the speeds] is found to be 2°.61 Hence
\[ \Theta : \text{ZG} = 0;57,40 : 0;39,51, \]
\[ \text{EG} : \text{GZ} = 2;35,11 : 0;39,51, \]
and \( \text{EG} : \text{GZ} = 1;43,4. \)

Furthermore \( \text{GA} : \text{AH} = 61;10 : 43;10, \)
\[ \text{DG} : \text{HG} = 104;20 : 18;0, \]
and \( \text{DG} : \text{HG} = 1878;0. \)

Dividing [1878 by 1:43,4], we get 1093;16,23, the square root of which, 33;3,53, multiplied into the above ratio of \( \Theta Z : \text{ZG} \), gives, in terms of the above sizes of \( \text{GA} \) and \( \text{AZ} \) [i.e. 61;10 and 43;10],
\[ \Theta Z = 31;46,44°, \]
\[ \text{GZ} = 21;57,38° \]
in the same units,
and, by addition, \( \Theta \Theta = 53;44,22°. \)

Hence, where hypotenuses \( \text{AZ} \) and \( \text{AG} \) are each respectively 120°,
\[ \Theta \Theta = 88;20,34° \]
and \( \Theta \Theta = 105;25,44°. \)

The [corresponding] arcs are:
\[ \text{arc } \Theta Z = 94;48,54° \]
and arc \( \Theta \Theta = 122;56,27°. \)

Accordingly \( \angle Z \Theta = 47;24,27° \)
and \( \angle G \Theta = 61;28,14°. \)

And, by subtraction, \( \angle Z \Theta \), which represents the [amount of] retrogradation due to the planet's speed, is \( [90° - \angle G \Theta ] = 28;31,46° \), while \( \angle ZAH \), which represents the [motion in] apparent anomaly, is \( [\angle G \Theta - \angle Z \Theta ] = 14;3,47°. \)

To the latter correspond [motions of] 20;19,3° in corrected longitude and 21;9,3° in mean longitude, according to the ratios [of the speeds] at apogee.62

Thus half of the retrogradation comes to
\[ [28;31,46° - 20;19,3° ] = 8;12,43° \]
and about 21°.

The total retrogradation is
16:25,26° and 43°.

According to our calculations for near least distance, the equation [corresponding to an argument of 1°] for correcting [the speeds] is found to be the same amount, 2°.63 Hence

60For a true centrum (x) of 20;35,19 the distance of the centre of the epicycle is 61;10,6° (≈ 61;15° - 5), and for \( \kappa = 180° - 20;35,19° \) the distance is 58;49,41° ≈ 58;49° + 5°.

61The increment between successive values of the equation in the anomaly table for Venus (XI 11) is 14' for 6° of argument near the apogee, hence 21° for 1°. However, one should take the increment between 18° and 24°, which is 15', leading to 21° for 1°.

62Cf. p. 567 n.31. Computation: 14;3,47° × 1/0;39,51 = 21° [accurately 21;10,26°], to which corresponds an equation of 0;50° [accurately 0;50,30°]. 14;3,47° - 0;50° = 13;13,47° ≈ 13;13°. 13;13° × 1/0;37,31 = 21;9,3°, and 21;9,3° - 0;50° = 20;19,3°.

63This corresponds to an increment of 7° for an increment of 3° in the argument. In the anomaly table for Venus (XI 11), near perigee, the increment is 7° between 165° and 162° and between 159° and 156°, but between 162° and 159°, which is the proper interval (x = 20°), it is only 6°.
XII.5. Venus' retrogradation at least distance

\[
\begin{align*}
Z\Theta:ZG &= 1;2,20 : 0;35,11, \\
EG:GZ &= 2;39,51 : 0;35,11, \\
\text{and} \ EG.GZ &= 1;33,44.
\end{align*}
\]

Furthermore \(GA:AD = 58;50 : 43;10,\)
\[
\begin{align*}
DG:GH &= 102;0 : 15;40, \\
\text{and} \ DG.GH &= 1598;0.
\end{align*}
\]

H487 Dividing \([1598 \text{ by } 1;33,44],\) we get \(1022;54,7,\) the square root of which, \(31;58;58,\) multiplied into the above ratio of \(\Theta Z:ZG,\) gives, in terms of the above sizes of \(GA\) and \(AZ\) \([\text{i.e.} 58;50 \text{ and } 43;10],\)
\[
\begin{align*}
\Theta Z &= 33;13,36^\circ, \\
GZ &= 18;45,16^\circ \text{ in the same units,}
\end{align*}
\]

and, by addition, \(G\Theta = 51;58,52^\circ.\)

Hence, where hypotenuses \(AZ\) and \(AG\) are each \([\text{respectively}]\) \(120^\circ,\)
\[
\begin{align*}
Z\Theta &= 92;22,3^\circ, \\
\text{and} \ G\Theta &= 106;1,23^\circ.\quad 64
\end{align*}
\]

The [corresponding] arcs are:
\[
\begin{align*}
\arct Z\Theta &= 100;39,34^\circ, \\
\text{and} \ arct G\Theta &= 124;8,22^\circ.
\end{align*}
\]

Accordingly \(\angle ZA\Theta = 50;19,47^\circ\)
\[
\begin{align*}
\text{and} \ \angle GA\Theta &= 62;4,11^\circ.
\end{align*}
\]

And, by subtraction, \(\angle ZGA,\) which represents the [amount of] retrogradation due to the planet's speed, is \([90^\circ - \angle GA\Theta =] 27;55,49^\circ,\) while \(\angle ZAH,\) which represents the [motion in] apparent anomaly, is \([\angle GA\Theta - \angle ZA\Theta =] 11;44,24^\circ.\) To the latter correspond [motions of] \(20;53,30^\circ\) in corrected longitude, and \(20;4,30^\circ\) in mean longitude, according to the ratios [of the speeds] at perigee.\(^{65}\) Accordingly half of the retrogradation comes to
\[
[27;55,49^\circ - 20;53,30^\circ =] 7;2,19^\circ \text{ and about } 20^\text{jd}.
\]

The total retrogradation is
\[
14;4,38^\circ \text{ and } 40^\text{jd}.
\]

H488 6. \{Demonstration of the retrogradations of Mercury\}

Again, in the case of Mercury \[\text{see Fig. 12.12], according to our calculations for mean distance,}\n\[
\begin{align*}
\Theta Z:ZG &= 1 : 3;9,8,\quad 66 \\
EG:GZ &= 5;9,8 : 3;9,8, \\
\text{and} \ EG.GZ &= 16;14,27.
\end{align*}
\]

Furthermore, \(GA:AH = 60 : 22\frac{1}{2},\)
\[
\begin{align*}
DG:GH &= 82;30 : 37;30, \\
\text{and} \ DG.GH &= 3093;45.
\end{align*}
\]

\(^{64}\) Calculation gives \(106;1,26^\circ,\) and perhaps one should correct to that, which is the reading of Is. However, an arc of \(124;8,22^\circ\) agrees better with a chord of \(106;1,23^\circ.\)

\(^{65}\) Cf. p. 567 n.31. Computation: \(11;44,24^\circ \times 1/0;35,11 = 20;1,15^\circ \simeq 20^\circ.\) To \((180^\circ - 20^\circ)\) corresponds an equation of \(0;49^\circ.\) \(11;44,24^\circ + 0;49^\circ = 12;33,24^\circ \simeq 12;33^\circ.\) \(12;33^\circ \times 1/0;37,31\)
\(\approx 20;4^\circ [\text{accurately } 20;4,16^\circ].\) \(20;4^\circ + 0;49^\circ = 20;53,30^\circ.\)

\(^{66}\) From the mean daily motions taken to 2 sexagesimal places \(\text{(IX 4),} 3;6,24/0;59,8 = 3;9,7,54 \approx 3;9,8.\).
Dividing \([3093;45 \text{ by } 16;14,27]\), we get \(190;29,31\), the square root of which, \(13;48,7\), multiplied into the above ratio of lines \(\Theta Z:ZG\), gives, in terms of the above sizes of \(GA\) and \(AZ\) [i.e. \(60\) and \(22;30\)],
\[
\Theta Z = 13;48,7^\circ, \\
ZG = 43;30,24^\circ, \\
\text{and, by addition, } \Theta G = 57;18,31^\circ.
\]

Hence, where hypotenuses \(AZ\) and \(AG\) are each [respectively] \(120^\circ\),
\[
Z\Theta = 73;36,37^\circ, \\
\text{and } \Theta G = 114;37,3^\circ.
\]

The corresponding arcs are:
\[
\text{arc } Z\Theta = 75;40,28^\circ \\
\text{and arc } \Theta G = 145;32,52^\circ.
\]

Accordingly \(\angle ZA\Theta = 37;50,14^\circ\) \\
and \(\angle \Theta AG = 72;46,26^\circ\).

And, by subtraction, \(\angle ZGA\), which represents the [amount of] retrogradation due to the planet's speed, is \([90^\circ - \angle \Theta AG = ] 17;13,34^\circ\), while \(\angle ZAH\), which represents the [motion in mean] anomaly, is \([\angle \Theta AG - \angle ZA\Theta = ] 34;56,12^\circ\).

To the latter corresponds a motion in [mean] longitude of \(11;4,59^\circ\), according to the above ratio [of the speeds],\(^67\) and half the retrogradation is found by subtraction as
\[
[17;13,34^\circ - 11;4,59^\circ = ] 6;8,35^\circ \text{ and about } 11\text{d}.
\]

The total retrogradation is computed as
\[
12;17,10^\circ \text{ and } 22\text{d}.
\]

\(^67\) \(34;56,12/3;9,8\) is indeed \(11;4,59\) (accurate to two places).
According to our calculations for near greatest distance, i.e. when the corrected longitude is about 11° from apogee (corresponding to a mean longitude of about 111°), the equation for correcting [the speeds] corresponding to 1° [of anomaly] is about 21°. Hence

\[
\begin{align*}
\Theta Z : ZG &= 0;57,40 : 3;11,28, \\
EG : GZ &= 5;6,48 : 3;11,28, \\
&\text{and } EG : GZ = 16;19,2.
\end{align*}
\]

Furthermore, GA : AH = 68;36 : 22;30,68

DG : GH = 91;6 : 46;6, \\
&\text{and } DG : GH = 4199;42,36.

Dividing [4199;42,36 by 16;19,2], we get 257;22,44, the square root of which, 16;2.35, multiplied into the above ratio of \(\Theta Z : ZG\), gives, in terms of the above sizes of GA and AZ [i.e. 68;36 and 22;30],

\[
\Theta Z = 15;25,9 \\
ZG = 51;11,43^\circ \\
\text{in the same units,}
\]

and, by addition, \(G\Theta = 66;36,52^\circ\).

Hence, where hypotenuses ZA and AG are each [respectively] 120°,

\[
\begin{align*}
Z\Theta &= 82;14,8^\circ \\
\text{and } G\Theta &= 116;31,36^\circ.
\end{align*}
\]

The corresponding arcs are:

\[
\begin{align*}
\text{arc } Z\Theta &= 86;31,4^\circ \\
\text{and arc } \Theta G &= 152;27,56^\circ. \\
\text{Accordingly } \angle ZA\Theta &= 43;15,32^\circ \\
\text{and } \angle \Theta AG &= 76;13,58^\circ.
\end{align*}
\]

And, by subtraction, \(\angle ZGA\), which represents the [amount of] retrogradation due to the planet's speed, is \(90^\circ - \angle \Theta AG = 13;46,2^\circ\), while \(\angle ZAH\), which represents the [motion in] apparent anomaly, is \(\angle \Theta AG - \angle ZA\Theta = 32;52,26^\circ\).71 To the latter correspond [motions of] 9;48,51° in corrected longitude and 10;16,51° in mean [longitude], according to the ratios [of the speeds] at the apogee.72 Thus half the retrogradation is found by subtraction as \((13;46,2^\circ - 9;48,51^\circ) = 3;57,11^\circ\) and about 10°.

The total retrogradation is

\[
7;54,22^\circ \text{ and } 21^\text{d}.
\]

According to our calculations for near least distance (which occurs near the

---

68 In the table of anomaly for Mercury (XI 11), to an argument of 6° corresponds an equation of 17°, and to 12°, 32°. Thus to an increment of 6° corresponds an increment of 15°, or, to 1°, 21°. I have no explanation for the discrepancy.

69 The distance at apogee is 69°; hence Ptolemy assumes that the distance at the given situation is 24° less. For \(\aleph = 111^\circ\), the distance (\(p\)) is in fact 68;37°. It is about 68;36° for \(\aleph = 11;40^\circ\).

70 Ptolemy has committed a considerable computing error here: the arc of the chord 116;31,36° should be about 152;22°.

71 As noted by Heiberg and Manitius, 76;13,58 - 43;15,32 in fact equals 32;58,26. But Ptolemy's erroneous number is confirmed by the following calculations and by H500,23. It is worth noting that had Ptolemy used the correct arc of the chord 116;31,36° (cf. n.70), he would have found \(\angle \Theta AG = 76;11^\circ\) and \(\angle ZAH = 32;55^\circ\), which is closer to the text, but still not in perfect agreement.

72 Cf. p. 567 n.31. Computation: 32;52,26° \times 1/3;11,28 \approx 10;16^\circ, to which corresponds an equation of 0;28° (accurately 0;27,45°), 32;52,26° - 0;28° = 32;24,26°, which divided by 3;9,6 gives 10;16,51°. 10;16,51° - 0;28° = 9,48,51°.
XII 6. Mercury’s retrogradation at least distance

elongations of 120° in mean motion from the apogee), the equation for correcting [the speeds], derived from entering [the table] at around 11° either side of the perigees is approximately 1 1/2°.\(^73\) Hence
\[
\begin{align*}
\Theta Z:ZG &= 1;1,30 : 3;7,38, \\
EG:GZ &= 5;10,38 : 3;7,38, \\
\text{and } EG:GZ &= 16;11,25.
\end{align*}
\]
Furthermore, \(GA:AH \approx 55;42 : 22;30,\)\(^74\)
\[
\begin{align*}
DG:GH &= 78;12 : 33;12, \\
\text{and } DG:GH &= 2596;14,24.
\end{align*}
\]
Dividing \([2596;14,24 \text{ by } 16;11,25\), we get \(160;21,29\), the square root of which, \(12;39,48\), multiplied into each member of the above ratio of \(\Theta Z:ZG\), gives, in terms of the above sizes of \(GA\) and \(AZ\) [i.e. 55;42 and 22;30],
\[
\begin{align*}
\Theta Z &= 12;58,47° \\
ZG &= 39;36,4° \text{ in the same units,}
\end{align*}
\]
and, by addition, \(G\Theta = 52;34,51°\).

Hence, where hypotenuses \(AZ\) and \(AG\) are each [respectively] 120°,
\[
\begin{align*}
\Theta Z &= 69;13,31° \\
\text{and } G\Theta &= 113;16,48°.
\end{align*}
\]
The corresponding arcs are:
\[
\begin{align*}
\text{arc } \Theta Z &= 70;27,44° \\
\text{and } \text{arc } G\Theta &= 141;28,14°.
\end{align*}
\]
Accordingly \(\angle \Theta AZ = 35;13,52°\)
\[
\text{and } \angle \Theta AG = 70;44,7°.
\]
And, by subtraction, \(\angle ZGA\), which represents the [amount of] retrogradation due to the planet’s speed, is \([90° - \angle \Theta AG = ] 19;15,53°\), while \(\angle ZAH\), which represents the [motion in] apparent anomaly, is \([\angle \Theta AG - \angle \Theta AZ = ] 35;30,15°\). To the latter correspond [motions of] 11;39,30° in corrected longitude, and 11;21,30° in mean [longitude], according to the above ratios [of the speeds near the perigee].\(^75\) Thus half of the retrogradation is found by subtraction as
\[
[19;15,53° - 11;39,30° = ] 7;36,23° \text{ and about 11 1/4°.}
\]
The total retrogradation is
\[
15;12,46° \text{ and } 23°.
\]
The amounts [of the retrogradations] we have demonstrated agree very closely with those derived from the actual phenomena associated with each planet.

\(^73\) From the table of anomaly for Mercury (XI 11) it can be seen that 1 1/4° is a compromise between the two values derived on either side of the perigee: to \(K = 108°\) corresponds an equation of 2;56°, and to \(K = 111°, 2.53°\). Here, then, an increment of 1° produces 1°. For \(K = 129°\) and 132° one finds 2;24° and 2;18° respectively, and hence, for an increment of 1°, 2°.

\(^74\) Cf. p. 580 n.69. Here, for a distance of 11 1/4° in mean motion from ‘perigee’ (at \(K = 120°\)), one finds, for \(K = 131\frac{1}{4}°, \rho = 55;41,58°\) (text 55;42°). On the other side of the perigee, however, for \(K = 108\frac{1}{4}°, \rho = 55;45,50°\).

\(^75\) Cf. p. 567 n.31. Computation: 35;30,15° \times 1/3;7,38 = 11;21,11°, to which corresponds an equation of 18° [in fact 11;21,11° before the perigee leads to an equation of +15°, and 11;21,11° after it to -23°, i.e. 18° is, again, a compromise]. 35;30,15° + 0;18° = 35;48,15°, which divided by 3;9,8 gives 11;21,30°. 11;21,30° + 0;18° = 11;39,30°.
We used the following method to find the motions in longitude at greatest and least distances.\(^{76}\)

For example, in the case of Mars [XII 4 p. 574], we showed that, near the greatest distance,\(^{77}\) the apparent arc of the epicycle from either of the stations to opposition (i.e. the arc as viewed from the centre of the ecliptic) is 22; 13, 19°. To the latter corresponds (according to the ratio 1 : 1;3,11) a motion in mean longitude of about 21;10°.\(^{78}\) But the latter does not represent [the actual mean motion] accurately, since the ratios of the speeds which we set out for the stations do not remain unchanged throughout the whole period of retrogradation. However, it is close enough to the truth so that the equation corresponding to it (which is about 3;45°)\(^{79}\) is not significantly different [from the true equation]. So we subtracted that [3;45°] from the 22;13, 19° of the epicycle (since at greatest distance the apparent motion on the epicycle is greater than the mean motion), and [thus] found that the corresponding mean motion in anomaly from either of the stations to opposition is 18;28, 19°. To this, according to the ratio of the mean motions [0;52,51 : 1] corresponds a motion in mean longitude of 20;58,21°.\(^{80}\) So we adopted that as the accurate value instead of the

\(^{76}\) There is no need to assume, with Neugebauer (note in Manitius, revised edition, p. 301) that the following passage has been displaced in antiquity from its rightful place in XII 4. For the method applies to all planets, not just Mars. It is quite in Ptolemy's manner to attach an explanation or justification of a particular method as an appendix at the end of his general treatment. Cf. V 19 pp. 267-73 and VI 4 p. 282.

\(^{77}\) See Fig. R. The planet is at opposition (P) when the epicycle is at apogee, and at second station (S) when the epicycle is at a mean centrum from apogee. Then 'the apparent arc of (motion on) the epicycle' is XS, and 'the mean motion on the epicycle' (which differs from it by the equation e) is ZS.

\(^{78}\) Accurately 21;6,8°.

\(^{79}\) Accurately 3;46,15°.

\(^{80}\) Accurately 20;58,15°.
Furthermore, to enable us to investigate conveniently at what point on the epicycle each planet is when it produces the appearance of being stationary, for distances in the interval between mean distance and greatest or least distance as well, we have constructed for this purpose a table with 31 lines and 12 columns. The first two of these columns will contain the numbers of the mean longitude at intervals of ° (corresponding to the arrangement of the other tables). The following 10 columns will contain the distances in corrected anomaly from the apparent apogee of the epicycle for each of the 5 planets: in each case the first column [of the pair for that planet] will contain the amount for first station, and the second column the amount for second station. We obtained the amounts for these [entries] too from the [numbers] demonstrated above for mean, least and greatest distances, and from the increments at distances in between these, which we happen to have determined already in [our computations of] the minutes to be tabulated in the eighth column of the tables for anomaly. For in demonstrating the amount of the maximum equation of anomaly corresponding to each entry in mean motion, one simultaneously demonstrates the distance of the epicycle, which is the principal factor affecting the difference in [the position of] the stations.

But first, since the retrogradation which we demonstrated for near apogee and perigee represent, not the stations which occur when the centre of the epicycle is precisely at apogee and perigee, but those when it is a certain specified distance [from them], we used the latter to determine, for each planet, the amount corresponding to the actual apogee and perigee, as follows.

In the case of Saturn and Jupiter, since the distances of the epicycle at actual apogee and perigee do not differ significantly from those at the elongations from apogee and perigee used above, we entered the amounts of anomaly (counted from apparent apogee of the epicycle) derived for those elongations on the appropriate lines, i.e. we entered the amount for apogee on the line with the argument '360', and the amount for perigee on the line with the argument '180'. We showed that for Saturn [XII 2, pp. 567-9] the distance [in anomaly] from the perigee of the epicycle at apogee of the eccentric is about 67° 15', and at perigee of the eccentric about 64° 31'; and that for Jupiter [XII 3, pp. 571-2] it is 55° 55' at apogee and 52° 49' at perigee. For convenience in use, we entered the

81 See HAMA 202-06, Pedersen 349-51.
82 Reading μεθοδευόμεν (with D,Ar) at H494,20 for μεθοδεύομεν ('we construct').
83 Cf. XI 10 p. 547. It was necessary for Ptolemy to compute the distances of the centre of the epicycle all round the orbit in order to calculate the 'minutes of interpolation' in the planetary anomaly tables.
amounts [in anomaly] corresponding to these, counted from the apogee of the epicycle, on the appropriate lines in the 4 columns following the [argument columns of] longitude: on the line with the argument ‘360’ (for the apogee) [we entered], in the third column, ‘112;45°’ for the first station of Saturn, and, in the fourth column, ‘247;15°’ for its second station; similarly, in the fifth column, ‘124;5°’ for Jupiter’s first station, and, in the sixth column, ‘235;55°’ for its second station. And on the line with the argument ‘180’ (for the perigee) [we entered], following the same order, ‘115;29°’ and ‘244;31°’, and similarly ‘127;11°’ and ‘232;49°’.

In the case of Mars, we showed [XII 4, pp. 573-4] that when the epicycle centre is 20;58° in mean [longitude] from the apogee of the eccentric, the planet performs its stations at a distance of 22;13° [in anomaly] from the apparent perigee of the epicycle; and the [corresponding] amount [of anomaly] at mean distance is 16;51°, so that the difference is 5;22°. Furthermore, where the mean distance is 60°, the greatest distance is 66° and the difference between greatest and mean is 6°, while at the above distance from the apogee [of 20;58°] the distance is 65;40° and the difference between this and the mean is 5;40°. So, multiplying 6 into 5;22 and dividing the result by 5;40, we find that the difference with respect to the mean distance at the actual apogee is about 5;41°. Thus we calculate the distance [in anomaly] from the apparent perigee of the epicycle as [16;51° + 5;41° =] 22;32°, and from the apogee as, for the first station, 157;28°, which we enter in the seventh column on the line with ‘360’, and, for the second station, 202;32°, which we enter in the eighth column on the same line.

Similarly [see p. 575], when the epicycle centre is 16;53° in mean [longitude] from the perigee [of the eccentric, [Mars] performs its stations at a distance of 11;11° [in anomaly] from the apparent perigee of the epicycle, so that the difference [in anomaly] from that for mean distance is [16;51° − 11;11° =] 5;40°. And, in the same units [as before], the least distance is 54° (with a difference from the mean of 6°), and at the above elongation from the perigee of the eccentric it is 54;20°, with a difference from the mean of 5;40°. Thus at the actual perigee we get the total difference [in anomaly from the mean] as [5;40° × 6 ÷ 5;40 =] 6°. Hence the amount [of anomaly] from apparent perigee of the epicycle is [16;51° − 6° =] 10;51°, and from the apogee, for the first station, 169;9°, and for the second 190;51°, which we enter in the appropriate columns on the line with ‘180’.

In the case of Venus, we showed [XII 5, pp. 576-7] that when it is 21;9° in mean longitude from the apogee [of the eccentric], the planet performs its stations at a distance of 14;4° [in anomaly] from the apparent perigee of the epicycle, while the [corresponding] amount at mean distance is 12;52°, so that the difference is 1;12°. And, where the mean distance is 60°, the greatest distance is 61;15°, and the difference from the mean 1;15°, while at the above elongation from the apogee the distance is 61;10° and the difference from the mean 1;10°. So, again, multiplying 1;15 into 1;12 and dividing the result by

---

84 Cf. p. 573 with n.50. One should probably read ἀνά τοῦ ἀνωγείου (with D) at H497.21 (cf. H499.11) Corrected by Manitius.
1;10, we find the difference [in anomaly] at the actual apogee with respect to that for the mean distance as 1;17°. Thus we calculate the distance [in anomaly] from the apparent perigee of the epicycle as [12;52° + 1;17° =] 14;90°, and from the apogee as, for the first station, 165;51°, which we enter in the ninth column on the line with '360', and, for the second station, 194;9°, which we enter in the tenth column on the same line.

Similarly [see p. 578], when the epicycle is about 20° in mean longitude from perigee of the eccentric, [Venus] performs its stations at a distance [in anomaly] of 11;44° from the apparent perigee of the epicycle, so that the difference with respect to [that for] mean distance is [12;52° - 11;44° =] 1;8°. And the least distance is 58;45° where the mean is 60°, and their difference is 1;5°, while the distance at the above elongation from the perigee is 58;50° in the same units, and the difference from the mean 1;10°. So, multiplying 1;15 into 1;8 and dividing the result by 1;10, we find the difference [in anomaly] at the actual perigee with respect to the mean distance as 1;13°. Hence the amount of anomaly from the apparent perigee of the epicycle is [12;52° - 1;13° =] 11;39°, and from the apogee, for the first station, 168;21°, and, for the second station, 191;39°, which we enter in the same columns [i.e. the ninth and tenth respectively] opposite the number 35 '180'.

In the case of the planet Mercury, we showed [XII 6, pp. 579-80] that when the epicycle is 10;17° in mean longitude from the apogee of the eccentric, the planet performs its stations at a distance [in anomaly] from the apparent perigee of the epicycle of 32;52°, while the [corresponding] amount at mean distance is 34;56°, so that the difference is 2;4°. Furthermore, where the mean distance is 60°, the greatest distance is 69° and the difference between them 9°, while at the above elongation from the apogee the distance is 68;36°, and the difference from the mean 8;36°. By the same procedure as before, multiplying 9 into 2;4 and dividing the result by 8;36, we find the difference [in anomaly] at the actual apogee with respect to that for the mean distance as about 2;10°. Thus we calculate the distance [in anomaly] from apparent perigee of the epicycle as [34;56° - 2;10° =] 32;46°, and from the apogee as, for the first station, 147;14°, which we enter in the eleventh column opposite the number '360', and for the second station 212;46°, which we enter in the twelfth column on the same line.

Similarly [see p. 581], when the epicycle is 11;22° in mean [longitude] from the perigee, the planet performs its stations at a distance [in anomaly] from the apparent perigee of the epicycle of 35;30°, so that the difference from that for mean distance is [35;30° - 34;56° =] 34'. And the least distance is 55;34° where the mean is 60°, and their difference is 4;26°, while at the above elongation from the perigee the distance is about 55;42°, and the difference from the mean 4;18°. So, again, multiplying 4;26 into 0;34 and dividing the result by 4;18, we find the difference [in anomaly] at the actual perigee with respect to that for the mean distance as 0;35°. Hence the distance in anomaly from the apparent perigee of

κατά τόν τῶν ἄρθρων. One would expect κατά τοῦ τῶν ἄρθρων (cf. e.g. H499, 1-2, 22), and that occurs (at least as an alternative reading) in L, Ger. But the same expression occurs at H501, 14 and 502, 12.

Cf. p. 580 with n.69.
the epicycle is \([34;56° + 0;35° =] 35;31°\), and from the apogee, for the first station, 144;29°, and for the second station 215;31°. We enter the latter in the same [i.e. eleventh and twelfth] columns, in this case, however, not opposite the number '180' of longitude, but opposite '120' and '240', since we have shown that the points of the planet Mercury's eccentre closest to the earth are at those positions.

Now that the above has been set out, the increments for the positions in between [apogee and perigee] can be obtained using the same methods.

To take an example, let us set ourselves the task of finding the entries (in apparent anomaly) for first station when the mean position in longitude is 30° from the apogee. At this situation the distance of the epicycle, for a mean distance in every case of 60°, calculated by the methods explained previously, is (as we stated before)\(^8\) as follows:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance (°')</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>63;2''</td>
</tr>
<tr>
<td>Jupiter</td>
<td>62;26''</td>
</tr>
<tr>
<td>Mars</td>
<td>65;24''</td>
</tr>
<tr>
<td>Venus</td>
<td>61;6''</td>
</tr>
<tr>
<td>Mercury</td>
<td>66;35''</td>
</tr>
</tbody>
</table>

Hence the differences of each with respect to the mean (using the above order, to avoid repetition) are

\[
\begin{align*}
3;2'' & 2;26'' & 5;24'' & 1;6'' & 6;35''
\end{align*}
\]

But the differences between the distance at actual apogee and the mean, since the above amounts for the distance are in all cases greater than the mean, are, in the same units,

\[
\begin{align*}
3;25'' & 2;45'' & 6;0'' & 1;15'' & 9;0''
\end{align*}
\]

Now the total differences in apparent anomaly between apogee and mean distance come to (using the same order)\(^8\)

\[
\begin{align*}
1;23° & 1;33° & 5;41° & 1;17° & 2;10°
\end{align*}
\]

We multiply each of the latter in turn into the difference between the distance at that point and the mean for the planet in question (e.g. [for Saturn we multiply] 1;23 into 3;2), and divide the result by the difference between greatest distance [and mean], (e.g. [for Saturn] by 3;25), and thus get for the above position in longitude, for each planet, the following amounts of difference in anomaly with respect to that for mean distance:

\[
\begin{align*}
1;14° & 1;22° & 5;7° & 1;8° & 1;35°
\end{align*}
\]

The distances [in anomaly] from the apparent apogee of the epicycle at the mean distances are:\(^9\)

\[
\begin{align*}
114;8° & 125;38° & 163;9° & 167;8° & 145;4°
\end{align*}
\]

The [corresponding amount] at greatest distance is greater than the above for Mercury, but less for the other planets. So for Mercury we add the difference which we found for the distance in question to that for the mean distance, but for the other planets we subtract it, and get the following amounts, in apparent anomaly.

---

\(^8\) XI 10 p. 547. See that chapter for the method of calculating these quantities.

\(^9\) Saturn (p. 567) Apogee 67;15°, mean 65;52°, difference 1;23°. Jupiter (p. 571) Apogee 55;55° mean 54;22°, difference 1;33°. For the other amounts see pp. 584, 585, and 585. Although Ptolemy does not explicitly say so, logic demands, and the tables confirm, that for positions of the epicycle between mean distance and perigee one takes the corresponding differences in anomaly between mean distance and perigee (namely 1;21, 1;33, 6;0, 1;13 and 0;35) and interpolates accordingly. Cf. HAMA 204 bottom.

\(^{10}\) For the following amounts see L ZAH on pp. 565, 570, 573, 576, and 579, where in each case the supplements (i.e. the distances from apparent perigee) are given.
anomaly from the apogee of the epicycle, which are entered in the columns for first station opposite 30° of mean longitude:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Venus</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>112;54°</td>
<td>124;16°</td>
<td>158;2°</td>
<td>166;0°</td>
<td>146;39°</td>
</tr>
</tbody>
</table>

We can immediately complete the columns for second station, by entering, for each [planet], the difference from 360° of the amount for first station, [putting the result] in the column for second station on the same line. Thus at the above position [we enter]

| Saturn   | 247;6°  | 235;44° | 201;58° | 194;0°  | 213;21° |

It is easy to see that if, for the sake of greater convenience, we should choose to enter, not the anomaly, taken with respect to the apparent apogee of the epicycle, but the uncorrected anomaly, taken with respect to the mean [epicyclic apogee], we can immediately derive this too, by taking in the table of anomaly the equation (combined [from the 3rd and 4th columns]) corresponding to each argument of mean longitude, and subtracting it from the amount we found for the apparent anomaly on the 180° of the eccentric counted from apogee, but adding it for [longitudes from apogee] of more than 180°.

The layout of the table is as follows.

8. {Table of Stations}[^91]

[See p. 588.]

9. {Demonstration of the greatest elongations from the sun of Venus and Mercury}[^92]

Now that we have gone through the theorems concerning retrogradations, next in the logical sequence is to demonstrate the greatest elongations of the planets Venus and Mercury from the sun, in each of the zodiacal signs, as derived from the above hypotheses. In setting out [the tables] for these, we have taken [the elongations] with respect to the apparent position of the sun, and assumed that the actual planets are at the beginning of the [respective] signs, and that the positions of their apogees with respect to the solstitial and equinoctial points are those which obtain in our time, namely, for Venus, in 8 25°, and, for Mercury, in = 10°. It will be easy for those who come after us to correct for the change in the greatest distances due to the shift in the apogees, using the same methods, and in any case the change remains negligible for a very long time.

In order to make it easy to understand the method of our approach [to this problem], by way of example we must demonstrate, for Venus first, the greatest

[^90]: Deleting the word στίγου at H504.20. If kept, this would mean 'on each line'. But, first, Ptolemy does not use ἐκεῖ in this sense, but κατά; secondly, it is hideously clumsy to follow ἐκεῖστον στίγου by κατά τὰν αὐτῶν στίγῳ; and thirdly one needs a reference to each planet (exactly as at H504.1). This is an ancient interpolation, since it is in all mss.

[^91]: For Mars, argument 138° (H507.28), D,Ar have the readings 167;10° (also A') and 192;50°, which are more correct than the 167;8°, 192;52° adopted by Heiberg, and should perhaps be preferred. However, errors of as much as 2° occur elsewhere in the Mars table.

**Table of Planetary Stations**

<table>
<thead>
<tr>
<th>Common Numbers</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Venus</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
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<td>112 45</td>
<td>247</td>
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<td>208 21</td>
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<td>112 45</td>
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<tr>
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<td>112 58</td>
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<td>180</td>
<td>180</td>
<td>115 29</td>
<td>244</td>
<td>153</td>
<td>208 21</td>
</tr>
</tbody>
</table>

H509 morning and evening elongations (as defined above) when the planet is at the spring equinox, [namely] at the beginning of Aries.

Let [Fig. 12.13] the line through A, the apogee of the eccentric, be ABGDE, on which B is taken as the centre of uniform motion, G as the centre of the eccentric carrying the epicycle, and D as the centre of the ecliptic. Draw GZ as radius of the eccentric, describe the epicycle HΘ about Z, and from D draw DΘ as tangent on the side of the epicycle which represents morning [visibility] and is in advance of it[s centre]. Join BZH and ZΘ, and drop perpendiculars GK, GL and BM.

Then, since DA points towards 8 25° and DΘ towards the beginning of Aries,

\[
\angle AD\Theta = \begin{cases} 
55^\circ & \text{where 4 right angles } = 360^\circ \\
110^\circ & \text{where 2 right angles } = 360^\circ;
\end{cases} 
\]

and \( \angle DGK = 70^\circ \) (complement).
Therefore, in the circle about right-angled triangle GDK,

arc GK = 110°
and GK = 98;18'' where hypotenuse GD = 120°.

Therefore where GD = 1;15° and the radius of the epicycle, ZΘ = 43;10°

\[ \text{GK} = L \Theta = 1;1^p, \]

and, by subtraction [of LΘ from ZΘ], ZL = 42;9°,
where GZ, the radius of the eccentre, is taken as 60°.

Therefore where hypotenuse GZ = 120°, ZL = 84;18°,
and, in the circle about right-angled triangle GZL,

\[ \text{arc ZL} = 89;16°. \]

\[ \therefore \angle ZGL = 89;16°\,\text{where} \,2 \,\text{right angles} = 360°. \]

But \( \angle DGK = 70° \) in the same units, and \( \angle LGK \) is right.

Therefore, by addition, \( \angle ZGD \) is found to be [89;16 + 70 + 180] = 339;16°,
and, by subtraction [from 2 right angles], \( \angle AGZ = 20;44°. \)

Therefore, in the circle about right-angled triangle BGM,

\[ \text{arc BM} = 20;44° \]

and arc GM = 159;16° (supplement).

Therefore the corresponding chords

\[ \begin{align*}
\text{BM} &= 21;35° \\
\text{and GM} &= 118;2^p
\end{align*} \]
where hypotenuse BG = 120°.

Therefore where BG = 1;15°, and GZ, the radius of the eccentric, is 60°,

\[ \begin{align*}
\text{BM} &= 0;13^p \\
\text{GM} &= 1;14^p
\end{align*} \]

and, by subtraction [of GM from GZ], MZ = 58;46°.

Hence hypotenuse BZ \( \{= \sqrt{BM^2 + MZ^2} \} = 58;46° \) in the same units.

Therefore, where BZ = 120°, BM = 0;27°,
and, in the circle about right-angled triangle BZM,
arc BM = 0;26°.
\[ \therefore \angle BZG = 0;26°\text{°} \text{ where 2 right angles = 360°} \]
And we showed that \( \angle AGZ = 20;44°\text{°} \) in the same units.
Therefore, by addition, \( \angle ABZ \), which represents the mean motion in longitude,
is \[ \begin{cases} 
21;10°\text{°} \text{ where 2 right angles = 360°} \\
10;35° \text{ where 4 right angles = 360°.}
\end{cases} \]
Therefore the mean position of the sun will be 10;35° in advance of the apogee at A, and, obviously, will be in 8 14;25°.

And the true position of the sun will be in 8 15;14°. Therefore the planet, when it is at the beginning of Aries, will have a maximum morning elongation from the true sun of 45;14°.

Again, let there be drawn next [Fig. 12.14] the diagram with the tangent to the side of the epicycle which represents evening [visibility] and is towards the rear of the epicycle [centre], while the planet, as before, is taken as being at the beginning of Aries.

By what was shown above, \( \angle AD\Theta \) will remain the same,
and \( \angle DGK = 70°\text{°} \text{ where 2 right angles = 360°} \text{°} \)
and \( GK = L\Theta = 1;1°\)
where GZ, the radius of the eccentric, is 60°,
and Z\Theta, the radius of the epicycle, is 43;10°.

Therefore, by addition, \( ZL[= Z\Theta + L\Theta] = 44;11°\text{°} \) in the same units.
And it is obvious that, where hypotenuse [of triangle GZL] \( GZ = 120°\),
\( ZL = 88;22°\),
and, in the circle about right-angled triangle GZL,
arc ZL = 94;51°.
\[ \therefore \angle ZGL = 94;51°\text{°} \text{ where 2 right angles = 360°} \text{°} \]
and \( \angle ZGK = 85;9°\text{°} \) (complement).
So, by addition, $\angle ZGD (= \angle BGM) [= \angle DGK + \angle ZGK] = 155;9^\circ$ in the same units.

Hence, in the circle about right-angled triangle $BGM$,

$$\text{arc } BM = 155;9^\circ$$

and $\text{arc } GM = 24;51^\circ$ (supplement).

Therefore the corresponding chords

$$BM = 117;11'$$

and $GM = 25;49'$ where hypotenuse $BG = 120^\circ$.

Therefore, where $BG = 1;15^\circ$,

$$BM = 1;13^\circ$$

$MG = 0;16^\circ$,

and, by addition, $MZ = 60;16^\circ$.

Hence hypotenuse $BZ [= \sqrt{BM^2 + MZ^2}] = 60;17^\circ$ in the same units.

Therefore, where $BZ = 120^\circ$, $BM = 2;25^\circ$.

and, in the circle about right-angled triangle $BZM$,

$$\text{arc } BM = 2;19^\circ$$

$\therefore \angle BZM = 2;19^\circ$ where 2 right angles $= 360^\circ$.

And $\angle BGZ = 204;51^\circ$ in the same units.

since $\angle DGZ$ was shown to be $155;9^\circ$ in those units.

Therefore, by addition, $\angle ABZ$, which represents the mean motion in longitude,$^93$

comes to $\begin{cases} 207;10^\circ & \text{where 2 right angles} = 360^\circ \\ 103;35^\circ & \text{where 4 right angles} = 360^\circ \end{cases}$.

Therefore the sun's mean position will be at $[8 25^\circ - 103;35^\circ =] = 11;25^\circ$ and its true position at $= 13;38^\circ$.

Thus the greatest evening elongation of the planet from the true sun, when, as before, it is at the beginning of Aries, will be $46;22^\circ$.

In the case of the planet Mercury, in order to have a more convenient approach to the demonstrations of its missing phases which we shall give further on,$^94$ let us set ourselves the task of finding the maximum elongation of the planet from the true sun, as evening star when it is at the beginning of Scorpius, and as morning star when it is at the beginning of Taurus.

Now, according to our hypothesis for Mercury, when the apparent position of the planet is given, the mean position in longitude cannot be found, since line $GZ$ does not remain the same constant length,$^95$ always equal to the radius of the eccentric (as it does in the hypothesis for the other [planets]). But if the mean position in longitude is given, the apparent position can be demonstrated. So we assume, for each [zodiacal] sign, two positions in [mean] longitude which can bring the planet [at greatest elongation] near the beginning of the sign in

---

$^93$ Reading τῆς ὀμαλῆς κατὰ μῆκος παρόδου (with D'G. Ar) at H513.15-16 for the nonsensical τῆς ὀμαλῆς κατὰ μῆκος παρόδου. Corrected by Manilius.

$^94$ The reference is to XIII 8 (p. 644).

$^95$ For the other planets (e.g. Venus, Fig. 12.14) this denotes the distance from the centre of the eccentric to the centre of the epicycle, but for Mercury Ptolemy seems to be referring to a figure such as Fig. 9.9, where it denotes the distance from the epant point to the centre of the epicycle. These two amounts are indeed trigonometrically comparable. Ptolemy is correct in stating that, for Mercury, one cannot find the mean position from the true, at least by Euclidean geometry.
question, the first in advance [of the beginning of the sign], and the second to the rear [of it]; we compute the greatest elongations at the chosen positions, and thence\(^6\) find the greatest elongation which occurs at the actual beginning of the sign. This will be easily comprehensible from the [particular] problems we have set ourselves to solve: and first for the greatest evening elongation at the beginning of Scorpius.

Let [Fig. 12.15] the diameter through the apogee A be ABGD, on which G is taken as the centre of the ecliptic, and B as the centre of the epicycle’s uniform motion. First let the epicycle centre be imagined as being precisely at the apogee, so that the mean position in longitude of the sun will be \(\simeq 10^\circ\), and its true longitude \(\simeq 8^\circ\). On centre A describe the epicycle ZH, draw GH as tangent to the side of it representing evening, and drop perpendicular AH.

Then, since in our previous treatment [IX 9, p. 459] it was shown that where GA, the greatest distance, is 69°, AH, the epicycle radius, is 22\(^{i\circ}\), where hypotenuse [of right-angled triangle AGH] AG = 120°,

\[
AH = 39;8^\circ,
\]

and, in the circle about right-angled triangle AGH,

\[
\text{arc } AH = 38;4^\circ,
\]

\[
\angle AGH = \begin{cases} 38;4^\circ & \text{where 2 right angles} = 360^\circ \\ 19;2^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}
\]

And GA is at \(\simeq 10^\circ\).

Therefore the planet will have a position of \(\simeq 29;2^\circ\), its maximum elongation from the true sun being 21;2°.

\(^6\)By linear interpolation.
XII 9. Computation of Mercury's greatest elongations

Again, let [Fig. 12.16] the distance in mean longitude from the apogee be 3°: thus the mean sun will be at \( \triangle 13° \), and the true sun at \( \triangle 11;4° \). Draw BE and on centre E describe the epicycle ZH. As before, draw the tangent GH, and join EG, EH. Then at the situation in question, i.e. with \( \angle ABE \) taken as 3°, by our previous methods one can show that the angle corrected for the eccentricity, \( \angle AGE = 2;52° \), and the distance of the epicycle in that situation, \( EG \approx 68;58° \) where EH, the radius of the epicycle, is 22;30°.

Therefore, where hypotenuse \( EG = 120° \), \( EH = 39;9° \). Therefore, in the circle about right-angled triangle GEH,

\[
\text{arc } EH = 38;5°.
\]

and \( \angle EGH = \begin{cases} 38;5° & \text{where 2 right angles} = 360° \\ 19;3° & \text{approximately, where 4 right angles} = 360° \end{cases} \)

Hence, by addition, \( \angle AGH = 21;55° \) in the same units.

So when the planet is at \( \pi 1;55° \), its greatest elongation from the true sun will be \( [\pi 1;55° - \triangle 11;4° =] 20;51° \).

And we showed that when it is at \( \triangle 29;2° \), its greatest elongation from the true sun will be 21;2°.

Thus the difference between the longitudes is 2;53°, and the difference between the greatest elongations is 11°, and so to the 0;58° from the first position

---

97 If the text is to be trusted here, this must be the meaning of της παρά την ἐκκεντρότητα διαφορᾶς. But the normal reference of such an expression would be to the equation (of centre) itself, not to the angle corrected by the equation. I strongly suspect that the phrase is interpolated (it is in the whole ms. tradition).

98 By trigonometrical calculation. \( EG = 68;58;25° \), \( \angle AGE = 2;52;10° \).
XII.9. Computation of Mercury’s greatest elongations

to the beginning of Scorpius corresponds [a decrement in greatest elongation of] about 4’, which we subtract from 21;2° to get the greatest evening elongation from the true sun [when the planet is] precisely at the beginning of Scorpius as 20;58°.

Next, to find the greatest morning elongation at the beginning of Taurus, let us suppose first that the mean position in longitude is 39° towards the rear from the perigee. Thus the mean sun is at 8 19°, and the true sun at 8 19;38°. Let there be drawn [Fig. 12.17] a figure similar [to the preceding], in which the epicycle is described to the rear of the perigee, and the tangent is drawn to the morning side of the epicycle.

Then at the position in question, i.e. with $\angle DBZ$ taken as 39°, by the method previously described one can show that

$\angle DGE = 40;57°$.

and that the distance at that moment.

$GE = 55;59°$ where the radius of the epicycle, $EH = 22;30°$.

Therefore where hypotenuse [of right-angled triangle $GEH$] $GE = 120°$.

$EH = 48;14°$

and, in the circle about right-angled triangle $GEH$.

$arc \ EH = 47;24°$.

$\therefore \angle EGH = \begin{cases} 47;24° & \text{where 2 right angles = 360°} \\ 23;42° & \text{where 4 right angles = 360°} \end{cases}$

And, by subtraction [from $\angle DGE$], $\angle HGD = 17;15°$ in the same units.

Therefore when the planet Mercury has a longitude of $\odot 27;15°$, its greatest morning elongation from the true sun will be \[8 19;38° - \odot 27;15° = \] 22;23°.

*For $\overline{K} = 219°$, $\rho = 55;59.1°$, and $\overline{K} = 220;55;57°$, hence $\angle DGE \approx 40;56°$. 

**Fig. 12.17**
Again, let it be assumed to have a distance in mean longitude from the perigee, on the same side, of 42°. Thus the sun will have a mean longitude of 8 22° and a true longitude of 8 22;31°.

Then at this position, i.e. with ∠ DBZ taken as 42°, one can show that

∠ DGE = 44;4°,

and that the distance at that moment,

GE = 55;53''°° where the radius of the epicycle, EH = 22;30°.

Therefore, where hypotenuse EG = 120°, EH = 48;19°,

and, in the circle about right-angled triangle EGH,

arc EH = 47;30°.

∴ ∠ EGH = \begin{cases} 47;30° & \text{where 2 right angles} = 360° \\ 23;45° & \text{where 4 right angles} = 360°, \end{cases}

and, by subtraction [from ∠ DGE], ∠ HGD = 20;19° in the same units.

Therefore when the planet Mercury has a longitude of 8 0;19°, its greatest morning elongation from the true sun will be [8 22;31° - 8 0;19° =] 22;12°.

And we showed that when it has a longitude of 8 27;15°, its greatest elongation (similarly defined) will be 22;23°.

So, again, since the difference between the longitudes is 3;4°, and the difference between the greatest elongations is 11', to the 2;45° from the longitude at the first position to the beginning of Taurus correspond approximately 10'. So, subtracting the latter from the 22;23°, we get the greatest morning elongation from the true sun [when the planet is] at the beginning of Taurus as 22;13°.

Q.E.D.

In the same way we computed the greatest morning and evening elongations for both planets by calculation at [the beginning of] the other signs, and constructed a small table for them, with 12 lines (equal in number [to the signs]) and 5 columns. At the beginning we put, in the first column, the first points of the signs, starting with Aries. In the following 4 columns we put the corresponding computed greatest elongations from the true sun: the second contains the morning elongations of the planet Venus, the third its evening elongations, the fourth the morning elongations of Mercury, and the fifth its evening elongations. The table is as follows.

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100 Reading ΒΕ Ψ at H519.13 for ΒΕ Ψ (55;50°). Calculation (for Κ = 222°) gives Ρ = 55;52,58°. Although Ptolemy is capable of a computing error of this amount, he did not in fact make it, for the following calculations are consistent with 55;53° and not with 55;50° (thus 22:30 x 120/55:50 = 42:21 1, whereas 55:53 leads to 48:19, as the text). The error, though scribal, is old, since it is shared by all mss.

101 Literally 'of 19' of the first degree of Taurus'.
### 10. {Greatest elongations with respect to the true sun}\(^{102}\)

<table>
<thead>
<tr>
<th>Beginning of the Sign</th>
<th>VENUS</th>
<th></th>
<th>MERCURY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As Morning Star</td>
<td>As Evening Star</td>
<td>As Morning Star</td>
</tr>
<tr>
<td>♈</td>
<td>45 14</td>
<td>46 22</td>
<td>24 14</td>
</tr>
<tr>
<td>♉</td>
<td>45 17</td>
<td>45 31</td>
<td>22 13</td>
</tr>
<tr>
<td>♊</td>
<td>45 34</td>
<td>44 49</td>
<td>20 18</td>
</tr>
<tr>
<td>♋</td>
<td>45 56</td>
<td>44 25</td>
<td>18 17</td>
</tr>
<tr>
<td>♌</td>
<td>46 20</td>
<td>44 31</td>
<td>16 35</td>
</tr>
<tr>
<td>♍</td>
<td>46 38</td>
<td>44 55</td>
<td>16 8</td>
</tr>
<tr>
<td>♎</td>
<td>46 45</td>
<td>45 41</td>
<td>17 46</td>
</tr>
<tr>
<td>♏</td>
<td>46 47</td>
<td>46 30</td>
<td>21 32</td>
</tr>
<tr>
<td>♐</td>
<td>46 50</td>
<td>47 13</td>
<td>26 9</td>
</tr>
<tr>
<td>♑</td>
<td>47 7</td>
<td>47 35</td>
<td>28 37</td>
</tr>
<tr>
<td>♒</td>
<td>45 41</td>
<td>47 34</td>
<td>28 17</td>
</tr>
<tr>
<td>♓</td>
<td>45 20</td>
<td>47 7</td>
<td>28 24</td>
</tr>
</tbody>
</table>

\(^{102}\) Correction to Heiberg: omit (with G,Ar) the column of argument before the entries for Mercury. Ptolemy's own description indicates that it was not in the original.

There are occasional computing errors of up to 5' in the entries. For Venus, ♉, evening, the printed version of the *Handy Tables* (Halma III p. 32) has 47;37 (computed 47;39), but this greater accuracy seems coincidental, as the version in *Vat. Gr. 1291*, f. 90', agrees with the *Almagest*. For Mercury, ♒, evening, there is a serious computing error, as noted *H.A.M.A.* 234 n.10. I find 18;53, but all mss. known to me agree in 19;14.
1. {On the hypotheses for the positions in latitude of the 5 planets}\(^1\)

The following two topics still remain to [complete] the treatment of the 5 planets: their position in latitude with respect to the ecliptic, and the discussion of their elongations at their first and last visibilities with respect to the sun. For the second topic the latitudinal distances of each must also be taken into account first, since some considerable differences in the first and last visibilities occur due to that factor. So we shall again first set out the hypotheses which we assign to the inclination of the circles of all [five] in common.

Now [first], just as each [planet] appears to perform a twofold anomaly in longitude, each exhibits a twofold difference in latitude, one [varying] with respect to the parts of the ecliptic, and due to the eccentric, the other with respect to [its elongation from] the sun, and due to the epicycle. Therefore in every case we suppose that the eccentric is inclined to the plane of the ecliptic, and that the epicycle is inclined to the plane of the eccentric. However, as we said [IX 6, p. 443], no noticeable difference occurs in the longitudinal position or the demonstrations of the anomalies on account of such small inclinations, as we shall show later.\(^2\) [Secondly,] from individual observations of every planet, [we see that] the planets appear exactly in the plane of the ecliptic when the corrected longitude is approximately a quadrant from the northern or southern limit of the eccentric, and at the same time the corrected anomaly is approximately a quadrant from its own apogee.\(^3\) So we suppose the inclinations of the eccentric to take place at the centre of the ecliptic (just as for the moon), and with respect to the diameters through the northern and southern limits; and [we suppose] that the inclinations of the epicycles take place with respect to that diameter of the epicycle which points towards the centre of the ecliptic, on which its apparent apogee and perigee are observed.

Moreover, in the case of the 3 planets Saturn, Jupiter and Mars, we have observed that when their longitudinal positions are in the section of the eccentric farther from the earth they are always\(^4\) north of the ecliptic, and are more

\(^1\) On chs. 1 and 2 see *HAM.I* 206-7, Pedersen 355-61.
\(^2\) See XIII 4 pp. 608-22.
\(^3\) I.e. from the true apogee of the epicycle.
\(^4\) One would expect καί (text Ṿ), and καί was apparently read by al-Hajjāj. If one keeps the text, one has to understand ‘through [the centre of the ecliptic] and the northern or southern limits’.
\(^5\) Excising τὸ πλαίσιον at H525.23, with Ar. It is a gloss (‘for the most part’) put in by a commentator to qualify ἀντί: since the northern limit does not quite coincide with the apogee (except for Mars), the planets are not *always* north of the ecliptic when on the semi-circle containing the apogee.
northerly for positions at the perigee of the epicycle than for those at the apogee; but that when their longitudinal positions are in the section of the eccentre nearer the earth, quite the opposite, they appear south of the ecliptic. And [we have observed] that the northern limit of the eccentre is, for Saturn and Jupiter, around the beginning of the sign of Libra, and, for Mars, around the end of Cancer, almost exactly at its apogee. From these [observations] we conclude that the parts of their eccentres in the above-mentioned regions of the zodiac are inclined towards the north, and the diametrically opposite parts [depressed] by an equal amount towards the south, and that the parts of the epicycle nearer the earth are always inclined in the same direction as the eccentre, while the diameter [of the epicycle] at right angles to the diameter through its apogee always remains parallel to the plane of the ecliptic.

In the case of Venus and Mercury, however, we have observed that [firstly], when their longitudinal positions are at the apogee or perigee of the eccentre, then positions at the perigee of the epicycle do not differ at all in latitude from positions at the apogee [of the epicycle]: rather they are either north or south of the ecliptic by an equal amount, always north for Venus, always south, on the contrary, for Mercury; whereas their positions at the greatest elongations differ [in latitude] from each other by the greatest amount (that is, the morning greatest elongations differ from the evening greatest elongations), while they differ from the positions at apogee and perigee of the epicycle (i.e. from the difference [in latitude] due to the eccentre) by an equal amount. [but] in opposite directions: the greatest elongation which is towards the rear [of the epicycle centre] and in the evening is, for Venus, more northerly [than the morning one] at the apogee of the eccentre and more southerly at the perigee, while for Mercury the opposite is true, it is more southerly at the apogee [of the eccentre] and more northerly at the perigee. [Secondly, we have observed that.] when their corrected longitudinal positions are at the nodes, then a distance of a quadrant on either side of apogee or perigee of the epicycle brings [the planet] into the plane of the ecliptic, whereas positions at the perigee [of the epicycle] have the greatest difference [in latitude] from positions at the apogee: for Venus this inclination is towards the south at the node on the semi-circle on which the equation is subtractive, and towards the north at the opposite [node]; for Mercury the opposite is again true: at the node on the subtractive semi-circle the inclination is towards the north, at the opposite one towards the south. From this too, then, we conclude that the inclination of the eccentre is also variable, and that its variation has the same period as the epicycle [on the...]

---

6 Excising τὸ πλείστῳ τὸτε at H526.1. This would have to mean 'the amount by which they are more northerly for apogee positions than for perigee positions is greatest at that point', where τὸτε refers to the apogee of the eccentre. But in fact the point where this occurs is not the apogee, but the northern limit, and in any case this refinement is simply not appropriate here.

7 I.e. if the eccentre is north of the ecliptic, the perigee of the epicycle is north of the eccentre. and if it is south, south.

8 At the positions in question (at apogee or perigee of the eccentre) the diameter of the epicycle through apogee and perigee of the epicycle lies in the plane of the eccentre, hence the latitudinal effect comes entirely from the inclination of the eccentre.

9 This nomenclature is used, rather than 'ascending' and 'descending' (as for the moon and the outer planets), because the effect of the inclination of the eccentre is always in one direction (north for Venus and south for Mercury). Cf. Maniitius p. 328 n.a) and Pedersen 376.
eccentre]: when the epicycle is in the nodes, the eccentre is in the same plane as
the ecliptic, but when [the epicycle] is at apogee or perigee, the eccentre produces
the greatest difference in the epicycle's latitude, making it most northerly for
Venus and most southerly for Mercury. [We also conclude that] the epicycle
brings about two variations [in latitude]: it produces the greatest inclination of
the diameter through the apparent apogee at the nodes of the eccentre, and the
greatest 'slant' (let us use this term to distinguish this kind of angular variation)
of the diameter at right angles to the former at the apogee and perigee of the
eccentre. Contrariwise, it brings the first [diameter] into the plane of the
eccentre at its [the eccentre's] apogee and perigee, and brings the second
diameter into the plane of the ecliptic at the above-mentioned nodes.

2. (On the type of the motions in inclination and slant according to the hypotheses)\(^{10}\)

The general structure of the hypotheses, then, which we infer is as follows. The
eccentric circles of [all] 5 planets are inclined to the plane of the ecliptic about
the centre of the ecliptic. But in the case of the 3 planets Saturn, Jupiter and
Mars the eccentre has a fixed inclination, so that diametrically opposite
positions of the epicycle have opposite directions in latitude, whereas in the case
of Venus and Mercury the eccentre moves together with the epicycle in the
same latitudinal direction, for Venus always to the north, for Mercury always to
the south. The epicycle [for all 5 planets] has the diameter through its apparent
apogee moved from a starting-point in the plane of the eccentre, by a small
circle which we may suppose attached to the end [of the diameter] nearer the
earth. This circle is of a size corresponding to the appropriate [maximum]
deviation in latitude, is perpendicular to and centred in the plane of the
eccentre, and revolves with uniform motion, with a period equal to that of the
motion in longitude, from one end of the intersection of its own plane and the
plane of the epicycle towards the north (by hypothesis), carrying with it the
plane of the epicycle: in its revolution through the first quadrant it carries the
epicycle's plane, obviously, to the northern limit, in the second back to the
plane of the eccentre, in the third to the southern limit, and in its return to [the
end of] the remaining quadrant back to the original plane. We also [infer] that
the origin and point of return of this revolution is for Saturn, Jupiter and Mars
the ascending node, for Venus the perigee of the eccentre, and for Mercury the
apogee of the eccentre.\(^{11}\) The diameter [of the epicycle] at right angles to the
aforementioned, in the case of the 3 [superior] planets, as we said [p. 598],
always remains parallel to the plane of the ecliptic, or at any rate is not inclined
to it by a significant amount, but in the case of Venus and Mercury it too
is carried from a starting-point in the plane of the ecliptic by a small circle
which we may suppose attached to the rearward end, which is again of a
size corresponding to the appropriate [maximum] deviation in latitude.

\(^{10}\)On the mechanism imagined by Ptolemy (and in particular the 'small circles') the best
discussion is by Riddell, 'Latitudes of Venus and Mercury', despite occasional inaccuracies due to
his use of Taliaferro's faulty translation.

\(^{11}\)It is essential to change Helberg's punctuation from a comma to a full stop at H530,13.
perpendicular to the plane of the ecliptic, and centred on the diameter\textsuperscript{12} parallel to the ecliptic. This circle revolves, with a speed equal to that of the other [small circle], from one end of the intersection of its plane and the plane of the epicycle towards the north, again by hypothesis, and carries with it the evening [i.e. rearward]\textsuperscript{13} end of the aforementioned diameter in the same order as before. For this too the origin and point of return of the similar type of revolution is, in the case of Venus, at the node in the additive semi-circle, and, in the case of Mercury, at the node in the subtractive semi-circle.

However, we must make the following assumption concerning those small circles which produce the motions in latitude of the epicycles: they too are, indeed, bisected by the planes about which we declare that the variations in latitude take place; for that is the only way in which it can come about that their [the epicycles'] motions in latitude are equal on both sides [of the planes]. However, their revolution in uniform motion takes place, not about their own centres, but about some other point which will produce in the small circle an eccentricity corresponding to [the eccentricity] of the planet in longitude in the ecliptic. For since the times of revolution on the ecliptic and the small circle are, by hypothesis, equal, and the arrivals at the quadrants in both [circles] also correspond to each other, according to the [observational] phenomena, if the [uniform] revolution of the small circle were to take place about its own centre, the desired result would not be achieved; since [in that case] each of the quadrants of the small circle would be traversed in an equal time, while the quadrants of the ecliptic traversed by the epicycle would not be, because of the eccentricity assumed for each planet. But if [the uniform revolution of the small circle takes place] about a point placed similarly to the [centre of uniform motion] in the eccentric, the returns in the inclinations will also traverse the corresponding quadrants of the ecliptic and the small circle in equal times.\textsuperscript{14}

Now let no one, considering the complicated nature of our devices, judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what [could one compare] more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself\textsuperscript{15}? Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions,\textsuperscript{16} but if this does not succeed, [one should apply hypotheses] which do fit. For provided that each of the phenomena is duly saved by the hypotheses, why should anyone think it strange that such

\textsuperscript{12}Cf. Manitius' note p. 331 b). If 'diameter' is to make any sense here, it must be a diameter of the epicycle which is parallel to the ecliptic (at a certain point in the orbit), and notionally remaining there all the time, even when the epicycle is 'slanted'. Cf. HAMA 1279 Fig. 219a (where the line through A is parallel to the ecliptic), and Riddell Fig. 4 and p. 101.

\textsuperscript{13}πρὸς ἑσπέραν, literally 'toward evening', which one would expect to mean 'western'. But the sense demands 'eastern', and, if the text is correct, one must interpret it, with the Arabic translators, as 'the side of the epicycle where the planet appears as evening star', cf. H511,22, τὰ ἑσπέρια καὶ ἔκπομα τοῦ ἐπικύκλου.

\textsuperscript{14}It is essential to correct Heiberg's punctuation of this passage by deleting the comma after τεταρτημόριον (H532,9) and inserting one after ἐκκέντρου (H532,8).

\textsuperscript{15}I.e. the substance of the heavenly bodies, the 'fifth essence'. Cf. p. 36 n.8.

\textsuperscript{16}On this principle of simplicity see p. 136 n.17.
complications can characterise the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another? Thus, quite simply, all the elements can easily pass through and be seen through all other elements, and this ease of transit applies not only to the individual circles, but to the spheres themselves and the axes of revolution. We see that in the models constructed on earth the fitting together of these [elements] to represent the different motions is laborious, and difficult to achieve in such a way that the motions do not hinder each other, while in the heavens no obstruction whatever is caused by such combinations. Rather, we should not judge 'simplicity' in heavenly things from what appears to be simple on earth, especially when the same thing is not equally simple for all even here. For if we were to judge by those criteria, nothing that occurs in the heavens would appear simple, not even the unchanging nature of the first motion, since this very quality of eternal unchangingness is for us not [merely] difficult, but completely impossible. Instead [we should judge 'simplicity'] from the unchangingness of the nature of things in the heaven and their motions. In this way all [motions] will appear simple, and more so than what is thought 'simple' on earth, since one can conceive of no labour or difficulty attached to their revolutions.

3. *On the amount of the inclination and slant for each [planet]*

From the above considerations one may infer the general situation and arrangement of the inclinations of the [various] circles. But [concerning] the actual size for each planet of the arc cut off by the inclination on the great circle drawn perpendicular to the plane of the ecliptic through the poles of the inclined circle (with respect to which [great circle] the positions in latitude are measured), this is readily calculated in the case of Venus and Mercury from the apparent positions in latitude at the situations described.

For when their motion in longitude brings them to apogee or perigee of the eccentric, if the planet's position is at perigee or apogee of the epicycle, they appear, as we said, (operating from nearby observations), an equal amount either north or south of the ecliptic: Venus always about $\frac{1}{2}^\circ$ north, and Mercury always $\frac{1}{2}^\circ$ south. Hence [we conclude that] the inclinations of the eccentres are of that size for each. But if they are at a greatest elongation from the sun, both planets appear about $5^\circ$ (in the mean) farther north or south than at the opposite greatest elongation: for Venus has an apparent difference in latitude of the kind mentioned [i.e. between greatest morning and evening elongations] of negligibly less than $5^\circ$ at the apogee of the eccentric, and negligibly greater than $5^\circ$ at the perigee, while Mercury has about [less and greater than $5^\circ$ in

17 On chs. 3 and 4 see HAM.1 208-16, Pedersen 361-85, Riddell, 'Latitudes of Venus and Mercury'.
18 'inclined circle': deferent or epicycle as the case may be.
19 From 'nearby observations' because the planets are invisible when precisely at apogee or perigee of the epicycle. Correct Heiberg's punctuation by inserting a comma after oξ έφαγεν ('as we said'), which cannot refer to the use of nearby observations, but only to the fact that the planet is north or south etc. (as p. 599).
latitudinal difference at apogee and 180° from apogee respectively]. Hence the slant of the epicycle to either side of the plane of the eccentre subtends about 2\(^\circ\) in the mean, of the [great] circle orthogonal to the ecliptic. From this [quantity] the size of the angles formed by the slant of the epicycle to the plane of the eccentre [for each planet] can be found, as will become clear in our proofs concerning them in what follows [XIII 4, p. 625] (so as not to fragment, at this point, our discussion of the inclinations, which will treat the five planets in common).

But when their corrected motion in longitude brings them to the nodes and [hence] very nearly to mean distance: then Venus, when its position is near the apogee of the epicycle, appears 1° north or south\(^29\) of the ecliptic, and, when its position is near the perigee, about 6\(^\circ\); hence the inclination of its epicycle too cuts off 2\(^\circ\) of the great circle drawn through its poles in the way described; for we find from the [table for] epicyclic anomaly that at mean distance that amount [2\(^\circ\)] subtends at the observer's eye an angle of 1;2° for [the planet at] the apogee of the epicycle, and 6:22° for [the planet at] the perigee.\(^21\) As for Mercury, when its position is near the apogee of the epicycle, as one can calculate from the phases nearest to it, it is north or south of the ecliptic\(^22\) by 1\(^\circ\), and, when near the perigee, about 4\(^\circ\); hence the inclination of its epicycle comes to 6\(^\circ\). For again we find from the [table for] epicyclic anomaly that at the distances of greatest inclination, that is when the corrected longitude is a quadrant from apogee, that amount [6\(^\circ\)] subtends, at the observer's eye, 1;46° for [the planet at] the apogee of the epicycle, and 4;5° for [the planet at] the perigee.\(^23\)

In the case of the other planets, Saturn, Jupiter and Mars, there is no method for finding the sizes of the inclination immediately [from the observational data], since both inclinations, that connected with the eccentre and that connected with the epicycle, are always intermingled; however, once again, from the latitudinal positions observed at perigee and apogee of eccentre and epicycle, we determine each inclination separately in the following manner.

[See Fig. 13.1.] In the plane orthogonal to the ecliptic let the intersection with it of the plane of the ecliptic be AB, and of the plane of the eccentre, GD. Let point E be the centre of the ecliptic, and at the intersection of the planes, [that of the eccentre and that orthogonal to the ecliptic], draw,\(^24\) in the defined plane, about G the apogee of the eccentre, and about D the perigee, equal circles ZH\(\Omega\)K and LMNX to represent the circles through the poles of the epicycles. On these circles let the planes of the epicycles [be drawn] on lines HGK and MDX, inclined, obviously, at equal angles at G and D. From E, the centre of...
the ecliptic, at which the observer's eye is. draw straight lines joining it to the apogees and perigees of the epicycles. EH and EM to the apogees, and EK and EX to the perigees. It is clear that points K and X will represent the positions at opposition, and H and M those at conjunction.

For Mars, then, we obtained the positions in latitude round about the oppositions occurring at the apogee of the eccentre (that is, round about point K of the epicycle), and also round about the oppositions occurring at the perigee of the eccentre (that is, round about point X of the epicycle), since the difference between them is quite noticeable. At the oppositions near the apogee it is 41° to the north of the ecliptic, and at those near the perigee about 7° to the south. Thus

\[ \angle AEK = 41° \]
\[ \text{and } \angle BEX = 7° \]

With that as data, we find the angle formed by the inclination of the eccentre, namely \( \angle AEG \), and that formed by the inclination of the epicycle, namely \( \angle HGZ \), in the following manner.

Since it is easy to see from our demonstrations of the anomalies of Mars that, if one considers the angles subtended at the observer's eye by equal arcs of the epicycle near its perigee, those for positions near the apogee of the eccentre bear to those for positions near the perigee [of the eccentre] a ratio of approximately 5:9, and since

\[ \text{arc } \Theta K = \text{arc } NX. \]

\[ 25 \text{ For the derivation of this ratio from the anomaly table see } HAMA 209-10, \text{ Pedersen 363 (with the correction Toomer [3], 141).} \]
it follows that \( \angle GEK : \angle DEX = 5:9 \).

H540  So, since angles AEK and BEX are given, and the ratio of \( \angle GEK : \angle DEX \) is given, and \( \angle AEG = \angle BED \),

if we form the difference between the magnitudes of the whole [angles, i.e. \( \angle AEK \) and \( \angle BEX \)], and the difference between [the members of] the ratio [i.e. 5 and 9], take the fraction which the first [difference] is of the second, and take that fraction of each [member of the] ratio, we will get the magnitude corresponding to each part of the ratio. This can be proven by means of an arithmetical lemma.\(^{26}\)

So, since the magnitudes are 4\( \frac{1}{2} \) and 7, and their difference 2\( \frac{1}{2} \), and the ratio is 5:9 and the difference 4.

and 2\( \frac{1}{2} \) is two-thirds of 4,

we take two-thirds of 5 and 9 [respectively], and get

\[
\angle GEK = 3\frac{1}{2}^0 \text{ and } \angle DEX = 6^0.
\]

Accordingly, by subtraction,

\[
\angle AEG = \angle BED = 1^0, \text{ the inclination of the eccentric.}
\]

Hence arc \( \Theta K \), representing the inclination of the epicycle, is 2\( \frac{1}{2}^0 \), for from the table of anomaly we find that that amount \( [2\frac{1}{2}^0] \) corresponds approximately to the quantities we found for the angles \( GEK \) and \( DEX \).\(^{27}\)

In the case of Saturn and Jupiter, we find that the [latitudinal] positions occurring near the apogee of the eccentric are not sensibly different from those diametrically opposite, near the perigee. So we computed the required results in another way, by comparing the [latitudinal positions] near apogee of the epicycle with those near perigee. It has become clear to us from individual observations that at positions near first and last visibilities the maximum deviation to north and south is about 2\( ^0 \) for Saturn and 1\( ^0 \) for Jupiter, while for positions near opposition [the maximum latitudinal deviation] is about 3\( ^0 \) for Saturn and 2\( ^0 \) for Jupiter. Now for these planets too it is obvious from the [table for] anomaly that, if one considers the angles subtended at the observer’s eye by equal arcs near apogee and perigee of the epicycle, the angles subtended by arcs near apogee bear a ratio to those subtended by arcs near perigee of 18:23 for Saturn, and 29:43 for Jupiter;\(^{28}\)

and arcs \( ZH \) and \( \Theta K \) of the epicycle are equal.

So \( \angle ZEH : \angle ZEK = \left\{ \begin{align*}
18:23 \text{ for Saturn} \\
29:43 \text{ for Jupiter.}
\end{align*} \right. \)

But \( \angle HEK \), which is the difference between the two latitudes [at apogee and

\(^{26}\) Given two magnitudes \( A, B \), and the ratio \( 1:m \) of two other magnitudes, \( C, D \) such that\n
\( A = x + C, B = x + D, \) the lemma states that

\( C = \frac{1 \times (B - A)}{(m - 1)}, \) \( D = \frac{m \times (B - A)}{(m - 1)}. \)

**Proof:** Since \( D/C = m/l, \) \( (D - C)/C = (m - 1)/l. \)

But \( D - C = B - A \)

\( \therefore C = \frac{1 \times (B - A)}{(m - 1)}; \)

\( D = \frac{m \times (B - A)}{(m - 1)}. \)

\(^{27}\) For the method see p. 602 n.21. Here, from Table XI 11, cols. 5-7, for argument \( \alpha = (180^0 - 24^0) \) at greatest and least distance respectively, one finds \( (5;45 - 1;16) \times 21/3 \approx 3;22^0 \) and \( (5;45 + 2;20) \times 21/3 \approx 6;4^0 \) (text 349 and 60).

\(^{28}\) See H.AMA 211, where however one should change to \( \frac{c_8(183)}{c_8(3)} = \frac{7}{4} \) for Saturn and \( \frac{3}{2} \) for Jupiter, in exact agreement with Ptolemy.
perigee of the epicycle], is, by subtraction, 1° for both planets. Therefore, if we divide that 1° in the above ratios, we get

\[ \angle ZEH = \begin{cases} 0;26° \text{ for Saturn} \\ 0;24° \text{ for Jupiter} \end{cases} \]

and

\[ \angle ZEK = \begin{cases} 0;34° \text{ for Saturn} \\ 0;36° \text{ for Jupiter} \end{cases} \]

So, by subtraction [from \( \angle AEG \)], the inclination of the eccentric

\[ \angle AEG = \begin{cases} 2;26° \text{ for Saturn} \\ 1;24° \text{ for Jupiter} \end{cases} \]

Instead of these, to achieve greater symmetry, we have adopted the round numbers 2° and 1°. Then arc \( \theta K \), representing the inclination of the epicycle, can immediately be computed as 4° for Saturn and 2° for Jupiter. For again, in the tables of anomaly for each planet, those were the amounts which correspond approximately to the quantities we found for angles \( \angle ZEH \) and \( \angle ZEK \).

Q.E.D.

4. \{Construction of tables for the individual positions in latitude\}

From the above, then, we established the generally applicable quantities of the greatest inclinations of centres and epicycles. But in order that we may be able to conveniently and systematically find the positions in latitude for a given moment for the individual distances [from apogee] as well, we constructed 5 tables for the 5 planets. Each contains the same number of lines as the tables for anomaly [i.e. 45], and 5 columns. The first 2 of these columns comprise the arguments, in the same way as in those tables for anomaly; the third column contains the latitudinal distances from the ecliptic corresponding to the particular degrees of [motion on] the epicycle, under the assumption of greatest inclination - for Venus and Mercury this is the inclination at the nodes of the eccentric, and for the other 3 planets the inclination at the northern limit of the eccentric. For the latter the fourth column will contain the similar corresponding amounts at the southern limit, and in the case of these 3 planets the maximum deviation to north and south of the eccentres too has also been included in the computation. The way in which we determined these quantities for Venus and Mercury again rested on a single theorem [for both], as follows.

[See Fig. 13.2] In the plane orthogonal to the ecliptic let \( ABG \) be the intersection with it of the plane of the ecliptic, and \( DBE \) the intersection [with it] of the plane of the epicycle. Let \( A \) be the centre of the ecliptic, \( B \) the centre of the epicycle, and \( AB \) the distance of the epicycle at the greatest inclination. About \( B \) describe the epicycle \( DZEH \), and draw diameter \( ZBH \) perpendicular

\[ \text{See p. 602 n.21. Here, from Table XI 11, col. 6 for Saturn, } 0;36 \times 4\frac{1}{6} = 0;27° \text{(text 0;26°), and } 0;23 \times 4\frac{1}{3} = 0;34,30° \text{ (text 0;34°); for Jupiter } 0;58 \times 2\frac{1}{6} = 0;24,10° \text{ (text 0;24°); } 0;43 \times 2\frac{1}{3} = 0;35,50 \text{ (text 0;36).} \]

\[ \text{Note that } G \text{ is not a point on the epicycle, as might appear from Fig. 13.2 and from the corresponding figure for Mercury, Fig. 13.4. To make the various planes in this three-dimensional figure clearer it has been redrawn as Fig. 5.} \]
to DE. Let the plane of the epicycle too be taken as perpendicular to the assumed plane [that orthogonal to the plane of the ecliptic], so that when lines are drawn in it perpendicular to DE, all will be parallel to the plane of the ecliptic, excepting only ZH, which will lie in the plane of the ecliptic.

Then let the problem be, given the ratio of $AB$ to $BE$, and the amount of the inclination (i.e. of $\angle ABE$), to find the positions of the planets in latitude when (to take an example) they are at a distance of 45° (where [the circumference of] the epicycle is 360°) from the perigee of the epicycle, $E$. [We choose 45°] because we intend to demonstrate at the same time the differences in the positions in longitude produced by these [maximum] inclinations, and these differences ought to reach their maximum at about halfway between the perigee $E$ and the positions $Z$ and $H$, since at those points [the longitudes so computed] are identical with the longitudes produced by neglecting the inclination.

So let arc $E\Theta$ be cut off in the above amount of 45°, and drop $\Theta K$, perpendicular to $BE$, and $KL$, $\Theta M$ perpendicular to the plane of the ecliptic. Join $\Theta B$, $LM$, $AM$ and $A\Theta$. 
It is immediately obvious that

[1] quadrilateral $\text{LK}\Theta\text{M}$ has parallel sides and right angles
(since $\text{K}\Theta$ is parallel to the plane of the ecliptic); and

[2] the equation in longitude is comprised by $\angle \text{LAM}$, and

[3] the position in latitude is comprised by $\angle \Theta\text{AM}$
(since angles $\text{ALM}$ and $\text{AM}\Theta$ too turn out to be right angles, because line $\text{AM}$ lies in the plane of the ecliptic).

But now we must demonstrate the numerical amounts of the required positions to be computed for each of the above planets, and first for Venus.

Since arc $\text{EO} = 45^\circ$ where \{the circumference of\} the epicycle is $360^\circ$,

$\angle \text{EO}\Theta$ (since it is at the centre of the epicycle) = \begin{cases} 45^\circ & \text{where 4 right angles = } 360^\circ \\ 90^\circ & \text{where 2 right angles = } 360^\circ \end{cases}$

Therefore, in the circle about right-angled triangle $\text{B0K}$,

arc $\text{BK} = \text{arc } \text{K}\Theta = 90^\circ$.

So the corresponding chords $\text{BK} = \text{K}\Theta = 84;52'$, where hypotenuse $\Theta\text{B} = 120'$.

Therefore where $\Theta\text{B}$, the radius of the epicycle, is $43;10''$,

and AB, the mean distance, is $60''$

(for the greatest inclination of the epicycle occurs at approximately that point),

$\text{BK} = \text{K}\Theta = 30;32''$.

Again, since the angle of inclination,

$\angle \text{ABE}$ is taken as \begin{cases} 2;30'' & \text{where 4 right angles = } 360^\circ \\ 5;50'' & \text{where 2 right angles = } 360^\circ \end{cases}$

in the circle about right-angled triangle $\text{BLK}$,

arc $\text{LK} = 5^\circ$

and arc $\text{BL} = 175^\circ$ (supplement).

So the corresponding chords

$\text{KL} = 5;14'$, where hypotenuse $\text{BK} = 120'$.

$\text{BL} = 119;53'$.

31 See Fig. S, which makes most of Ptolemy's statements obvious. In particular, since $\text{M}$ is in the ecliptic (by construction) and $\angle \text{AM}\Theta$ is constructed as a right angle, $\text{LM}$, $\text{K}\Theta$ and $\text{BH}$ are all parallel, so $\angle \text{ALM}$ is a right angle.
Therefore, where hypotenuse $BK = 30;32^\circ$, and $AB = 60^\circ$,

- $KL = 1;20^\circ$,
- $BL = 30;30^\circ$,

and, by subtraction [of BL from AB], $AL = 29;30^\circ$.

But, in the same units, $LM = K\Theta = 30;32^\circ$.
Therefore hypotenuse $AM = \sqrt{AL^2 + LM^2} = 42;27^\circ$ in the same units.

Therefore, where hypotenuse $AM = 120^\circ$, $LM = 86;19''$,
and the equation in longitude at that point,

$$\angle LAM = \begin{cases} 92;0^\circ \text{ where 2 right angles} = 360^\circ \\ 46;0^\circ \text{ where 4 right angles} = 360^\circ \end{cases}.$$ 

Similarly, where $AM = 42;27^\circ$,
- $\Theta M = KL = 1;20^\circ$;
and $\Theta M^2 + AM^2 = A\Theta^2$,
so $A\Theta = 42;29^\circ$ in the same units.

Therefore, where hypotenuse $A\Theta = 120^\circ$,
- $\Theta M = 3;46^\circ$,
and the angle of the deviation in latitude,

$$\angle \Theta AM = \begin{cases} 3;36^\circ \text{ where 2 right angles} = 360^\circ \\ 1;48^\circ \text{ where 4 right angles} = 360^\circ \end{cases}.$$ 

That $[1;48^\circ]$ is what we shall put in the third column of the table for Venus on the line containing '135°'.

In order to make a comparison of the difference in the equation of longitude which results [from the above computation], let there be drawn [Fig. 13.3] the corresponding figure without any inclination of the epicycle. Then we showed that

- $BK = K\Theta = 30;32^\circ$ where $AB = 60^\circ$,

so, by subtraction, $AK = 29;28^\circ$;
and $AK^2 + K\Theta^2 = A\Theta^2$,

so $A\Theta = 42;26^\circ$ in the same units.

Therefore, where hypotenuse $A\Theta = 120^\circ$, $K\Theta = 86;21^\circ$,
and the angle of the equation in longitude,

$$\angle \Theta AK = \begin{cases} 92;3^\circ \text{ where 2 right angles} = 360^\circ \\ 46;2^\circ, \text{ approximately, where 4 right angles} = 360^\circ \end{cases}.$$ 

And with the inclination it was shown to be $46^\circ$.

Therefore the equation in longitude, computed according to the inclination, was less by $2^\circ$.

Q.E.D.\textsuperscript{32}

Again, to enable us to demonstrate the [latitudinal] positions for Mercury too, let there be drawn a figure [Fig. 13.4] similar to the one before the last, with arc $E\Theta$ taken as the same size, $45^\circ$. Hence again

$BK = K\Theta = 84;52^\circ$ where hypotenuse $B\Theta = 120^\circ$.

Therefore, where the radius of the epicycle, $B\Theta = 22;30^\circ$,

\textsuperscript{32} Accurately, one finds $45;59^\circ$ (to the nearest minute) with the inclination, and $46;0^\circ$ without it. Ptolemy's inaccuracy here is mysterious, since for the table of anomaly (XI 11), argument $135^\circ$ at mean distance, he found (presumably by an identical computation) the better value $45;59^\circ$. 
and $AB$, the distance at which the greatest inclinations occur, is $56;40^\circ$ (all of which we have previously demonstrated),

$$BK = KO = 15;55^\circ$$ in the same units.

Again, since by hypothesis the angle of the inclination of the epicycle,

$$\angle ABE = \begin{cases} 6;15^\circ & \text{where 4 right angles = 360°} \\ 12;30^\circ & \text{where 2 right angles = 360°} \end{cases},$$

in the circle about right-angled triangle $BKL$,

$$\text{arc } KL = 12;30^\circ$$

and $\text{arc } BL = 167;30^\circ$ (supplement).

So the corresponding chords

$$KL = 13;4^\circ$$

and $BL = 119;17^\circ$, where hypotenuse $BK = 120^\circ$.

Therefore where $BK$, as we showed, is $15;55^\circ$,

and $AB$, by hypothesis, is $56;40^\circ$,

$$KL = 1;44^\circ,$$

$$BL = 15;49^\circ,$$
and, by subtraction [from AB], $AL = 40;51^p$ in the same units.

And $LM = K\Theta = 15;55^p$.

And since $AL^2 + LM^2 = AM^2$,

$AM = 43;50^p$ where line $LM = 15;55^p$.

Therefore, where hypotenuse $AM = 120^p$, $LM = 43;34^p$,

and the angle of the equation in longitude,

$$\angle LAM = \begin{cases} 42;34^o & \text{where 2 right angles} = 360^o \\ 21;17^o & \text{where 4 right angles} = 360^o \end{cases}$$

Similarly, where $AM = 43;50^p$,

$\Theta M = KL = 1;44^p$;

and $AM^2 + \Theta M^2 = A\Theta^2$,

so $A\Theta = 43;52^p$ in the same units.

Therefore, where hypotenuse $A\Theta = 120^p$,

$\Theta M = 4;44^p$.

and the angle of the deviation in latitude,

$$\angle \Theta AM = \begin{cases} 4;32^o & \text{where 2 right angles} = 360^o \\ 2;16^o & \text{where 4 right angles} = 360^o \end{cases}$$
That [2;16°] is what we shall enter in the third column of the table for Mercury on the same line, namely that containing the argument '135°'.

In order again to make a comparison of the equation, let there be drawn [Fig. 13.5] the figure without the inclination [of the epicycle]. Then we showed that, where line $AB = 56;40'$,

$$ discouraged $$

and, by subtraction, obviously, $AK = 40;45'$ in the same units;

$$ discouraged $$

so $AO = 43;45'$ where $OK = 15;55'$. 

Therefore, where hypotenuse $AO = 120'$, $OK = 43;39'$, and the angle of the equation in longitude,

$$enerated$$

But we showed that with the inclination it was 21;17°. Therefore here too the equation in longitude computed according to the inclination was less, by 3'.

Q.E.D.

Such, then, is the method by which we computed the positions in latitude at the greatest inclinations for these two planets. For the greatest inclinations occur when the eccentre is in the same plane as the ecliptic. For the remaining 3 planets, however, we computed [those positions] by means of a theorem which

Fig. 13.5
requires a different diagram, since [for these] the greatest inclination of the epicycle occurs when the inclination of the eccentre is also at a maximum, and it would benefit us to have the positions in latitude resulting from both inclinations computed together.

[See Fig. 13.6 and cf. Fig. T.] In the plane orthogonal to the ecliptic, again, let the intersection with it of the plane of the ecliptic be AB, the intersection of the plane of the eccentre AG, and the intersection of the plane of the epicycle DGE. Let A be taken as the centre of the ecliptic, and G as the centre of the epicycle, and let the epicycle DZEH be described about G in such a way, again,
that when lines are drawn perpendicular to DE, diameter ZGH lies in the plane of the eccentric and parallel to the plane of the ecliptic, while the other perpendiculars are parallel to both the above planes. Similarly, let arc EΘ be cut off in the same amount of $45^\circ$, and drop perpendicular ΘK from Θ (the point at which the planet is located), and also drop perpendiculars ΘL, KB from points Θ and K to the plane of the ecliptic. Join BL and AL. Then let the problem be, to find the equation in longitude, represented by $\angle BAL$, and the position in latitude, represented by $\angle LAΘ$.

So draw perpendicular KM from K to AG, and join GΘ, AK and AΘ. Let us again take it as given, from what was proved before, that $GK = KΘ = 84;52''$ where hypotenuse $GΘ = 120^\circ$.

Then first, for Saturn:

Since we showed that the radius of the epicycle is 6;30'' where the mean distance is 60°,

$$GK = KΘ = 4;36'' \text{ where hypotenuse } GΘ = 60^\circ.$$

And since, by hypothesis, the angle of the inclination of the epicycle,

$$\angle AGE = \begin{cases} 4;30^\circ \text{ where 4 right angles } = 360^\circ \\
9^\circ \text{ where 2 right angles } = 360^\circ 
\end{cases},$$

in the circle about right-angled triangle GKM,

$$\text{arc } KM = 9^\circ$$

and arc $GM = 171^\circ$ (supplement).

So the corresponding chords

- $KM = 9;25''$ where hypotenuse $GK = 120^\circ$
- $GM = 119;38''$ in the same units.

Therefore, where $GK = 4;36''$,

- $KM = 0;22''$
- $GM = 4;35''$

Now at the greatest inclination on the semi-circle containing the apogee, AG, representing the distance [when the epicycle is] near the beginning of Libra,\textsuperscript{34} is computed, by means of the theorems which we went through before, in treating the anomalies, as 62;10'' in the same units.\textsuperscript{35} Hence, by subtraction [of GM from AG],

$$AM = 57;35'' \text{ where line } MK = 0;22'';$$

hence hypotenuse $AK = \sqrt{AM^2 + MK^2} = 57;35''$ in the same units.

Therefore, where hypotenuse $AK = 120^\circ$, $KM = 0;46''$,

- and $\angle KAM = 0;44^oo \text{ where 2 right angles } = 360^oo$.

But, by hypothesis, the angle of the inclination of the eccentric,

$$\angle BAG = \begin{cases} 2;30^\circ \text{ where 4 right angles } = 360^\circ \\
5^oo \text{ where 2 right angles } = 360^oo 
\end{cases}.$$ 

Therefore, by addition, $\angle BAK = 5;44^oo \text{ where 2 right angles } = 360^oo$.

Therefore, in the circle about right-angled triangle BAK,

- $BK = 5;44^o$
- and arc $AB = 174;16^o$ (supplement).

\textsuperscript{34} Cf. XIII 1 p. 598.

\textsuperscript{35} Accurately, 62;8,21'' when the centre of the epicycle is at a true longitude of $\equiv 0^\circ$ (the apogee being in $m, 20^\circ$, cf. XIII 6 p. 635).

\textsuperscript{36} Reading KAM for KAM (misprint in Heiberg's text) at H554.11. Corrected by Manitius.
So the corresponding chords
\[ \begin{align*}
    BK &= 6;0^p \\
    \text{and } AB &= 119;51^p
\end{align*} \]
where hypotenuse AK = 120^p.

Therefore, where line AK = 57;35^p,
\[ \begin{align*}
    BK &= 2;53^p, \\
    AB &= 57;31^p, \\
    \text{and } BL &= K\Theta = 4;36^p \text{ [p. 613].}
\end{align*} \]

And since \( AB^2 + BL^2 = AL^2 \),
\[ \begin{align*}
    AL &= 57;42^p \text{ in the same units.}
\end{align*} \]

Similarly, since \( L\Theta = BK = 2;53^p \) in the same units,
\[ \begin{align*}
    \text{and } AL^2 + L\Theta^2 &= A\Theta^2, \\
    A\Theta &= 57;46^p.
\end{align*} \]

Therefore, where hypotenuse \( A\Theta = 120^p, \Theta L = 5;59^p, \)
and the angle of the deviation in latitude,
\[ \angle \Theta AL = \left\{ \begin{align*}
    5;44^0 & \text{ where 2 right angles = 360^0} \\
    2;52^0 & \text{ where 4 right angles = 360^0}
\end{align*} \]

That [2;52°] is what we shall enter in the third column of the table for Saturn opposite '135°'.

But at the greatest inclination on the semi-circle containing the perigee, since
AG, representing the distance [when the epicycle is] near the beginning of
Aries, is computed as 57;40^p,\(^{17}\)
where, as we demonstrated [p. 613], \( KM = 0;22^p \) and \( GM = 4;35^p, \)

\[ \text{hence, by subtraction, } AM = 53;5^p. \]

And hypotenuse AK = 53;5^p in the same units, since it is negligibly
greater than line AM.

Therefore, where hypotenuse \( AK = 120^p, \)
\[ \begin{align*}
    KM &= 0;50^p, \\
    \text{and } \angle KAM &= 0;48^0 \text{ where 2 right angles = 360^0}.
\end{align*} \]

But, by hypothesis, \( \angle BAG = 5^o \) in the same units.

So, by addition, \( \angle BAK = 5;48^0 \) where 2 right angles = 360^0.

Therefore, in the circle about right-angled triangle BAK,
\[ \begin{align*}
    \text{arc } BK &= 5;48^0, \\
    \text{and arc } AB &= 174;12^0 \text{ (supplement).}
\end{align*} \]

So the corresponding chords
\[ \begin{align*}
    BK &= 6;4^p \\
    \text{and } AB &= 119;51^p
\end{align*} \]
where hypotenuse AK = 120^p.

Therefore, where line AK = 53;5^p,
\[ \begin{align*}
    BK &= 2;41^p, \\
    \text{and } AB &= 53;1^p.
\end{align*} \]

And since \( AB^2 + BL^2 = AL^2, \)
\[ \text{and BL was shown to be 4;36^p in the same units,} \]
\[ AL = 53;13^p \text{ in the same units.} \]

\(^{17}\) Accurately, 57;44,48^p when the centre of the epicycle is at a true longitude of 10°. Precisely
opposite a distance of \( p = 62;10^p \) is the distance \( (63;25 \times 56;35/62;10 =) 57;43^p \). It is obvious that
Ptolemy has rounded to the nearest convenient number, whatever method of computation he used.
Therefore, where hypotenuse $AL = 120^\circ$, $BL = 10;23^\circ$,
and the angle of the equation in longitude,
\[
\angle BAL = \begin{cases} 
9;56^\circ \text{ where 2 right angles} = 360^\circ \\
4;58^\circ \text{ where 4 right angles} = 360^\circ.
\end{cases}
\]
Again, where line $AL = 53;13^\circ$,
\[
\Theta L = KB = 2;41^\circ,
\]
and $AL^2 + \Theta L^2 = A\Theta^2$,
so $A\Theta = 53;17^\circ$.
Therefore where hypotenuse $A\Theta = 120^\circ$, $\Theta L = 6;3^\circ$,
and the angle of the deviation in latitude,
\[
\angle A\Theta L = \begin{cases} 
5;46^\circ \text{ where 2 right angles} = 360^\circ \\
2;53^\circ \text{ where 4 right angles} = 360^\circ.
\end{cases}
\]
That [2;53°] is what we shall enter in the fourth column of the table opposite 135°.

Then in order to compare the equations in longitude for the inclination nearer the perigee, let the diagram with no inclination be drawn again [Fig. 13.7]. Then, where the distance at that point,
\[
AG = 57;40^\circ,
\]
$GK (= K\Theta)$ is given as 4;36°,
and, by subtraction, $AK = 53;4^\circ$ in the same units;
but $AK^2 + K\Theta^2 = A\Theta^2$,
so $A\Theta = 53;16^\circ$.

---

Fig. 13.7
Therefore, where hypotenuse $A\Theta = 120^\circ$, $K\Theta = 10;22^\circ$, and the angle of the equation in longitude,

$$\angle \Theta AK = \begin{cases} 9;54^\circ & \text{where 2 right angles} = 360^\circ \\ 4;57^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}$$

But when the inclinations [of eccentric and epicycle] were taken into account it was shown to be 4;58°. So the equation in longitude computed according to both inclinations was 1′ greater.

Q.E.D.

Let there again be drawn [Fig. 13.8], first, the diagram for the inclinations, representing the ratios established for Jupiter.

Hence, where the radius of the epicycle, $G\Theta = 11;30^\circ$,

$GK (= K\Theta)$ is computed as $[84:52 \times 11:30/120 =] 8;8^\circ$.

Then, since the angle of the inclination of the epicycle,

$$\angle AGE = \begin{cases} 2;30^\circ & \text{where 4 right angles} = 360^\circ \\ 5^\circ & \text{where 2 right angles} = 360^\circ, \end{cases}$$

in the circle about right-angled triangle $GKM$,

arc $KM = 5^\circ$ 

and arc $GM = 175^\circ$ (supplement).

So the corresponding chords

$$KM = \begin{cases} 5;14^p \end{cases}$$

and $GM = 119;53^p$, where hypotenuse $GK = 120^\circ$. 

Fig. 13.8
Therefore, where line $GK = 8^\circ 8'$, and $AG$, the distance near the beginning of Libra, is $62^\circ 30'$, 38

$KM = 0^\circ 21'$,

$GM = 8^\circ 8'$,

and, by subtraction, $MA = 54^\circ 22'$. Hence hypotenuse $AK$, being negligibly greater than $MA$, is $54^\circ 22'$ in the same units.

Therefore, where hypotenuse $AK = 120^\circ$, $KM = 0^\circ 46'$, and $\angle KAM = 0^\circ 44''$ where 2 right angles $= 360^\circ$.

But, by hypothesis, the angle of the inclination of the eccentre,

$$\angle BAG = \begin{cases} 1^\circ 30' & \text{where 4 right angles} = 360^\circ, \\ 3^\circ 0' & \text{where 2 right angles} = 360^\circ. \end{cases}$$

Therefore, by addition, $\angle BAK = 3^\circ 44''$ where 2 right angles $= 360^\circ$.

Therefore, in the circle about right-angled triangle $BAK$,

$arc KB = 3^\circ 44'$

and $arc AB = 176^\circ 16'$ (supplement).

So the corresponding chords

$KB = \begin{cases} 3;54' \\ 3^\circ 54' \end{cases}$ where hypotenuse $AK = 120^\circ$.

Therefore, where line $AK = 54;22'$,

$KB = 1;46'$

and $AB = 54;20'$.

And, from what was demonstrated previously, $BL = 8;8'$ in the same units. Hence, where hypotenuse $AL = 120^\circ$, $L\Theta = 3;52'$, and the angle of the deviation in latitude,

$$\angle \Theta AL = \begin{cases} 3;42'' & \text{where 2 right angles} = 360^\circ, \\ 1;51'' & \text{where 4 right angles} = 360^\circ. \end{cases}$$

That $[1;51']$ is what we shall enter in the third column of the table for Jupiter opposite $135^\circ$.

In the same way, $AG$, when it represents the distance at the beginning of Aries, is computed as $57;30'$, 39 where, as we demonstrated, $KM = 0;21'$ and $GM = 8;8'$; hence, by subtraction, $AM(= AK$ which is negligibly greater) is $49;22'$ in the same units.

Therefore, where hypotenuse $AK = 120^\circ$, $KM = 0;51'$, and $\angle KAM = 0^\circ 49''$ where 2 right angles $= 360^\circ$.

---

38 Accurately, $62;34;36'$ when the centre of the epicycle is at a true longitude of $\approx 0^\circ$ (the apogee being in $\pi 10^\circ$, cf. XIII 6 p. 635).

39 Accurately $57;24;31'$. The values of Ptolemy for both distances (cf. n.38) would fit better an elongation from the apogee of $-24^\circ$ and $(180^\circ - 24^\circ)$, rather than the $-20^\circ$ which he specifies in XIII 6. But if one were to take the precise position of the apogee in his time, $\pi 11^\circ$, this would give $-19^\circ$ with even worse agreement with the text.
Therefore, by addition, \( \angle BAK \) \( [= \angle KAM + 3^\circ] \) = 3;49° in the same units.

Therefore, in the circle about right-angled triangle \( AKB \),

\[
\text{arc } KB = 3;49^\circ
\]

and arc \( AB = 176;11^\circ \) (supplement).

H561 So the corresponding chords

- \( BK = 3;59^\circ \)
- \( AB = 119;56^\circ \)

where hypotenuse \( AK = 120^\circ \).

Therefore, where line \( AK = 49;22^\circ \),

- \( KB = 1;39^\circ \)
- \( AB = 49;20^\circ \).

Hence, since \( BL = 8;8^\circ \) in the same units,

\[
\text{and } AB^2 + BL^2 = AL^2,
\]

\( AL = 50;0^\circ \) in the same units.

Therefore, where hypotenuse \( AL = 120^\circ \), \( BL = 19;31^\circ \), and the angle of the equation in longitude,

\[
\angle BAL = \begin{cases} 18;44^\circ \text{ where 2 right angles} = 360^\circ \\ 9;22^\circ \text{ where 4 right angles} = 360^\circ \end{cases}
\]

Again, where line \( AL = 50;0^\circ \),

- \( \Theta L \) \( [= KB] = 1;39^\circ \)
- \( \text{and } AL^2 + \Theta L^2 = A\Theta^2 \)

so \( A\Theta = 50;2^\circ \).

Therefore, where hypotenuse \( A\Theta = 120^\circ \), \( L\Theta = 3;57^\circ \), and the angle of the deviation in latitude,

\[
\angle \Theta AL = \begin{cases} 3;46^\circ \text{ where 2 right angles} = 360^\circ \\ 1;53^\circ \text{ where 4 right angles} = 360^\circ \end{cases}
\]

That \( [1;53^\circ] \) is what we shall enter in the fourth column of the table opposite the same \( `135^\circ` \).

In order to compare the equations in longitude, let the diagram with no inclinations be drawn again [Fig. 13.9]. Then at the distance in question, where \( \Theta K = GK = 8;8^\circ \),

H562 the whole line \( AG = 57;30^\circ \),

and, by subtraction, \( AK = 49;22^\circ \) in the same units.

But \( AK^2 + K\Theta^2 = A\Theta^2 \),

so \( A\Theta = 50;2^\circ \) in the same units.

Therefore, where hypotenuse \( A\Theta = 120^\circ \), \( \Theta K = 19;30^\circ \), and the angle of the equation in longitude,

\[
\angle \Theta AK = \begin{cases} 18;42^\circ \text{ where 2 right angles} = 360^\circ \\ 9;21^\circ \text{ where 4 right angles} = 360^\circ \end{cases}
\]

And when the inclinations were taken into account it was shown to be \( 9;22^\circ \). So the equation in longitude computed according to both inclinations was, again, greater by only a single minute.)

Q.E.D.

Next, to determine the quantities for Mars, let there be drawn, first, the diagram for the inclinations [Fig. 13.10], and let \( GK (= K\Theta) \) be computed as \( [84;52 \times 39;30/120 = ] 27;56^\circ \), where the radius of the epicycle, \( G\Theta = 39;30^\circ \).
Then, since the angle of the inclination of the epicycle,
\[ \angle AGE = \begin{cases} 2;15^\circ & \text{where 4 right angles } = 360^\circ \\ 4;30^\circ & \text{where 2 right angles } = 360^\circ \end{cases} \]
in the circle about right-angled triangle GMK,
arc KM = 4;30°
and arc GM = 175;30° (supplement).
So the corresponding chords
\[
\begin{align*}
KM &= 4;43'' \\
GM &= 119;54''
\end{align*}
\]
Therefore, where line GK = 27;56'',
and AG, the greatest distance, is 66°,\(^{10}\)
\[
\begin{align*}
KM &= 1;6'' \\
GM &= 27;54''
\end{align*}
\]
and, by subtraction, AM = 38;6°.
Hence hypotenuse AK \(= \sqrt{AM^2 + KM^2} \) = 38;7° in the same units.
Therefore, where hypotenuse AK = 120°,
KM = 3;28°,
and \(\angle KAM = 3;19^\circ \) where 2 right angles = 360°.
But, by hypothesis, the angle of the eccentric's inclination, \(\Omega \) 0°.

\(^{10}\) I.e. the northpoint is taken as coinciding with the apogee, both being placed in the (rounded) \(\Omega \) 0°.
\[ \angle BAG = \begin{cases} 1^\circ \text{ where 4 right angles } = 360^\circ \\ 200^\circ \text{ where 2 right angles } = 360^\circ . \end{cases} \]

Therefore, by addition, \( \angle BAK = 5;19^\circ \) where 2 right angles = 360°.

So, in the circle about right-angled triangle BAK,

- arc \( KB = 5;19^\circ \)
- and arc \( AB = 174;41^\circ \) (complement).

So the corresponding chords

- \( BK = 5;34^p \) where hypotenuse \( AK = 120^\circ \).
- and \( AB = 119;52^p \).

Therefore, where line \( AK = 38;7^p \),

- \( KB = 1;46^p \)
- and \( AB = 38;5^p \).

But line \( BL \) \( [= K\Theta = GK] = 27;56^p \) in the same units.

And, since \( AB^2 + BL^2 = AL^2 \),

- \( AL = 47;14^p \).

Similarly, since \( \Theta L = 1;46^p \) in the same units,

- and \( AL^2 + \Theta L^2 = A\Theta^2 \),

- \( A\Theta = 47;16^p \) in the same units.

Therefore, where hypotenuse \( A\Theta = 120^p \), \( \Theta L = 4;29^p \),

and the angle of the deviation in latitude,

- \( \angle \Theta AL = \begin{cases} 4;18^\circ \text{ where 2 right angles } = 360^\circ \\ 2;9^\circ \text{ where 4 right angles } = 360^\circ . \end{cases} \)
That \( [2;9^\circ] \) is what we shall enter in the third column of the table for Mars opposite \('135^\circ'\).

In the same way, for the inclinations at least distance:

\[
AG = 54^\circ \text{ where, as was shown,}
\]

\[
KM = 1;6^\circ
\]

and \( GM = 27;54^\circ \).

Thus, by subtraction, \( AM = 26;6^\circ \),

and hypotenuse \( AK \) \( = \sqrt{KM^2 + AM^2} \) \( = 26;7^p \) in the same units.

Therefore, where hypotenuse \( AK = 120^p \), \( KM = 5;3^p \),

and \( \angle KAM = 4;49^\circ\) where 2 right angles = \(360^\circ\).

Hence, by addition, \( \angle BAK = 6;49^\circ\) in the same units.

Therefore, in the circle about right-angled triangle \( ABK \),

\[
arc BK = 6;49^\circ
\]

and arc \( AB = 173;11^\circ \) (supplement).

So the corresponding chords

\[
BK = \begin{cases} 7;8^p & \text{ where hypotenuse } AK = 120^p. \\ 119;47^p & \end{cases}
\]

Therefore, where line \( AK = 26;7^p \),

\[
BK = 1;33^p
\]

and \( AB = 26;4^p \).

And line \( BL \) is, again, \( 27;56^p \) in the same units.

And, since \( AB^2 + BL^2 = AL^2 \),

\[
AL = 38;12^p.
\]

Therefore, where hypotenuse \( AL = 120^p \), \( BL = 87;45^p \),

and the angle of the equation in longitude,

\[
\angle BAL = \begin{cases} 94^\circ \text{ where 2 right angles = } 360^\circ \\
47^\circ \text{ where 4 right angles = } 360^\circ \end{cases}
\]

Similarly, where line \( AL = 38;12^p \), \( \Theta \) \( = BK \) \( = 1;33^p \),

and \( AL^2 + \Theta^2 = A\Theta^2 \),

so \( A\Theta = 38;14^p \).

Therefore, where hypotenuse \( A\Theta = 120^p \), \( \Theta = 4;52^p \),

and the angle of the deviation in latitude,

\[
\angle A\Theta L = \begin{cases} 4;40^\circ \text{ where 2 right angles = } 360^\circ \\
2;20^\circ \text{ where 4 right angles = } 360^\circ \end{cases}
\]

That \( [2;20^\circ] \) is what we shall enter in the fourth column of the table opposite the same \('135^\circ'\).

Again, if, in order to compare the equations in longitude, we set out the diagram without the inclinations [Fig. 13.11], at the least distance (where the difference must necessarily become most noticeable),

\[
AG:GK = (A\Theta) = 54 : 27;56.
\]

hence, by subtraction, \( AK = 26;4^p \),

and hypotenuse \( A\Theta \) \( = \sqrt{AK^2 + K\Theta^2} \) \( = 38;12^p \) in the same units.

Hence, where hypotenuse \( A\Theta = 120^p \),

\[
\Theta K = 87;45^p \text{ again [as } BL \text{ in the previous computation],}
\]

and the angle of the equation in longitude,

\[
\angle A\Theta K = \begin{cases} 94^\circ \text{ where 2 right angles = } 360^\circ \\
47^\circ \text{ where 4 right angles = } 360^\circ \end{cases}
\]
But that is the same size as was demonstrated by means of the calculations including the inclinations. Therefore the equation in longitude for Mars computed according to the inclinations of the circles [of epicycle and eccentre] did not differ at all.

\[ \text{Q.E.D.} \]

The fourth column in the two tables for Venus and Mercury will contain the positions in latitude produced by the greatest slants of their epicycles, which occur at the apogee and perigee of the eccentre. However, we have computed these separately, without the effect due to the inclination of the eccentre, since that would have required a greater number of tables and a more complicated method of calculation [from the tables]: for the [corresponding latitudinal] positions as morning-star and evening-star are not going to be equal to each other, and not even always on the same side [i.e. north or south] of the ecliptic; and in any case, since the inclination of the eccentre is not constant, the differences in the amount to be diminished with respect to the greatest inclination [of the epicycle] would not correspond to the differences in the amount to be diminished with respect to the greatest slant. However, if we separate the effects, we can determine each element in a more convenient way, as will become clear from the actual procedure which we shall adduce.

\[ ^{11} \text{Ptolemy means that one could not use a single coefficient column (} c_5 \text{ in } HAMA \text{) to compute the diminution with respect to maximum of both inclination and slant as a function of the planet's position on the epicycle.} \]
Let AB [see Fig. 13.12] be the intersection of the planes of the ecliptic and the epicycle. Let point A be taken as the centre of the ecliptic, and B as the centre of the epicycle, and let the epicycle GDEZH be described about it slanted to the plane of the ecliptic, i.e. so that straight lines drawn in the [two planes] perpendicular to the common section GH all form equal angles at the points on GH. Draw AE tangent to the epicycle, and AZD intersecting the epicycle at an arbitrary point, and drop from points D, E and Z perpendiculars Dθ, EK and ZL to GH, and perpendiculars DM, EN and ZX to the plane of the ecliptic. Join ΘM, KN, LX, and also AN and AXM (for AXM will be a straight line, since the three points [A, X, M all] lie in two planes, the plane of the ecliptic and the plane through AZD perpendicular to the ecliptic.

\[\text{Fig. 13.12}\]

It is obvious that, with the slant as depicted, the equations in longitude of the planet [at D and E respectively] will be represented by angles ΘAM and KAN, and the [positions] in latitude by angles DAM and EAN. We must demonstrate, first, that the position in latitude at the tangent point, \(\angle EAN\), is the maximum, just as the equation in longitude [is maximum at that point].

4\(^\text{See Fig. U for a redrawing of this three-dimensional figure. Note that Ptolemy's figure is an artificial one, since when the intersection of the planes of ecliptic and epicycle passes through the centre of the epicycle, the 'slant' is zero. But it is justified by the 'separation of the effects'.}
[Proof:] Since $\angle E\Delta K$ is the maximum,

$$KE:EA > \Theta D:DA = LZ:ZA.$$  

But $EK:EN = \Theta D:DM = LZ:ZX.$

for, as we said, the triangles formed by them [$EKN$, $D\Theta M$ and $ZLX$] have equal angles [at $GH$] and right angles at $M$, $N$ and $X$.

$$\therefore NE:EA > MD:DA = XZ:ZA.$$  

H570 And, again, the angles $DMA$, $ENA$ and $ZXA$ are right.

Therefore $\angle EAN > \angle DAM$, and hence, obviously,  

$\angle EAN$ is greater than any angle so formed.

It is immediately obvious that, when one considers the effect on the equations in longitude caused by the slant, the maximum difference is produced at the greatest deviations in latitude at $E$. For the differences [in the equation caused by the slant] are represented by the angles subtended by $(\Theta D - \Theta M)$, $(KE - KN)$ and $(LZ - LX)$ [when the planet is at $D$, $E$ and $Z$ respectively], and since the ratios of these lines [$\Theta D:\Theta M$ etc.] to each other and to the difference between them [$(\Theta D - \Theta M)$ etc.] remains the same, it follows that

$$(EK - KN) : EA > (\Theta D - \Theta M) : AD,$$ etc.  

And it is also immediately clear that, whatever the ratio between the greatest equation in longitude and the greatest deviation in latitude [due to the slant], that ratio holds between the equation in longitude for any position [of the planet] on the epicycle and the [corresponding] position in latitude.

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43 Ptolemy's argument here is fallacious, as pointed out by Pedersen 382. He seems to have been misled by his figure, which substitutes straight lines for arcs.
For $KE:EN = LZ:ZX = OD:DM$, and so on for the other points [on the epicycle].

Q.E.D.

Having established these preliminary points, let us first examine the size of the angle which is contained by the slant of the planes for each of the two planets. We take for granted what was noted at the beginning [of the discussion, p. 601], that both planets, when halfway between greatest and least distances, display a maximum difference [in latitude] between opposite positions on the epicycle of 5° to north or south: for Venus appears to [so] vary by slightly more than 5° at perigee and slightly less than 5° at apogee, while Mercury varies by about \( \frac{1}{2} \)° [more and less than 5° at 180° from apogee and apogee respectively].

So let [Fig. 13.13] ABG again be the intersection of ecliptic and epicycle. Describe the epicycle GDE about centre B, slanting to the plane of the ecliptic in the way described. From A, the centre of the ecliptic, draw AD tangent to the epicycle, and from D drop perpendicular DZ on to GBE, and perpendicular DH on to the plane of the ecliptic. Join BD, ZH and AH, and let \( \angle DAH \) be taken as comprising half the above deviation in latitude for each of the two

Fig. 13.13

\[44\] This too is fallacious, since Ptolemy has substituted chords for arcs (in modern terminology, has treated a relationship between the sines of angles as a relationship between the angles). See Pedersen 380-1. However, if one treats it as an approximation, it is a very reasonable one: see my remark on Pedersen, Toomer [3] 145.

\[45\] Cf. p. 623 n.42.
planets (thus it is 21°). Let our problem be, to find for each the amount of the slant between the planes, namely the size of $\angle DZH$.

For Venus, since, where the radius of the epicycle is 43;10°, the greatest distance is 61;15°, the least 58;45°, and the mean between them 60°,

$$AB:BD = 60 : 43;10.$$ 

And since $AB^2 - BD^2 = AD^2$, 

$$AD = 41;40°$$ in the same units.

Similarly, since $BA:AD = BD:DZ$, 

$$DZ = 29;58°$$ in the same units.

Furthermore, since, by hypothesis,

$$\angle DAH = \begin{cases} 2;30° & \text{where 4 right angles} = 360° \\ 5°° & \text{where 2 right angles} = 360°° \end{cases}$$

in the circle about right-angled triangle ADH, 

$$\text{arc } DH = 5°$$

and the corresponding chord $DH = 5;14°$ where hypotenuse $AD = 120°$.

Therefore, where line $AD = 41;40°$, $DH = 1;50°$. 

And $DZ$ was shown to be $29;58°$ in the same units. 

Therefore, where hypotenuse $DZ = 120°$, $DH = 7;20°$. 

and the angle of the slant,

$$\angle DZH = \begin{cases} 7°° & \text{where 2 right angles} = 360°° \\ 3;30° & \text{where 4 right angles} = 360°° \end{cases}$$ 

But since the amount by which $\angle DAZ$ exceeds $\angle HAZ$ represents the resulting difference in the equation in longitude, we must immediately compute this too, by finding the amounts of these angles. For we showed that, where line $DH = 1;50°$, hypotenuse $AD = 41;40°$ and $DZ = 29;58°$; 

and $AD^2 - DH^2 = AH^2$ 

while $ZD^2 - DH^2 = HZ^2$; 

so $AH = 41;37°$ 

and $HZ = 29;55°$ in the same units. 

Therefore, where hypotenuse $AH = 120°$, $ZH = 86;16°$. 

and $\angle ZAH = \begin{cases} 91;56°° & \text{where 2 right angles} = 360°° \\ 45;58° & \text{where 4 right angles} = 360°° \end{cases}$ 

Similarly, since $DZ = 86;18°$ where hypotenuse $AD = 120°$. 

$$\angle DAZ = \begin{cases} 91;58°° & \text{where 2 right angles} = 360°° \\ 45;59° & \text{where 4 right angles} = 360°° \end{cases}$$

Thus the equation in longitude computed according to the slant was less by one minute.

For Mercury [see Fig. 13.14], where the radius of the epicycle is 22;30°, the greatest distance, as we demonstrated, is 69°, and the distance diametrically opposite to that 57°; the mean between these two is calculated as 63° in the same units.

$$\text{So } AB:BD = 63 : 22;30.$$ 

And since $AB^2 - DB^2 = AD^2$, 

$$AD = 58;51°.$$ 

46 This neat result is achieved only by some devious rounding; computing accurately one finds 3;281°.
Similarly, since \( \frac{AB}{AD} = \frac{BD}{DZ} \).

\[
DZ = 21;1^p \text{ in the same units.}
\]

Again, since, by hypothesis,

\[ \angle DAH = 5^\circ \text{ where 2 right angles = } 360^\circ, \]

in the circle about right-angled triangle \( \triangle ADH \),

\[ \text{arc } DH = 5^\circ, \]

and the corresponding chord \( DH = 5;14^p \) where hypotenuse \( AD = 120^p \).

Therefore, where line \( AD = 58;51^p \), \( DH = 2;34^p \).

But we showed that \( DZ = 21;1^p \) in the same units.

Therefore, where hypotenuse \( DZ = 120^p \), \( DH = 14;40^p \), and the angle of the slant,

\[
\angle DZH = \begin{cases} 14^\circ & \text{where 2 right angles = } 360^\circ \\ 7^\circ & \text{where 4 right angles = } 360^\circ \end{cases}
\]

In the same way [as for Venus], in order to compare the angles of the equation [in longitude]:

again, where \( DH = 2;34^p \), we showed that

hypotenuse \( AD = 58;51^p \) and \( DZ = 21;1^p \).

And \( DA^2 - DH^2 = AH^2 \),

\( DZ^2 - DH^2 = HZ^2 \),

so \( AH = 58;47^p \)

and \( ZH = 20;53^p \) in the same units.

\(^{47}\text{Accurately, } 7;1^\circ.\)
Therefore, where hypotenuse \( AH = 120^\circ \), \( HZ = 42;38^\circ \),
and \( \angle ZAH = \begin{cases} 41;38^\circ & \text{where 2 right angles } = 360^\circ \\
20;49^\circ & \text{where 4 right angles } = 360^\circ. \end{cases} \)
In the same way, where hypotenuse \( AD = 120^\circ \), \( DZ \) is calculated as \( 42;50^\circ \),
and \( \angle DAZ = \begin{cases} 41;50^\circ & \text{where 2 right angles } = 360^\circ \\
20;55^\circ & \text{where 4 right angles } = 360^\circ. \end{cases} \)
So in this case the equation in longitude due to the slant was less by \( 6' \).

Q.E.D.

Next let us examine whether, if we take the above amounts of the slant as
given, we find the greatest latitudes at the greatest and least distances [derived
from them] to agree with those derived from our observations. In the same
figure [Fig. 13.15], let us now take as basis the greatest distance of Venus, i.e.

Fig. 13.15

\[
\text{AB:BD} = 61;15 : 43;10.
\]

Hence, since \( \text{AB}^2 - \text{BD}^2 = \text{AD}^2 \),
\[ \text{AD} = 43;27^\circ. \]

But \( \text{AB:AD} = \text{BD:DZ} \).
\[ \text{So DZ} = 30;37^\circ \text{ in the same units.} \]

Again, since, by hypothesis, the angle of the slant,
\[ \angle \text{DZH} = 7^\circ \text{ where 2 right angles } = 360^\circ \]

\[ \text{Ptolemy has fudged the calculations a little to get this result. Accurate computation gives} \]
\[ \angle \text{ZAH} = 41;33.58^\circ, \angle \text{DAZ} = 41;50.50^\circ, \text{ with a difference of 0;16.52^\circ, or about 8'.} \]
and [hence] \( DH = 7;20^p \) where hypotenuse \( DZ = 120^p \),
therefore, where line \( DZ = 30;37^p \), and \( AD = 43;27^p \),
\( DH = 1;52^p \).
So where hypotenuse \( AD = 120^p \),
\( DH = 5;9^p \).
and the greatest deviation in latitude,
\[
\angle DAH = \begin{cases} 
4;54^\circ \text{ where 2 right angles } = 360^\circ \\
2;27^\circ \text{ where 4 right angles } = 360^\circ .
\end{cases}
\]
But at the least distance, where the radius of the epicycle,
\( BD = 43;10^p \),
\( AB \) is given as \( 58;45^p \).
And \( AB^2 - DB^2 = AD^2 \).
so \( AD = 39;51^p \) in the same units.
Similarly, since \( AB:AD = BD:DZ \),
\( DZ = 29;17^p \) in the same units.
But \( DZ:DH \) is given as \( 120 : 7;20 \).
Therefore, where \( DZ = 29;17^p \) and \( AD = 39;51^p \),
\( DH = 1;47^p \).
Therefore, where hypotenuse \( AD = 120^p \), \( DH = 5;22^p \),
and the greatest deviation in latitude,
\[
\angle DAH = \begin{cases} 
5;8^\circ \text{ where 2 right angles } = 360^\circ \\
2;34^\circ \text{ where 4 right angles } = 360^\circ .
\end{cases}
\]
Thus [the greatest latitude] differs from the \( 2^\circ \) of [greatest] deviation in latitude assumed for the mean, being less at the apogee and greater at the perigee, but [in both cases] by an amount which is negligible to the senses; for at the greatest distance it was only three minutes less, and at the least distance four minutes more. Such [small differences] could not be at all easily detected from the observations.

Next [see Fig. 13.16] let us take the greatest distance of Mercury as basis, namely
\( AB:BD = 69 : 22;30 \).
Hence, by the same procedure as above,
\[
AD = \sqrt{AB^2 - BD^2} = 65;14^p ,
\]
and \( DZ = \frac{AD \times BD}{AB} = 21;16^p \) in the same units.
But in this case the angle of slant,
\( \angle DZH \) is given as \( 14^\circ \) where 2 right angles = \( 360^\circ \).
Hence we have \( DH = 14;40^p \) where hypotenuse \( DZ = 120^p \).
Therefore, where line \( DZ = 21;16^p \), and \( AD = 65;14^p \),
\( DH = 2;36^p \).
Therefore, where hypotenuse \( AD = 120^p \), \( DH = 4;47^p \),
and the greatest deviation in latitude,
\[
\angle DAH = \begin{cases} 
4;34^\circ \text{ where 2 right angles } = 360^\circ \\
2;17^\circ \text{ where 4 right angles } = 360^\circ .
\end{cases}
\]

The chord of \( 14^\circ \) (111) is \( 14;37,27^p \). But Ptolemy's \( 14;40^p \) is justified by p. 627, where the \( 7^\circ \) of the slant is derived from that value.
But at the least distance,\(^{50}\) 
\(AB:BD\) is given as 57 : 22:30.
and so, by the same procedure again,
\[ AD = 52:22^p \text{ in the same units} \]
and \(DZ = 20:40^p\).
And the slant is the same as before,
and hence \(ZD:DH\) is given as 120 : 14:40,
so where \(DZ = 20:40^p\) and \(AD = 52:22^p\),
\[ DH = 2:32^p. \]
Therefore, where hypotenuse \(AD = 120^p, DH = 5:48^p\),
and \(\angle DAH = \begin{cases} 5:32^o & \text{where 2 right angles = 360^o} \\ 2:46^o & \text{where 4 right angles = 360^o} \end{cases}\)
Thus the difference from the maximum deviation in latitude at the mean (which was taken as 2\(^{1}\)\(^o\) here too) was 13' in the negative direction at apogee and 16' in the positive direction at perigee. To represent these, we shall use a correction of \(\frac{1}{2}\) with respect to the mean in the calculations [from the table], in accordance with the perceptible difference derived from the observations.

Now that we have demonstrated the above, and also demonstrated that the ratio between the greatest equation in longitude and the greatest deviation in

\(^{50}\) Ptolemy is speaking loosely here. 57\(^p\) represents, not the least distance, (which is c. 55:34\(^p\) at 120\(^o\) from apogee, IX 9 p. 460), but the distance at the point opposite the greatest distance, i.e. strictly analogous to the situation for Venus. Cf. the use of ‘perigee’ below.
latitude also holds good at other points on the epicycle for the ratio between the individual equations in longitude and the [corresponding] individual positions in latitude, we immediately have a convenient method for computing the positions in latitude due to the slant to be entered in the fourth column of the tables for Venus and Mercury. However, as we mentioned, these positions are based only on the slant of the epicycles at mean distance: the difference due to the inclination of the eccentres, and also the difference due to [the approach towards] apogee or perigee for Mercury, will be found by means of a correction procedure in the computation [from the tables], for convenience of calculation.

For, at the mean distances as set out above, the greatest deviation due to the slant was shown to be 2;30° on either side of the ecliptic for both planets; and the greatest equation in longitude is approximately 46° for Venus and 22° for Mercury; and we already have, set out in the tables for anomaly of these planets, the equations corresponding to the individual positions on the epicycle. So we form the ratios between the latter and the greatest equation, take the same proportion of 2°, separately for each planet, and enter the results in the fourth column of the tables of latitude opposite the corresponding arguments.

We have produced the fifth column [in each table] in order to correct the positions in latitude for other positions [of the epicycle] on the eccentre, by using the sixtieths entered [in that column]. For since, as we said, the increase and decrease in the inclination and slant of the epicycle, through the action of the attached small circles, have a period precisely corresponding to the period of return on the eccentre, and since the amounts of all the inclinations and slants is not very different from that associated with the moon's inclined orbit, and the individual deviations in latitude, for such small inclinations, are, again, almost proportional, and since we already have the corresponding entries for the moon computed geometrically, we multiplied each of the entries in that table by 12 (because the maximum there is about 5°, and here we are making the maximum 60), and entered the results opposite the appropriate argument in the fifth column of each table.

The layout of the tables is as follows.

5. \textit{Layout of the tables for the computations in latitude}^33

\textit{[See pp. 632-4.]}
### Inclinations of Saturn

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<th>Argument (in Distance from Apogee)</th>
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### Inclinations of Jupiter

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### INCLINATIONS OF MARS

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Those [tables] thus established, we carry out the latitude computation for the 5 planets as follows.

For the 3 planets Saturn, Jupiter and Mars, we take the corrected longitude (for Mars just as it is, for Jupiter subtracting 20° and for Saturn adding 50°), and entering the argument [columns] of the appropriate table, find the sixtieths corresponding to it in the fifth column of the latitude, and write that down separately. Similarly, we enter the same argument [columns] with the corrected amount of the anomaly, and take the difference in latitude corresponding to it, in the third column if the corrected longitude falls within the first 15 lines, but in the fourth column if it falls within the lines after [the 15th]. We multiply this by the sixtieths we wrote down, and the result will give us the amount by which the planet is north of the ecliptic, if we took the difference in latitude from the third column, or south of it, if we took it from the fourth.

For Venus and Mercury we first enter with the corrected amount of the anomaly into the argument [columns] of the appropriate table, take the corresponding amounts in the third and fourth columns of the latitude, and write them down separately; we take them unchanged from all columns except the fourth column for Mercury, but for that, if the corrected longitude falls within the first 15 lines, we subtract a tenth part of the amount, whereas if the corrected longitude falls within the lines below [the 15th], we add a tenth part. Then we add to the corrected longitude, for Venus always 90°, and for Mercury always 270°, subtract [the 360° of] a circle if it comes to that [i.e. to more than 360°], enter with the result into the same argument [columns], and take the corresponding number of sixtieths in the fifth column. We multiply the latter into the amount we wrote down from the third column, and set out the result. The direction of this will be:

[A] if the longitude (with the addition as detailed above) falls within the first 15 lines, and

[1] the amount of the corrected anomaly falls within the first 15 lines: southerly
[2] the anomaly falls within the lines following [the 15th]: northerly;

[B] if the above-mentioned longitude falls within the lines below the 15, and

[1] the amount of the above-mentioned anomaly falls within the first 15 lines: northerly
[2] the anomaly falls within the lines following [the 15th]: southerly.

Next we again take the corrected longitude, just as it is for Venus, but with the addition of 180° for Mercury, enter with it into the same [columns of]

54 See HAMA 219-20, 222-6, and Appendix A, Example 15.
55 The 'corrected longitude' means 'the distance of the epicycle centre from apogee, as seen from the observer (i.e. corrected by the equation of centre)'. The amounts to be applied to it represent the (rounded) distance between apogee and northpoint of the inclined orbit.
56 I.e. the true anomaly $\alpha$, corrected for equation of centre.
57 The 'tenth part' represents the ratio $1^\circ : 21^\circ$. Cf. XIII 4 p. 630.
argument; take the sixtieths corresponding to this in the fifth column, multiply
them into the amount we wrote down from the fourth column, and set out the
result. The direction of this will be:
[A] if the longitude we entered with (as described above) falls within the first 15
lines, and
  [1] the corrected anomaly is $180^\circ$ or less: northerly
  [2] the anomaly is greater than $180^\circ$: southerly;
[B] if the longitude falls within the lines below the 15, and
  [1] the anomaly is $180^\circ$ or less: southerly
  [2] the anomaly is greater than $180^\circ$: northerly.

Then we take these same sixtieths which were found by the second entry with
the longitude, calculate the amount which is the same fraction of them as they
are of 60, and, for Venus, take $\frac{1}{4}$ of this and set it out too, always with a
northerly direction; but for Mercury we take $\frac{1}{3}$ of the amount and set it out, always in a southerly direction.\(^{58}\)

Thus, by combining the 3 quantities set out, we determine the apparent
position in latitude with respect to the ecliptic of these [two planets].

Now that we have dealt with the basic problem of the deviations in latitude of
the 5 planets, there remains the supplementary topic of the requisite theorems
for their first and last visibilities with respect to the sun. For, as we explained in
the treatise on the fixed stars [VIII 6, p. 413], it turns out that their distances
from the sun along the ecliptic are variously unequal, for both first and last
visibilities, for a number of reasons: the first of these is due to the fact that they
are of unequal size, the second due to the variation of the inclination of the
ecliptic to the horizon, and the third due to their positions in latitude.

For if we again imagine [see Fig. 13.17] segments of great circles, \(AB\) of the
horizon, and \(GD\) of the ecliptic,\(^{60}\) and take point \(E\) as their intersection at rising
or setting, points \(G\) and \(A\) in the direction of south [i.e. the meridian],\(^{61}\) and
point \(D\) as the sun’s centre, and we draw through \(D\) and the pole of the horizon
another great circle segment \(DBZ\), and suppose the planet to rise or set along the
horizon \(AEB\) (when it is situated on the ecliptic, it will do so, obviously, at \(E\);
when it is north of the ecliptic, at \(H\), and when it is south, at \(\Theta\)), and drop
perpendiculars \(HK\) and \(OL\) on to the ecliptic from points \(H\) and \(\Theta\), then we will
again\(^{62}\) have, in \(BD\), an arc which is equal to the amount which the sun must always be below the earth in order for the same [given] planet to be first or last
visible. For it is on a great circle so drawn [i.e. perpendicular to the horizon]

\(^{58}\) For an explanation of this procedure see H.A.M.I 224.
\(^{59}\) See H.A.M.I 234-8, Pedersen 386-8, with the correction Toomer [3], 145.
\(^{60}\) Reading κύκλου (with D.Ar) for μεγίστου κύκλου ("the great circle of the ecliptic") at
H490.18. Corrected by Manitius.
\(^{61}\) Ger adds ‘and points \(\Theta\) and \(H\) in the direction of south and north’, which makes good sense.
\(^{62}\) ‘again’ refers back to the similar situation with the fixed stars, VIII 6 p. 413.
that equal intervals below the earth must be taken in order for the identical obscuring effect of the sun's rays to take place.

First, then, this arc [BD] is, naturally, unequal for the various planets, which are unequal [in size], so, even if all other factors remain the same, the arc of the ecliptic subtending the right angle, i.e. the interval corresponding to ED, must vary, being, obviously, smaller for the larger planets, and greater for the smaller planets.

Similarly, even if BD remains the same for the same [given] planet, but the angle of inclination of the ecliptic, BED, varies either because there is a different zodiacal sign [crossing the horizon] or [the latitude of] the location is different, the arc of the [sun's] distance, ED, will again vary, and will become greater as the angle in question decreases and lesser as it increases.

In the same way, even if we join to the above condition [of BD being constant] the further condition that the inclination remains the same, but the planet does not lie on the ecliptic, but is either north of it at H or south of it at Θ, its first and last visibility will no longer take place at a distance [from the sun] of arc DE, but when it is north of the ecliptic, at the lesser distance DK, and when it is south, at the greater distance DEL.

Therefore, for our investigations of the particular cases, it is essential that there first be given, for each of the 5 planets, the universally applicable size of the arc corresponding to BD, from the more reliable observations of the phases. These would be those made in summer, round about Cancer, since at that season the atmosphere is thin and clear, and the inclination of the ecliptic to the horizon is symmetrical [at eastern and western horizons].

We find, then, by examining observations of [first] risings of this kind, that near the beginning of Cancer, in general,

---

63 This is Neugebauer's interpretation of 'symmetrical' (H.A.M.4 235), and it is confirmed by p. 639, 'when the beginning of Cancer is setting, it forms the same angle and inclination to the horizon as before [at rising].

64 For Saturn at least, these could hardly have been Ptolemy's own observations, as the requirement of a longitude near 0° takes us back to about the year 120, much earlier than any of Ptolemy's quoted observations. This is confirmed by the references to the Babylonians.
Saturn rises [i.e. is first visible] at a distance from the true sun of 14°
Jupiter at 12°
Mars at 14½°
Venus as evening star at 5½°, and
Mercury as evening star at 11½°.

With these data given, let the diagram of the preceding figure be drawn [Fig. 13.18]. (For such small arcs it will make no difference if, for convenience' sake, we substitute in our calculations the corresponding chords which are not sensibly different from them). Let point E be the intersection of ecliptic and horizon at the above-mentioned phases, at the beginning of Cancer, and rising

Fig. 13.18

for the 3 morning-star planets, Saturn, Jupiter and Mars, but, obviously, setting for the evening-stars, Venus and Mercury. Let us take as geographical latitude the parallel through Phoenicia, where the longest day is 14½ hours, since it is mainly on this parallel or round about it that the majority and most reliable of the observations of the phases have been made, those of the Babylonians almost on it, and those in Greece and Egypt round about it.⁶⁵

Now we find, by means of the procedure for angles [between ecliptic and horizon] previously demonstrated [II 11], that when the beginning of Cancer is rising at the latitude in question,

\[ \angle BED = 103° \] where 2 right angles = 360°,

and hence the ratio of the sides about the right angle,⁶⁷

\[ \frac{BD}{BE} \approx 94 : 75 \] where the hypotenuse [DE] = 120°.

⁶⁵ According to the Geography Babylon has a latitude of 35° (which corresponds closely to the standard Babylonian daylight ratio M:m = 3 : 2). In fact its latitude is about 32½°. The parallel with M = 14½° (and φ = 33;18°) is halfway between the climata of Lower Egypt (14° and 30;22°) and Rhodes (14½° and 36°).

⁶⁶ How Ptolemy got this angle remains mysterious: whether he used interpolation in the tables II 13 (cf. HAMA 236) or direct computation, he should have found (in round numbers) 53° = 106°. On the general problem of the angles between ecliptic and horizon in this chapter see HAMA 245-50.

⁶⁷ The text has 'right angles', 'hypotenuses' etc. because it is true for each planet.
By means of the procedure for the [planetary] latitude, we find that (considering now just the 3 [outer] planets), when they [first] rise near the beginning of Cancer, that is, when they are near the apogee of the epicycle, then at any distance from the apogee not exceeding \( \frac{1}{12} \)th [of the epicycle circumference], with no sensible error Saturn and Jupiter are practically on the ecliptic, while Mars is about \( \frac{1}{4} \)° north of the ecliptic. Therefore their distance from the sun along the ecliptic will be represented by \( DE \) for Saturn and Jupiter, and by \( DK \) for Mars, since it is north [of the ecliptic] by \( KH \), of the amount 12°.

And since \( KH : KE = 94 : 75 \),

\[
KE \approx 10' \text{ in the same units.}
\]

But \( DK \) is given for Mars as \( 14\frac{3}{4}° \),

so, by addition, \( DE = 14;40° \).

And for Saturn it is \( 14° \)

and for Jupiter \( 12\frac{1}{4}° \).

So, since \( ED : DB = 120 : 94 \),

we get, approximately, for \( DB \), the arc of the great circle drawn through the poles of the horizon.

\[
\begin{align*}
11° & \text{ for Saturn} \\
10° & \text{ for Jupiter} \\
11\frac{3}{4}° & \text{ for Mars.}
\end{align*}
\]

Similarly, for Venus and Mercury, when the beginning of Cancer is setting, it forms the same angle and inclination to the horizon as before; and we are given that, when these planets have their first visibility as evening-star in this part of the ecliptic, the distance of Venus from the true sun is \( 5\frac{1}{2}° \), while Mercury's is \( 11\frac{1}{2}° \). Therefore at their [first] risings the true sun will have a longitude of

\[
\begin{align*}
\Pi 24\frac{3}{4}° & \text{ for Venus} \\
\Pi 18\frac{3}{4}° & \text{ for Mercury,}
\end{align*}
\]

while the longitude of the mean sun will be about

\[
\begin{align*}
\Pi 25° & \text{ for Venus} \\
\Pi 19° & \text{ for Mercury.}
\end{align*}
\]

Therefore the planets will have these positions in mean longitude. And when, with these [mean] longitudes, the planets have apparent positions at the beginning of Cancer, we find that their distances from the apogee are about

\[
\begin{align*}
14° & \text{ for Venus} \\
32° & \text{ for Mercury.}
\end{align*}
\]

(This kind of computation can be carried out by means of the theorems on their anomaly which we set out before.) Accordingly, at these positions, we find that

68 At apogee of the epicycle the planet is at mean conjunction. So Ptolemy is considering elongations from the mean sun of up to one zodiacal sign.

69 See H.1.1.1 235,237.

70 From the anomaly tables, XI 11, given, for Venus, \( \lambda = 85°, \alpha = 14° \) and the apogee in \( \beta = 25° \),

then \( R = 30° \), leading to an equation of centre of \( 1;11°, \) so \( \alpha = 15;11° \), which leads to an equation of anomaly of \( 6;6\frac{1}{2}° \), so \( \lambda = 85° - 1;11° + 6;6\frac{1}{2}° = 95;56\frac{1}{2}° \approx 0°. \) For Mercury, with \( \lambda = 79°, \alpha = 32° \) and the apogee in \( \approx 10°, R = 249° \), leading to an equation of centre of \( 2;53°, \) so \( \alpha = 29;7° \), which leads to an equation of anomaly of \( 8;16°, \) hence \( \lambda = 79° + 2;53° + 8;16° = 90°90° \approx 0°. \)
Venus is about 1° north of the ecliptic, and Mercury about 1½° north.\textsuperscript{71}

These, obviously, are the amounts of KH [in Fig. 13.19].

So, since KH:EK = 94 : 75,

and 94 : 75 \approx \begin{cases} 1 : 1 \\ 12 : 11 \end{cases},

EK = \begin{cases} \frac{2}{3} \text{ for Venus} \\ 1\frac{3}{10} \text{ for Mercury} \end{cases}.

And in the same units, by hypothesis, the apparent distance of the planet from the sun,

\( \text{DK} = \begin{cases} 5\frac{1}{3} \text{ for Venus} \\ 11\frac{3}{4} \text{ for Mercury} \end{cases} \)

Therefore, by addition, DKE \approx \begin{cases} 6\frac{2}{3} \text{ for Venus} \\ 12\cdot4 \text{ for Mercury} \end{cases}.

So, since ED:BD is again 120 : 94, and that ratio is about the same as 6\frac{2}{3} : 5 and 12\cdot4 : 10,

we get for DB, the size of the normal distance,

5° for Venus and 10° for Mercury.

Q.E.D.

8. \textit{That the peculiar characteristics of the phases of Venus and Mercury are also in accordance with the hypotheses.}\textsuperscript{72}

Furthermore, it is in accordance with the hypotheses detailed above that the strange characteristics of the first and last visibilities of Venus and Mercury take

\textsuperscript{71}For the calculations confirming this see \textit{H.A.M.A} 237-8.

\textsuperscript{72}See \textit{H.A.M.A} 239-42. There is a reference to this in Proclus, \textit{Hypotyposis} I 17 (ed. Manitius p. 10).
place: namely that, for Venus, the interval from evening setting to morning rising is about 2 days round about the beginning of Pisces, but about 16 days round about the beginning of Virgo; and, for the planet Mercury, the phases as evening-star are missing, when one would expect it to appear round about the beginning of Scorpius, and the phases as morning-star, when round about the beginning of Taurus. We can come to understand that as follows; and first for Venus.

Let there be drawn a diagram [Fig. 13.20] similar to the preceding figure for the phases, and let point E represent, first, the point on the ecliptic at the beginning of Pisces (at this point Venus, when it is near the perigee of the epicycle, is about 6° north of the ecliptic). Let the diagram represent the evening setting [i.e. last visibility as evening-star]. In this \( \angleBED \), at the terrestrial latitude in question, is calculated as 154° where 2 right angles equal 360°.

And [in the right-angled triangles \( \triangleBED, \triangleKEH \)], where the hypotenuse is 120°, the greater of the sides about the right angle,

\[
[BD \text{ or } KH] \approx 117°,
\]

and the lesser, \([BE \text{ or } KE] \approx 27°\).

Hence, where the normal distance, \(DB = 5°\),

\[
\text{DE} = 5;8°.
\]

73 See *HAMA* 239, and cf. XIII 3 p. 602, when Venus is in the node and near the perigee of the epicycle its latitude is 6°. Since Venus' apogee is taken as 8° 25°, for a position of \(\chi 0°\) it is 27° from apogee or 5° from the node.

74 On the angles between ecliptic and horizon given by Ptolemy see *HAMA* 245-50. The (rounded) value here, 77°, can be found from the tables II 13, taking the values for \(\chi 0°\) at Clima III and Clima IV, 10.5° and 15.53°, taking the mean, 12.59°, and taking its complement, 77.1°. The other values given by Ptolemy, however, cannot be so derived.
But since the planet is $6^\circ$ north of the ecliptic (which amount is represented by arc KH),

and the ratio $117 : 27 \approx 6 : 1$,

and, by subtraction, KD, which represents the distance of the planet towards the rear from the sun at its evening setting, is

$$[5;8 - 1;30 =] 3;38^\circ.$$ 

Again, on the similar diagram [Fig. 13.21], since at the morning rising [i.e. at the planet's first visibility as morning-star]

$\angle BED = 69^\circ$ where 2 right angles = $360^\circ$,

and hence, where the hypotenuse [of the right-angled triangles] is $120^\circ$,

the lesser of the sides about the right angle, [BD or KH] $\approx 68^\circ$,

and the greater, [BE or KE] $\approx 99^\circ$;

and we calculate that $68 : 120 = 5 : 8;49$

and that $68 : 99 = 6\frac{1}{2} : 9;13$,

so we get DE $= 8;49^\circ$ in the same units,

and the difference [in longitude] due to the latitude,

KE $= 9;13^\circ$;

and, by subtraction, DK, [the planet's distance] from the sun, towards the rear (obviously), is 0;24$^\circ$.

And at its evening setting its distance, likewise towards the rear, was 3;38$^\circ$. Therefore during the interval from evening setting to morning rising it has moved a distance which is less than the sun's motion (that is, approximately, its own motion in [mean] longitude) by 3;14$^\circ$, which is due to its motion in advance on the epicycle. Now it is easy to determine from the table of anomaly that a motion in advance of that amount [3;14$^\circ$] is produced by a motion on the epicycle near its perigee of 1$^\circ$. And the planet traverses 1$^\circ$ in mean motion [in

---

75‘approximately’, because the sun’s motion is that of the true sun, while the planet’s mean motion in longitude is equal to that of the mean sun.

76From the table of anomaly, XI 11, Venus has an equation of anomaly of 7;38$^\circ$ for $\alpha = 177^\circ$ ($= 180^\circ - 3^\circ$); hence to 3;14$^\circ$ corresponds $3;14 \times 3/7;38 = 1;16,14^\circ$ $\approx 1^\circ$. Similarly, (below pp. 643-4), for $\alpha = 172^\circ$ we find an equation of 18;1$^\circ$ (text 18;2$^\circ$), and for $\alpha = 177^\circ$ an equation of 6;21$^\circ$ (text 6;38$^\circ$).
anomaly] in about 2 days. Hence it is clear that that [2 days] is the period of the above interval, in agreement with the phenomena.

Again, on the similar diagram [Fig. 13.22], let point E be taken as the beginning of Virgo (at this point, when Venus is at the perigee of the epicycle, it is south of the ecliptic by about the same amount, \(6^{\circ}\)). Let us consider, first, the evening setting, when

\[\angle BED = 69^\circ\] where 2 right angles = 360°.

![Fig. 13.22](image)

Thus where the hypotenuse [of right-angled triangle BED] is 120°, the lesser of the sides about the right angle, \([BD] = 68^\circ\), and the greater, \([BE] = 99^\circ\). Thus since the ratios [of BD:BE:DE] are the same as for the morning rising in Pisces, and the difference due to the latitude is equal [to its amount there], we get

\[\text{arc ED} = 8;49^\circ,\]

the difference [in longitude] due to the latitude, \(LE = 9;13^\circ\), and, by addition, \(DL\), the planet's distance from the sun towards the rear, is \(18;2^\circ\).

From the table of anomaly, as mentioned before, [the motion in anomaly] near the perigee of the epicycle corresponding to that amount [18;2°] of retrogradation with respect to the mean motion in longitude of sun and planet is about \(7;1^\circ\).

Similarly, at the morning rising at the beginning of Virgo, when

\[\angle BED = 154^\circ\] where 2 right angles = 360°,

and [hence], where the hypotenuse [of right-angled triangle BED] is 120°, the greater of the sides about the right angle, \([BD] = 117^\circ\), and the lesser, \([BE] = 27^\circ\);

and one again finds the same ratios as those set out for the evening setting in Pisces, so we get

\[DE = 5;8^\circ,\]

\(^{77}\) Cf. p. 641 n.73.
the difference [in longitude] due to the latitude, $EL = 1;30^\circ$, and, by addition, $DL$, the planet's distance from the sun in advance, is $6;38^\circ$. To this amount corresponds, in the same way as above, about $2\frac{1}{2}^\circ$ of [motion in anomaly] near the perigee of the epicycle.

Therefore the total amount of motion on the epicycle which the planet Venus will perform from evening setting to morning rising is $10^\circ$; and it traverses that amount in about 16 days, which, as stated above, is the amount agreeing with the phenomena.

Having demonstrated the above, we must apply our theory to the facts concerning the missing phases of Mercury, and [show], first, that at the beginning of Scorpius, even if it reaches its greatest elongation towards the rear from the sun, it cannot become visible as evening-star.

[Proof:] Let the diagram for the phases [Fig. 13.23] be drawn, with point E taken as the point on the ecliptic at the beginning of Scorpius at a [terrestrial latitude] such that at setting

\[ \angle BED = 69^\circ \text{ where } 2 \text{ right angles} = 360^\circ, \]

and [thus] where the hypotenuse [of right-angled triangle $BED$] is $120^\circ$,

the lesser of the sides about the right angle, $[BD] = 68^\circ$,

and the greater, $[BE] = 99^\circ$.

Therefore where the amount of the normal distance, $BD = 10^\circ$, $DE = 17;39^\circ$.

But when the planet is in the above situation, it is about $3^\circ$ south of the ecliptic.

So, according to the above ratios,

where $L\Theta$, the amount of the latitude, is $3^\circ$,

$LE = 4;22^\circ$,

and, by addition, $DEL = [17;39^\circ + 4;22^\circ] \approx 22^\circ$.

\[ ^{18} \text{A similar phrase is used of Mercury as early as Aristotle (Meteorologica 342b34) διά γὰρ τὸ μικρὸν ἐπανβαίνειν πολλὰς ἐκλειπεῖ θάνεις 'because it rises only a little above [the horizon] it misses many phases (appearances)'.} \]

\[ ^{79} \text{At XII 9 Ptolemy has calculated the maximum elongations for Mercury at } m_6 0^\circ \text{ and } B 0^\circ, \text{ in preparation, as he says (p. 591) for this problem.} \]

\[ ^{80} \text{For a computation of this see } H.A.M.I \text{ 241 n.11.} \]
Hence the planet must have that elongation \(22^\circ\) from the true sun in order to have its first visibility. But since its maximum elongation from the true sun when it is at the beginning of Scorpius is only \(20;58^\circ\), as we demonstrated previously [XII 9, p. 594] in our treatment of the greatest elongations, it is obvious that it is natural for phases of this kind to be missing.

Again, if we set out the same diagram for the phases [Fig. 13.24] and take point E as the beginning of Taurus at morning rising, when the planet, in accordance with the positions in question, is about \(3^\circ\) south of the ecliptic, and the ratios of the sides [of triangles BED, LEΘ] about the right angles are the same as those above,

then \(DE = 17;39^\circ\)

and, where the latitude \(ΘL = 3;10^\circ\),

\(LE = 4;37^\circ\).

Thus, by addition, \(DEL = 22;16^\circ\).

Thus here too the planet must have an elongation of that amount \(22;16^\circ\) from the true sun in order to have its first visibility. But since its maximum elongation [in this situation] does not exceed \(22;13^\circ\), as we demonstrated previously [p. 595], naturally, this kind of phase too is missing. Thus we have shown that the facts in question are in agreement with the hypotheses we set out as well as with the phenomena.

9. \{Method of determining the individual elongations from the sun of the first and last visibilities\}\(^82\)

It is immediately obvious [see Fig. 13.25] that if we take as fixed, for each planet, the normal arc \(arcus visionis\) BD, and are given the beginning of [each of] the [zodiacal] signs at the intersection E, and hence angle BED, there will also be given DE and the position in latitude of the planet at that elongation [i.e. DE],

\(^{81}\) See HAMA 241 n.11.

\(^{82}\) See HAMA 242-56.
namely KH or ΘL; thence will be given KE or EL [respectively], and also the [corresponding] apparent distance, DK or DL. In this way, (to avoid lengthening our discussion), we computed, for all the signs and for each of the 5 planets, but for only one [terrestrial latitude], the intermediate parallel used above, since that is sufficient in itself, the apparent distance from the true sun of the risings and settings [i.e. first and last visibilities], on the assumption that the planets themselves were located at the beginning of the signs. We have set these out below, putting them, too, for the user’s convenience, in 5 tables, [one] for [each of] the 5 planets, each containing 12 lines. The first 3 tables, for Saturn, Jupiter and Mars, are arranged in 3 columns: the first column contains the beginnings of the signs, the second the elongations at morning rising, and the third those at evening setting. The next 2 tables, for Venus and Mercury, are arranged in 5 columns: the first, as before, contains the beginnings of the signs, the second the elongations at evening rising, the third those at evening setting, and the fourth, again, those at morning rising, and the fifth those at morning setting. The layout of the tables is as follows.

[See p. 647.]

The basis of computation of these tables is in part unclear (see HAMA 242-56), hence I have not been able to recompute them to check the numbers. However, from Neugebauer’s computations, the following corrections to Heiberg have been made:

H606.6 Saturn, Morning Rising, Aries, κγ λ (with DK,Is) for κγ α (23;1°) (HAMA 248, n.11).
H606.7 Mars, Morning Rising, Taurus, κ τς (with DHKL) for κ η (20;8°) (HAMA 248 n.9 suggests 20;19°).

See also HAMA 255 for a suggestion to emend Venus, Morning Rising, Aries, to 2;0° from 3;0°.
### Tables for First and Last Visibilities of the 5 Planets

<table>
<thead>
<tr>
<th>Beginning of Sign</th>
<th>Saturn Morning Rising</th>
<th>Saturn Evening Setting</th>
<th>Jupiter Morning Rising</th>
<th>Jupiter Evening Setting</th>
<th>Mars Morning Rising</th>
<th>Mars Evening Setting</th>
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<td>12 10</td>
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### Venus

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<th>Morning Rising</th>
<th>Morning Setting</th>
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<td>3 16</td>
<td>0 30</td>
<td>8 33</td>
</tr>
<tr>
<td>Pisces</td>
<td>5 22</td>
<td>3 38</td>
<td>0 24</td>
<td>10 16</td>
</tr>
</tbody>
</table>

### Mercury

<table>
<thead>
<tr>
<th>Beginning of Sign</th>
<th>Evening Rising</th>
<th>Evening Setting</th>
<th>Morning Rising</th>
<th>Morning Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aries</td>
<td>9 58</td>
<td>9 43</td>
<td>23 58</td>
<td>23 38</td>
</tr>
<tr>
<td>Taurus</td>
<td>10 4</td>
<td>10 15</td>
<td>22 15</td>
<td>22 15</td>
</tr>
<tr>
<td>Gemini</td>
<td>10 18</td>
<td>11 47</td>
<td>18 0</td>
<td>16 44</td>
</tr>
<tr>
<td>Cancer</td>
<td>12 22</td>
<td>15 34</td>
<td>14 4</td>
<td>12 30</td>
</tr>
<tr>
<td>Leo</td>
<td>13 43</td>
<td>19 59</td>
<td>11 25</td>
<td>10 21</td>
</tr>
<tr>
<td>Virgo</td>
<td>18 1</td>
<td>23 13</td>
<td>10 21</td>
<td>9 39</td>
</tr>
<tr>
<td>Libra</td>
<td>22 49</td>
<td>23 16</td>
<td>9 51</td>
<td>10 0</td>
</tr>
<tr>
<td>Scorpius</td>
<td>20 1</td>
<td>22 1</td>
<td>9 44</td>
<td>10 19</td>
</tr>
<tr>
<td>Sagittarius</td>
<td>18 11</td>
<td>17 25</td>
<td>9 25</td>
<td>11 19</td>
</tr>
<tr>
<td>Capricorn</td>
<td>13 54</td>
<td>12 10</td>
<td>9 36</td>
<td>14 5</td>
</tr>
<tr>
<td>Aquarius</td>
<td>11 10</td>
<td>9 50</td>
<td>12 27</td>
<td>17 50</td>
</tr>
<tr>
<td>Pisces</td>
<td>10 11</td>
<td>9 43</td>
<td>19 15</td>
<td>21 46</td>
</tr>
</tbody>
</table>

### Epilogue of the treatise

We have now completed these additional topics, Syrus, and have shown the way to deal with almost all the topics which should, at least to my mind, be subjected to theory for the purposes of this kind of treatise, at any rate as far as the time up to our own days contributed to greater accuracy in our discoveries or in corrections of earlier discoveries, and as far as was suggested by a memorandum directed only toward scientific usefulness, and not towards ostentation. So at this point our present discussion can be terminated at an appropriate place and at the right length.

---

84 Cf. p. 37 n.11.
85 Cf. p. 37 n.12.
Appendix A

Examples of Computations

1 (a). II 4 p. 80. Given the terrestrial latitude (φ), compute the distance of the sun from the summer solstice as measured along the ecliptic (Δλ).
Example: φ = 4°15' (cf. II 6, second parallel, p. 83).

From Table I

<table>
<thead>
<tr>
<th>λ</th>
<th>δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>4°13,8°</td>
</tr>
<tr>
<td>11°</td>
<td>4°25,32°</td>
</tr>
</tbody>
</table>

Hence to a declination of 4°15' corresponds a longitude (counted from equinox) of 10°33.33°.
Therefore the distance from solstice, Δλ = (90° - 10°33.33°) = 79°26.27° (text: 79°1°).

1 (b). II 6 p. 89. Find the terrestrial latitude (φ) at which the sun does not set for a given period of time.
Example: Period of one month. Taking a month as 30 days, and assuming the sun to move 1° in the ecliptic, we find that the parallel in question cuts off 30° of the ecliptic, or 15° either side of the summer solstice.

From Table I

90° - 15° = 75° 22;59,41°.

Hence φ = 90° - δ = 67°0,19° (text: 67°).

2. II 9 p. 99. Given the longitude of the sun (λ⊙) and the terrestrial latitude (i.e. the 'clima'), find the length of day or night and the length of the seasonal hour.
Example: λ⊙ = 28°18'. Place: Babylon (cf. IV 11 p. 212). What is the length of night?
We use the rising-time table (II 8) for Rhodes (M = 14 1/2).
(a) First method.
Since it is night, we take the degree opposite the sun, Π 28;18°.

From the table:

<table>
<thead>
<tr>
<th>λ</th>
<th>δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>28°18'</td>
<td>69;27°</td>
</tr>
<tr>
<td>28°18'</td>
<td>286;50°</td>
</tr>
</tbody>
</table>

Difference (in order of signs), Δ = 217;23°.
Length of night in equinoctial hours is Δ/15: 14;29° (text: 14°29')
Length of 1 seasonal night-hour in time-degrees is Δ/12: 18;7° (text: 18°)
(hence length of 1 seasonal hour in equinoctial hours: 1;12,28°).
Appendix A. Examples 2-5

(b) Second method.
From rising-time table (II 8) at sphaera recta:

<table>
<thead>
<tr>
<th>a (Π 28;18°)</th>
<th>b (Π 28;18°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>88;9°</td>
<td>69;27°</td>
</tr>
</tbody>
</table>

Difference (Δ): 18;42°
Δ/6: 3;7°

Since Gemini is north of the ecliptic, add 15°:
18;7°
This is the length of 1 seasonal night-hour in time-degrees.

3. II 9 p. 104. Given the length of a seasonal hour in time-degrees, convert the time in seasonal hours to the time in equinoctial hours.
From Example 2 (q.v.), length of 1 seasonal night-hour: 18;7°.
What is 5½ seasonal hours after midnight in equinoctial hours?
5½ × 18;7/15 = 6;38, so the time is 6;38 a.m.
Ptolemy (l.c.) multiplies by 1/3 and gets 6½ equinoctial hours after midnight.

4. II 9 p. 104. Given the longitude of the sun (λ⊙), the terrestrial latitude, and the time in seasonal hours, find the point of the ecliptic which is rising (the 'horoscope').
Example (cf. VII 3 p. 336). λ⊙: m, 13;17° (text, 'about the middle of m')
Place: Alexandria. Time: 2½ seasonal hours after midnight.¹

Length of 1 night-hour (λ⊙ = m, 13;17°, M = 14°, cf. Example 2): 16;38°
Time from sunset: 8½ seasonal hours. 8½ × 16;38: 137;14°
From Table II 8 for Clima III: ρ (8 13;17°): 31;4°
(we take the point opposite the sun, since it is night) Sum 168;18°.
168;18° is the rising-time (at Clima III) of the horoscope: ρ (m 19:51°)
(text: 'about m 221°).

5. II 9 p. 104. Given the same data as in Example 4, find the point of upper culmination.
Total of seasonal hours from last midday: 6 day-hours plus 8½ night-hours.
Length of 1 day-hour: 13;22°
Length of 1 night-hour: 16;38°
6 × 13;22° + 8½ × 16;38° = 80;12° + 137;14° = 217;26°
Rising-time at sphaera recta of sun’s degree: α (m 13;17°) 220;46°
Sum: 78;12°

¹ Ptolemy (l.c.) gives 2½ equinoctial hours, which is approximately the same.
6. II 9 p. 104. Given the longitude of the horoscope at a given place, find the point of upper culmination.

Example: same data as in Example 4.

Rising-time of horoscope at Clima III: \( \rho (\mp 19;51^\circ) \):

\[
\begin{array}{c}
168;18^\circ \\
-90;0^\circ \\
\hline
78;18^\circ
\end{array}
\]

\(78;18^\circ = \alpha (\Pi 19;16^\circ)\) (text: \(\Pi 22;1^\circ\)).

The discrepancy from the result of Example 5 is due to the rounding to minutes of the tables and at every step of the computation.

7. III 8 p. 169. Given the date, compute the position of the sun.

Example (Cf. IV 11 p. 214). Date: Nabonassar 548, Mechir [VI] 9/10, 1\(\frac{1}{2}\) equinoctial hours after midnight.

From mean motion table, III 2:

\[
\begin{array}{c|c}
\hline
\Delta \lambda & \\
540^d & 228;42,48^\circ \\
7^d & 358;17,53^\circ \\
150^d & 147;50,43^\circ \\
8^d & 7;53,6^\circ \\
13^h & 0;32,2^\circ \\
0:20^h & 0:0,49^\circ \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\text{Sum } 547^d 158^d 13^h \\
743;17,21^\circ \rightarrow 23;17,21^\circ \\
\hline
\end{array}
\]

\(\kappa\) (epoch): + 265;15°

\(\kappa\): 288;32,21°

From Table III 6, for argument 288;32°, we find (by interpolation) the equation as 2;13,28°. This is additive, since \(\kappa\) falls in the second column. 288;32.21° longitude of apogee: + 65;30°

\[
\begin{array}{c}
\lambda: 354;2,21^\circ \\
\theta: +2;13,28^\circ \\
\hline
\lambda: 356;15,49^\circ \\
\end{array}
\]

or about \(\lambda\) 26;16° (text: \(\lambda\) 26;17°).

8. III 9 p. 171. Computation of the 'equation of time', \(E\) (given an interval in true solar days, find the interval in mean solar days).

Example (cf. IV 6 p. 198):

\(t_1\): Hadrian 17 (Nabonassar 880) Pauni [X] 20/21, 11;15 p.m.

\(t_2\): Hadrian 19 (Nabonassar 882) Choraik [IV] 2/3, 11 p.m.

From the solar tables (cf. Manitius I p. 437):

\[
\begin{align*}
\lambda (t_1): & \quad 42;21^\circ, \\
\lambda (t_2): & \quad 206;42^\circ \\
\lambda (t_2): & \quad \in 25;10^\circ.
\end{align*}
\]
Hence, from Table II 8 (rising-times at sphaera recta):

\[ \alpha (t_1): \quad 40;44^\circ \]
\[ \alpha (t_2): \quad 203;17^\circ. \]

\[ \Delta \lambda = \lambda (t_2) - \lambda (t_1) = 164;21^\circ \]
\[ \Delta \alpha = \alpha (t_2) - \alpha (t_1) = 162;33^\circ. \]

Since \( \Delta \lambda > \Delta \alpha \), we subtract \( E \) from the 'simple' interval, \( 1^\circ 166^d 23;45^h \), to get, for the interval in mean solar days, \( 1^\circ 166^d 23;37,48^h \) (text: \( 23^h = 23;37,30^h \)).

9. V 9 p. 239. Computation of the moon's latitude and longitude from the tables for a given date.
From the mean motion tables, IV 4:

<table>
<thead>
<tr>
<th>epoch value</th>
<th>( \lambda )</th>
<th>( \bar{\alpha} )</th>
<th>( \bar{\omega} )</th>
<th>( \bar{\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 450^d )</td>
<td>268;49°</td>
<td>354;15°</td>
<td>70;37°</td>
<td></td>
</tr>
<tr>
<td>( 15^d )</td>
<td>260;46,44°</td>
<td>323;26,5°</td>
<td>10;11,3°</td>
<td></td>
</tr>
<tr>
<td>( 6^d )</td>
<td>140;41,33°</td>
<td>250;46,52°</td>
<td>144;20,22°</td>
<td></td>
</tr>
<tr>
<td>( 14^h )</td>
<td>79;3,30°</td>
<td>78;23,24°</td>
<td>73;8,40°</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta \lambda = 128;13^\circ \]
\[ \bar{\alpha} = 209;3^\circ \]
\[ \bar{\omega} = 112;56^\circ \]
\[ 2\bar{\eta} = 250;48^\circ. \]

From anomaly table, V 8.

col. 3: \( c_3(2\bar{\eta}) = -13;4^\circ \)
true anomaly \( \alpha = \bar{\alpha} + c_3 = 209;3 - 13;4 = 195;59^\circ \)
col. 4: \( c_4(\alpha) = 1;30^\circ \)
col. 5: \( c_5(\alpha) = 0;55^\circ \)
col. 6: \( c_6(2\bar{\eta}) = 36,52 \)
equation \( c = c_4 + c_5,c_6 = +(1;30^\circ + 0;55^\circ \times 0;36,52) = +2;4^\circ \)
longitude = \( \Delta \lambda + c + \lambda_{\text{epoch}} = 128;13^\circ + 2;4^\circ + 41;22^\circ = 171;39^\circ \) (text: 171;30°).
\( \omega = \bar{\omega} + c = 112;56^\circ + 2;4^\circ = 115;0^\circ. \)
col. 7: latitude \( \beta(\omega) = -2;7^\circ \) (text: -2°).

10. V 19 p. 264. Computation of the parallax of the moon for a given time, place, solar longitude and lunar longitude, latitude and elongation, from the tables.
Example: time, 2\( \frac{1}{2} \) equinoctial hours after midnight (true local time Alexandria);
\( \lambda_\odot: \) \( \mu 13;17^\circ, \lambda_e: \mu 21;30^\circ, \beta_e: -2^\circ \) (cf. VII 3 p. 336 and Example 9).
From solar longitude and local time: culminating point: \( \lambda_1 19;11^\circ \) (cf. Example 5).
Distance of moon from meridian:
\[ \alpha (\mu 21;30^\circ) - \alpha (\Pi 19;11^\circ) = 172;12^\circ - 78;12^\circ = 94^\circ \]
\[ = 6;16^h \text{ east.} \]
Appendix A. Example 10

From Table II 13 (Clima III), arguments 6;16° (vertical) and 21;30° (horizontal), by interpolation in tables for Virgo and Libra:

arc 90°
east angle 172;30°.

Correction to arc and angle for moon's latitude (cf. V 19 p. 272):

Crd (2 × (180° - 172;30°)) = Crd 15° = 15;40°
Crd (180° - 15°) = Crd 165° = 118;58°.

Multiplying β by each of these and dividing by 120, we get 0;17° and 2;9° respectively. Then the corrected arc is given by

\[ \sqrt{(90° + 0;16°)^2 + (2;9°)^2} \approx 90;18°, \]

and the corresponding angle of correction from: 2;9 × 120 = 2;51°, which is the chord of ca. 2;44°, half of which is 1;22°.

Therefore the corrected angle is 172;30° - 1;22° = 171;8°.

We take the arc as exactly 90° (since otherwise the moon would be below the horizon).

Computation of total parallax.

From Table V 18, argument ζ = 90°.

Lunar parallax (α = 195;59°, η = 305;24°, cf. Example 9):

<table>
<thead>
<tr>
<th>col.</th>
<th>col.</th>
<th>col.</th>
<th>col.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0;53,34</td>
<td>0;10,17</td>
<td>1;19,0</td>
<td>0:25,0</td>
</tr>
</tbody>
</table>

with argument (360° - α)/2 (≈ 82°), from col. 7: minutes: 58,39
from col. 8: minutes: 58,31.

Parallax at syzygy: 0;53,34 + 0;10,17 × 0;58,39 = 1;3,37°
Parallax at quadrature: 1;19,0 + 0;25,0 × 0;58,31 = 1;43,23°

\[ \Delta = 0;39,46° \]

with argument (360° - η) = 54:36, from col. 9: minutes: 42,35.

Parallax: 1;3,37 + 0;39,46 × 0;42,35 = 1;32°.

Determination of longitudinal and latitudinal components of parallax.

Angle between hour-circle and ecliptic (see above): 171;8°.
This is greater than 90°, so we take the supplement. 8;52°.
Twice this is 17;44°, and the supplement of the latter 162;16°.
The chords of these angles are 18;30° and 118;34° respectively.
Latitudinal parallax: 1;32 × 17;44/120 = 0;13\frac{1}{2}°.
Longitudinal parallax: 1;32 × 118;34/120 = 1;31°.

Latitudinal parallax is southwards (zenith to the north of the culminating point).
Since latitudinal parallax is southwards and the angle greater than 90°, longitudinal parallax is positive.
Result: parallax in latitude: -0;13\frac{1}{2}° (text: -0;5°)
parallax in longitude: +1;31° (text: +1;0°).

Appendix A. Example 10

From Table II 13 (Clima III), arguments 6;16° (vertical) and 21;30° (horizontal), by interpolation in tables for Virgo and Libra:

arc 90°
east angle 172;30°.

Correction to arc and angle for moon's latitude (cf. V 19 p. 272):

Crd (2 × (180° - 172;30°)) = Crd 15° = 15;40°
Crd (180° - 15°) = Crd 165° = 118;58°.

Multiplying β by each of these and dividing by 120, we get 0;17° and 2;9° respectively. Then the corrected arc is given by

\[ \sqrt{(90° + 0;16°)^2 + (2;9°)^2} \approx 90;18°, \]

and the corresponding angle of correction from: 2;9 × 120 = 2;51°, which is the chord of ca. 2;44°, half of which is 1;22°.

Therefore the corrected angle is 172;30° - 1;22° = 171;8°.

We take the arc as exactly 90° (since otherwise the moon would be below the horizon).

Computation of total parallax.

From Table V 18, argument ζ = 90°.

Lunar parallax (α = 195;59°, η = 305;24°, cf. Example 9):

<table>
<thead>
<tr>
<th>col.</th>
<th>col.</th>
<th>col.</th>
<th>col.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0;53,34</td>
<td>0;10,17</td>
<td>1;19,0</td>
<td>0:25,0</td>
</tr>
</tbody>
</table>

with argument (360° - α)/2 (≈ 82°), from col. 7: minutes: 58,39
from col. 8: minutes: 58,31.

Parallax at syzygy: 0;53,34 + 0;10,17 × 0;58,39 = 1;3,37°
Parallax at quadrature: 1;19,0 + 0;25,0 × 0;58,31 = 1;43,23°

\[ \Delta = 0;39,46° \]

with argument (360° - η) = 54:36, from col. 9: minutes: 42,35.

Parallax: 1;3,37 + 0;39,46 × 0;42,35 = 1;32°.

Determination of longitudinal and latitudinal components of parallax.

Angle between hour-circle and ecliptic (see above): 171;8°.
This is greater than 90°, so we take the supplement. 8;52°.
Twice this is 17;44°, and the supplement of the latter 162;16°.
The chords of these angles are 18;30° and 118;34° respectively.
Latitudinal parallax: 1;32 × 17;44/120 = 0;13\frac{1}{2}°.
Longitudinal parallax: 1;32 × 118;34/120 = 1;31°.

Latitudinal parallax is southwards (zenith to the north of the culminating point).
Since latitudinal parallax is southwards and the angle greater than 90°, longitudinal parallax is positive.
Result: parallax in latitude: -0;13\frac{1}{2}° (text: -0;5°)
parallax in longitude: +1;31° (text: +1;0°).
Example: Date, Nabonassar 28, Thoth (cf. IV 6 pp. 191–2).
From Table VI 3, compute mean opposition:

<table>
<thead>
<tr>
<th>Days of Thoth</th>
<th>( \bar{\kappa} )</th>
<th>( \bar{\alpha} )</th>
<th>( \bar{\omega} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period: 26</td>
<td>9;55,35</td>
<td>267;58,12°</td>
<td>83;24,29°</td>
</tr>
<tr>
<td>Year: 2</td>
<td>8;15,53</td>
<td>7;39,36°</td>
<td>285;25,4°</td>
</tr>
</tbody>
</table>

Year: 28 18;11,28⁹ 275;37,48° 8;49,33° 276;55,59°.
Time of mean opposition: 18;11,28⁹ = Thoth 18/19, 4:35 p.m.
\( \bar{\omega} \) lies within ecliptic limits for lunar eclipse, which is therefore possible.
Computation of true opposition.

From Table III 6, \( c(\bar{\kappa}) \): +2;21° solar equation
From Table IV 10, \( c(\bar{\alpha}) \): -0;42° lunar equation.

True position in latitude: \( \omega = \bar{\omega} + c(\alpha) = 276;14° \) at mean opposition.
\( \Delta \lambda = 2;21° + 0;42° = 3;63° \).
Moon’s true hourly motion in longitude: \( 0;32,56 - 0;32,40 \times 45' = 0;30,24° \).
\( \Delta t = 3;3 \times \frac{11}{12} + 0;30,24 = 6;31\text{h} \).
True longitude of moon at mean syzygy is less than true longitude of sun (minus 180°). So we add \( \Delta t \) to the time of mean opposition to get the time of true opposition as 11:6 p.m. (text: 11:10 p.m.).
Motion over \( \Delta t \): 3;3 \times \frac{11}{12} = 3;18°.
We add this to the position in latitude: \( \omega = 279;32° \) at true opposition.
In 6;31° motion in anomaly is 3;33°, so at true opposition \( \bar{\alpha} = 12;22° \).
Computation of circumstances of eclipse.

From Table VI 8. II, argument 279;32°.

<table>
<thead>
<tr>
<th>At greatest distance</th>
<th>At least distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>Duration</td>
</tr>
<tr>
<td>2;32 digits</td>
<td>0:26,22°</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>2;10 digits and 0;13,13°</td>
</tr>
</tbody>
</table>

From III. argument 12;22°: sixtieths: 0;43.
Magnitude: 2;32 + 2;10 \times 0;0,43 = 2;34 digits (text: 3 digits observed).
Duration: 0;26,22 + 0;13,13 \times 0;0,43 = 0;26,31°.
To get time from beginning to middle of eclipse, we divide the duration, increased by a twelfth, by the moon’s true hourly motion:
\( 0;26,31 \times \frac{11}{12} + 0;30,24 = 0;57° \).
Beginning of eclipse (Alexandria) 10:9 p.m.
Eclipse middle 11:6 p.m.
End of eclipse 12:3 a.m.
Magnitude ca. 2½ digits.

12. VI 10. Given year, month and place, compute solar eclipse.
There is no example of a solar eclipse in the Almagest, so I have selected the eclipse of 364, June 16, which Theon observed at Alexandria, and gave as the example of computation in his commentary on the Almagest, first according to the Almagest, and again according to the Handy Tables (Basel edition pp. 332–339, cf. Rome [6]). A somewhat different calculation of the same eclipse also
appears in some mss. of Theon’s small commentary on the Handy Tables, and has been published in extenso by Tihon, ‘Calcul de l’eclipse’.

Example: Nabonassar 1112, Thoth, Alexandria.

From Table VI 3 compute mean conjunction:

<table>
<thead>
<tr>
<th>Days of Thoth</th>
<th>Ρ</th>
<th>Σ</th>
<th>Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period:</td>
<td>1101</td>
<td>22;41,45</td>
<td>19;11,56°</td>
</tr>
<tr>
<td>Year:</td>
<td>11</td>
<td>1;9,39</td>
<td>358;28,11°</td>
</tr>
</tbody>
</table>

Year 1112 23;51,24^d = 17;40,7° = 133;57,51° = 276;54,0°.
Time of mean conjunction: 23;51,24^d = Thoth 24, 8:34 a.m.
ω lies within ecliptic limits for solar eclipse, which is therefore possible.

Computation of true conjunction.

From Table III 6, c(Ω): -0;41° solar equation
From Table IV 10, c(α): -3;50° lunar equation.

True position in latitude: ω = Ω + c(Ω) = 273;4° at mean conjunction.
Δλ = -0;41° + 3;50° = 3;9°.
Moon’s true hourly motion in longitude: 0;32,56° + 0;32,40 × 3° = 0;34,56°
(Theon: 0;34,56°).
Δt = 3;9 × 1/6 + 0;34,56 = 5;52^h.
Time of true conjunction: 8:34 a.m. + 5;52^h = 2;26 p.m. (Theon: 2 + 1 + 10 hours after noon).
Motion over Δt: 3;9 × 1/6 ≈ 3;25°.
We add this to the position in latitude:ω = 276;29° at true conjunction.
In 5;52^h mean motion in anomaly is 3;12°, so at true conjunction α = 137;10°.
To find time of apparent conjunction at Alexandria we have first to find true local time, i.e. apply equation of time.

True longitude of sun at mean conjunction: Ρ + λ + c(Ρ) = 17;40° + 65;30° - 0;41° = 82;29°.
Motion of sun from mean to true conjunction: Δλ. 12 = 0;16°.
True longitude of sun at true conjunction: 82;45°.
Hence equation of time with respect to era Nabonassar (cf. Example 8 for method): +24 mins.
Time of true conjunction with respect to noon at Alexandria: 2;50 p.m.
Calculation of apparent conjunction.

(1) Parallax computation (cf. Example 10).
From Table II 13, Clima III, λ = Π 22;45°, 2;50 p.m.: zenith distance: 38;28° angle: 17;35°.
From Table V 18, ζ = 38;28°, α = 137;10° (latitude of moon neglected):
   total parallax of sun: 0;1,45°
   total parallax of moon: 0;39,35° (from cols. 3 and 4 only)
   difference in parallax: 0;37,50°.
Longitudinal parallax (for angle 17;35°): pλ = 0;36°.
Time from true to apparent conjunction is found by dividing the above by the true hourly velocity of the moon: 0;36 ÷ 0;34,56 = 1;2^h.
Hence time of apparent: 3;52 p.m.
(2) Second parallax computation, for corrected time.
From Table II 13, Clima III, $\lambda = \Pi 22;45^\circ$, 3:52 p.m.:
zenith distance: 51;48° angle: 18;32°.
In 1;2° motion in anomaly is about 0;33°, hence $\alpha$ for corrected time is
137;10° + 0;33° = 137;43°.
Neglecting lunar latitude, as before, from Table V 18, $\zeta = 51;48^\circ$, $\alpha = 137;43^\circ$:
total parallax of sun: 0;2,15°
total parallax of moon: 0;49,47°
difference in parallax: 0;47,32°.
Longitudinal parallax (for angle 18;32°): $p'_{\lambda} = 0;45^\circ$.
Computation of the 'epiparallax':
Difference between first and second longitudinal parallaxes,
\[ d = p'_{\lambda} - p_{\lambda} = 0;45^\circ - 0;36^\circ = 0;09^\circ. \]
Further increment, $f$, is found by $f:d = d:p$, hence $f = 0;9 \times 0;9 \div 0;36 \approx 0;2$, and
epiparallax $= d + f = 0;11^\circ$.
Final parallax in longitude: 0;36° + 0;11° = 0;47°.
To account for sun's motion add $\frac{1}{5}$th to this: $\frac{1}{5} \times 0;47^\circ \approx 0;51^\circ$.
Time from true to apparent conjunction: 0;51 + 0;34,56 $\approx$ 1;28 h.
Hence time of apparent conjunction: 2;50 h + 1;28 h = 4;18 p.m. (Theon: 4;1 h p.m.)
Position of moon at this time:
$\lambda$: $\Pi 22;45^\circ + 0;51^\circ = \Pi 22;46;26^\circ$
$\omega$: 277;29° + 0;51° = 277;30°
$\alpha$: 137;10° + 0;51° = 137;61°
Computation of circumstances of eclipse.
Computation of latitudinal parallax.
From Table II 13, Clima III. $\lambda = \Pi 23;36^\circ$, 4:18 p.m.:
zenith distance: 57;18° angle: 19;46°.
From Table V 18. with $\zeta = 57;18^\circ$, $\alpha = 138;1^\circ$:
total parallax of sun: 0;2,24°
total parallax of moon: 0;53,2°
difference in parallax: 0;50,38°.
Latitudinal parallax (cf. Example 10) for angle 19;46°: $p_{\beta} = 0;17^\circ$.
We convert this to a distance along the moon's orbit by multiplying it by 12:
$\Delta \omega = 12.p_{\beta} = 3;24^\circ$ (Theon uses the factor 11$\frac{1}{2}$ and gets 3;19°).
Since $\omega$ is 277;20°, the moon is just past the ascending node. The effect of the
deparallax is southwards, therefore its effect on $\omega$ is negative.
Final position of moon on orbit: 277;20° - 3;24° = 273;56°, apparent argument
of latitude.
From Table VI 8. I, argument 273;56°:
<table>
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<th>At least distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>Duration</td>
</tr>
<tr>
<td>4;8 digits</td>
<td>23;44,28 minutes</td>
</tr>
<tr>
<td>Magnitude</td>
<td>Duration</td>
</tr>
<tr>
<td>4;56 digits</td>
<td>26;18,52 minutes</td>
</tr>
</tbody>
</table>
of travel | of travel |
$\Delta$: 0;48 digits and 2;34,24 minutes.
From III, argument $\alpha = 138;1^\circ$: sixtieths: 51,39.
Magnitude: 4;8 + 0;48 $\times$ 0;51,39 = 4;49 digits.
Duration: 23;44,28 + 2;34,24 $\times$ 0;51,39 = 25;57 minutes of travel.
We increase the latter by 1/10th, to account for the sun's motion: 28;7', and divide by the moon's hourly velocity, 0;34,56°, to get half-duration of the eclipse: 0;28,7 + 0;34,56 = 0;48,18° ('Theon: 1/10th + 1/10th = 0;48°').

Thus circumstances of eclipse (neglecting variation of zenith distance during the eclipse):

- **Magnitude**: 4;49 digits (Theon: 4;39,18 digits)
- **Beginning of eclipse, Alexandria**: 3;30 p.m. (Theon: 3;32 p.m.)
- **Mid-eclipse, Alexandria**: 4;18 p.m. (Theon: 4;20 p.m.)
- **End of eclipse, Alexandria**: 5;6 p.m. (Theon: 5;8 p.m.).

('Theon goes on to calculate the differences in beginning and end of eclipse because of the variation in the zenith distance, cf. Almagest VI 10 pp. 312-13. These amount to 12 minutes earlier and 7 minutes later respectively, verifying Ptolemy's statements about the effect on the intervals).

Using modern tables (those in P. V. Neugebauer, *Astronomische Chronologie*), I find:

- **Maximum phase at Alexandria**: 5.6 digits
- **Times of phases at Alexandria**: beginning: 15;18°
  middle: 16;28°
  end: 17;24°.

---

13. VI 13 p. 319. Given the circumstances of an eclipse (magnitude and times of principal phases), compute the 'inclination' (πρόσνευσίς, i.e. point on the horizon towards which the line joining the centres points).

We take as example the solar eclipse of Example 12 (364 June 16 = Nabonassar 1112, Thoth 24), beginning of eclipse (first contact).

Given: time at Alexandria, 3;30 p.m.; magnitude, 4:49 digits.

First, find the rising-point of the ecliptic (cf. Example 4).

The longitude of the sun is Π 22:45° (Example 12 p. 655).

Time in seasonal hours at Alexandria (cf. Example 2): 3° after noon.

Hence rising-point of ecliptic: π, 10°; and setting-point is therefore 8 10°.

From Fig. 6.7, azimuth of 8 10° at Clima III:

<table>
<thead>
<tr>
<th>8 0° 13:33° N. of W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Π 0° 23:53° N. of W.</td>
</tr>
</tbody>
</table>

Hence 8 10° is 17° N. of W.

From Table VI 12, col. 2 argument 4;49 digits: 37;41°.

Moon is north of ecliptic (ω is somewhat more than 270° in Example 12).

Hence this angle is set off to the north of the setting-point.

So point of 'inclination' on the horizon is 17° + 37;41° = 54;41° N. of W.

---

14. XI 12 p. 554. Compute the longitude of a planet from the tables for a given time.

Example: Mars, Nabonassar 886, Epiphii [XI] 15/16, 9 p.m. (cf. X 8, where Mars is observed for this moment).
Appendix A. Examples 14-15

From mean motion tables, IX 4, find mean longitude and mean anomaly:

<table>
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<tr>
<th>epoch</th>
<th>$\lambda$</th>
<th>$\bar{\alpha}$</th>
</tr>
</thead>
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<tr>
<td>810°</td>
<td>138:15,13°</td>
<td>24:48,59°</td>
</tr>
<tr>
<td>72°</td>
<td>92:17,21°</td>
<td>250:12,21°</td>
</tr>
<tr>
<td>3°</td>
<td>213:50,43°</td>
<td>145:25,31°</td>
</tr>
<tr>
<td>10° (300°)</td>
<td>157:13,4°</td>
<td>138:28,21°</td>
</tr>
<tr>
<td>14°</td>
<td>7:20,13°</td>
<td>6:27,43°</td>
</tr>
<tr>
<td>9°</td>
<td>0:11,47°</td>
<td>0:10,23°</td>
</tr>
</tbody>
</table>

$\bar{\lambda} = 252:40°$  $\bar{\alpha} = 172:46°$ (as X 8 p. 500).

Apogee position at epoch: $\bar{\omega} = 16:40°$

motion of apogee in 886° (at 1° in 100°): 8:52°

hence apogee position at date: 115:32°.


From anomaly table (XI 11):

with argument $\bar{\omega}$, find equation of centre from col. 3 and col. 4:

$137:8° - 9,3 - 0,41 = 8:22°$ (cf. X 8, $\angle$ ZBE = 16:44°).

Since $\bar{\omega}$ is in the first column (less than 180°), we subtract the latter from $\bar{\lambda}$ and add it to $\bar{\alpha}$:

$\lambda' = 252:40 - 8:22 = 244:18°$, $\alpha = 172:46 + 8:22 = 181:8°$.

With argument $\alpha$, take the equation from col. 6: $c_6(181:8°) = 2.10°$.

With argument $\bar{\omega}$, take the 'sixtieths' from col. 8: $c_8(137:8°) = 37.9$

Since $\bar{\omega}$ is between mean distance and perigee ($c_8$ positive), we take the increment from col. 7: $c_7(181:8°) = 0:53°$.

Then equation of anomaly $c = c_6 + c_8.c_7 = 2:10° + 0:53° \times 0:37,9 = 2:43°$.

(cf. X 8, $\angle$ BEX = 5:26°).

Since $\alpha$ is greater than 180° (in second column of argument), this equation is negative.

Therefore $\lambda = \lambda' - c = 244:18° - 2:43° = 241:35°$ (X 8: observed: $\varphi 1\frac{1}{2}°$).

15. XIII 6. Compute latitude of planet, given 'corrected longitude' (see p. 635 n.55: distance of epicycle center from apogee, $\kappa_0$) and 'corrected anomaly' ($\alpha$).

(a) Outer planet. Example: Jupiter, Nabonassar 507 XI 18, 6 a.m. (cf. XI 3 p. 522)

Given: $\kappa_0 = 290:40°$, $\alpha = 72:3°$.

$\omega = \kappa_0 - 20° = 270:40°$: $c_5(\omega \ ) = 0,43$ (Table XIII 5).

$\omega > 270°$, so we enter col. 3: $c_3(72:3°) = 1:21°$.

$\beta = c_3.c_5 = 1:21 \times 0:0,43 \approx +0,1°$ (northerly since we took $c_5$).

Text says that Jupiter occulted $5$ Cnc, which according to the star catalogue (XXV 5) had a latitude of $-0°$. Thus there is a discrepancy of $8°$. Tuckerman (- 240 Sept. 4) gives $\beta \approx +0,14°$. Since $5$ Cnc was, by modern calculations, almost exactly on the ecliptic at the time of the observation, there could not have been an occultation.
Appendix A. Example 15

(b) Inner planet. Example: Mercury, Nabonassar 486 IV 18, 6 a.m. (cf. IX 7 p. 450)
Given: \( \kappa_0 = 129;44^\circ, \alpha = 239;15^\circ. \)

Table XIII 5, argument \( \alpha: c_3 = 1;27^\circ, \)
\[ c_4 = 2;29^\circ. \] Since \( 90^\circ < \kappa_0 < 270^\circ, \) we add to the latter \( 1/2 \)th of itself:
\[ c'_4 = 2;29^\circ + 0;15^\circ = 2;44^\circ. \]

\[ \kappa' = \kappa_0 + 270^\circ = 39;44^\circ. \] \( c_5(\kappa') = 45,55. \)

\[ \beta_1 = 1;27^\circ \times 0;45,55 = 1;7^\circ. \]

Condition A2 (p. 635) holds, since \( \kappa' < 90^\circ, 90^\circ < \alpha < 270^\circ, \) so \( \beta_1 \) is northerly.

\[ \kappa'' = \kappa_0 + 180^\circ = 309;44^\circ. \] \( c_5(\kappa'') = 38,11. \)

\[ \beta_2 = c'_4 \times c_5 = 2;44 \times 0;38,11 = 1;44^\circ. \]

Condition A2 (p. 636) holds, since \( 270^\circ < \kappa'' < 360^\circ, \alpha > 180^\circ, \) so \( \beta_2 \) is southerly.

\[ \beta_3 = 0;45^\circ \times c_5(\kappa''), c_5(\kappa'') = 0;18^\circ. \] This is southerly.

\[ \beta = \beta_1 + \beta_2 + \beta_3 = +1;7^\circ - 1;44^\circ - 0;18^\circ = -0;55^\circ. \]

Text says Mercury was '3 moons to the north' of \( \delta \) Cap. In the star catalogue (XXXI 24) this has a latitude of \(-2^\circ\); so according to the observation Mercury's latitude should be about \(-1^\circ\), a discrepancy of about \(1^\circ\) with the computation.

From Tuckerman, for \( -261 \) Feb. 12, 6 a.m. Alexandria, I find a latitude of about \(+0;8^\circ\).
Appendix B

 Corrections to Heiberg's text

This is a list of all corrections to the Greek text of the standard edition which I have adopted in making the translation (for certain types of corrections omitted see Introduction p. 4). For each item I give the reference in Heiberg's text, the correction (usually the reading of Heiberg followed, after a colon, by the reading I adopt), and the page and note in which I make and, where necessary, justify the correction.

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Appendix C

How did Ptolemy derive the mean motions for the five planets?

Our discussion concerns only the mean daily motions in anomaly, since the mean daily motions in longitude are not derived independently: for Venus and Mercury the latter are identical with that of the sun, while for the outer planets they are found by subtracting the mean daily motions in anomaly from the sun’s mean daily motion.

The answer to the above question would seem to be provided by those chapters entitled ‘On the correction of the periodic motions [of each planet]’, IX 10 (Mercury), X 4 (Venus), X 9 (Mars), XI 3 (Jupiter) and XI 7 (Saturn). In every case Ptolemy determines the position of the planet on the epicycle at one of his own observations, and also at an ‘ancient’ observation (approximately 400 years earlier). From the (Babylonian) period relations stated in IX 3 he computes how many integer revolutions in anomaly have occurred between the two observations: this plus the increment in degrees derived from the two observations gives the total motion of the planet in anomaly. Division of the latter by the interval in days and fractions of a day between the two observations gives the mean daily motion in anomaly, and Ptolemy explicitly states in every case that this was the basis of the mean daily motion used in the tables (IX 4).

However, if one does the computations implied in the above chapters using Ptolemy’s numbers, in no case does one find agreement with the mean daily motions in anomaly which he actually lists, as the following shows.

Ptolemy’s mean daily motions in anomaly (IX 3 pp. 424-5)

\[
\begin{align*}
\alpha &: 0;57,7,43,41,43,40^\circ/; \\
\beta &: 0;54,9,2,46,26,0^\circ/; \\
\gamma &: 0;27,41,40,19,20,58^\circ/; \\
\delta &: 0;36,59,25,53,11,28^\circ/; \\
\epsilon &: 3;6,24,6,59,35,50^\circ/
\end{align*}
\]

p. 543 \(\alpha\) travels 35,11,51;27° in 36,57,59;45" − 0;57,7,43,41,44,18% \[1a\]

p. 524 \(\beta\) travels 34,31,45;45° in 38,15,32,57,30" − 0;54,9,2,45,8,48% \[2a\]

p. 504 \(\gamma\) travels 19,13,1;43° in 41,38,1,40" − 0;27,41,40,19,28,7% \[3a\]

1 Cf. Newton pp. 320-1, 325-7, where the discrepancy is described almost correctly, but implausible consequences drawn.

2 In these and subsequent computations the last place is rounded on the basis of one more computed place.

3 Ptolemy gives an increment of ‘1 day’, implying 6 a.m. for the first observation and 10 p.m. for the second. If we assume (improbably) that the second was in fact 10;25 p.m. (cf. p. 484 n.32), and
The worst of these discrepancies, that for Jupiter, does not produce an error of as much as one minute of arc in 400 years. Hence it is clear that Ptolemy had no motive for ‘fudging’ here (and also that it is strictly illegitimate to derive a mean motion to the sixth sexagesimal fractional place from observations separated by only 400 years). But, although his observations are essentially in agreement with the mean daily motions he uses, the latter cannot be derived from them, not at least by the method he states.7

An alternative possibility is suggested by the way the derivation of the mean motions is presented in IX 3. There Ptolemy expresses them in the form of ‘corrections’ to the period relations, e.g. ‘for Saturn, 57 returns in anomaly correspond to 59 tropical years plus 1 1/2 days’. These are reduced to degrees and days, e.g. ‘Saturn travels (in anomaly) 20520° in 21551;18’. It is plausible to suppose that the latter are actually primary, i.e. the corrections ‘plus 1 1/2 days’ etc. are derived from the equivalences between days and degrees together with the parameter ‘one tropical year equals 365;14,48°’.8 These equivalences can be derived from the pairs of observations in IX 10 etc., combined with the Babylonian period relations, as follows.

Example: Saturn. From Hipparchus Ptolemy knew the Babylonian period relation, 57 returns in anomaly take place in 59 years, i.e. that the planet travels (57 × 360)° in approximately (59 × 365;14,48)°. He knew from his pair of observations that it travels 35.11,51:27° in 36,57,59,45bas. From the latter equivalence he could derive a ‘correction’ to the period of days in the former, by multiplying 36,57,59:45 by (57 × 360) and dividing the result by 35.11,51:27. This produces 5,59,11,17,59,55...°, or (rounded to the nearest sixtieth) 21551;18°, as in IX 3. The corresponding calculations for the other planets are:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Correction</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>35.11,51:27</td>
<td>5,59,11,17,59,55...°</td>
</tr>
<tr>
<td></td>
<td>36,57,59,45</td>
<td>21551;18°</td>
</tr>
</tbody>
</table>

Text in IX 3 has 28857:53, emended by me to 28857:43 (cf. n.8).

The increment actually 16,25°, this would make the interval 41,38,1;41,2,30°, leading to 0,27,41,40,18,46,32°, which is even more discrepant.

1But see p. 479 n.21. The interval, which Ptolemy rounds to integer days, should probably be 1 1/2 or 1 1/4 hours less. These corrections lead to daily motions of 0,36,59,25,51,36,24° and 0,36,59,25,32,29,19°, of which the second is much closer to, but still not identical with, the tabulated daily motion.

2Applying the equation of time of -23 mins. to Ptolemy’s observation, i.e. taking the increment as 13,7°, instead of 13,1°, leads to a daily motion of 3,6,24,7,3,2°, which is even more discrepant.

3Assuming that we correct the interval for Venus as in n.4.

4In case anyone should conjecture that Ptolemy computed the times of the observations more precisely than he states (with e.g. corrections for equation of time), I note that in order to get Ptolemy’s mean daily motion accurate to the sixth sexagesimal fractional place directly from the observations, these would have to be recorded to an accuracy of seconds, which is totally implausible.

5This works well for all planets except Mars (where the text figure, ‘28857,53°’ is certainly corrupt. I have emended ‘53’ to ‘43’, but ‘42’ would give perfect agreement with the above hypothesis) and Mercury, where ‘+1 3/8°’ should rather be ‘+1 3/4°’. But, rather than emending to ‘1 3/8°’ (which is possible), we can regard ‘1 3/4°’ as simply a small inaccuracy.
Appendix C. Derivation of planetary mean motions

\[ 41,30,52 \times (5 \times 360) + 25,35,38;25 = 48,39;40,5,19. \ldots \text{d} \text{ or (rounded)} \]
\[ 2919;40;52,52,52 \text{ as in IX 3.} \]
\[ 40,50,13;33,45 \times (145 \times 360) + 2,6,52,6;53 = 4,40,2;24,1. \ldots \text{d} \text{ or (rounded)} \]
\[ 16802;24;24,24,24,24 \text{ as in IX 3.} \]

From these ‘corrected period relations’ the mean daily motions can now be derived:

\[ 20520^\circ \text{ in } 21551;18^h \text{ leads to } 0;57,7,43,41,43,39,41. \ldots \text{d}, \text{ in agreement with [1].} \]
\[ 23400^\circ \text{ in } 25927;37^d \text{ leads to } 0;54,9,2,42,55,52. \ldots \text{d}, \text{ in disagreement with [2], and worse than [2a].} \]
\[ 13320^\circ \text{ in } 28857;41^d \text{ leads to } 0;27,41,40,18,39,12. \ldots \text{d}, \text{ in disagreement with [3], and worse than [3a].} \]
\[ 1800^\circ \text{ in } 2919;40^d \text{ leads to } 0;36,59,25,53,11,27,36. \ldots \text{d}, \text{ in agreement with [4].} \]
\[ 52200^\circ \text{ in } 16802;24^d \text{ leads to } 3;6,24,6,59,35,49,55. \ldots \text{d}, \text{ in agreement with [5].} \]

Thus, perverse as this procedure may appear, it could theoretically be used to derive Ptolemy’s mean motions for Saturn, Venus and Mercury. However, it fails miserably for Jupiter and Mars, which casts doubt on the validity of this explanation in general.

Let us suppose, instead, that Ptolemy found his mean daily motions by some other method. Then the equivalences ‘Saturn travels 20520\degree \text{ in } 21551;18^h \text{ etc. can be directly derived by division of 20520 by 0;57,7,43,41,43,40, etc.} \]
and the pairs of observations in IX 10 etc. are simply used as a check. E.g. for Saturn Ptolemy found from the observations an increment of 351;27\degree \text{ in } 364;219\text{.}

From the mean motion tables one finds, for the latter interval, 351;26,59\degree. The corresponding numbers for the other planets are:

\[ 377^h 128^d - 1^h \text{ observations } 105;45^\circ \text{ tables } 105;45,48^\circ \]
\[ 410^d 231^d \text{ observations } 61;43^\circ \text{ tables } 61;42,55^\circ \]
\[ 409^d 167^d \text{ observations } 338;25^\circ \text{ tables } 338;27,48^\circ \]
\[ 402^d 283^d 13^h \text{ observations } 246;53^\circ \text{ tables } 246;53,28^\circ \]

Thus the observations can in every case be regarded as justifying the mean motions used, within the accuracy attainable. On this assumption, Ptolemy had derived his mean motions from some other source, and simply did not bother to

\[ ^9 \text{Taking an interval } 1^h \text{ or } 1^h \text{ hours less (see n.4) makes no difference to the first sexagesimal fractional place.} \]
\[ ^{10} \text{Taking the sexagesimal fraction of the day as } 42,43 \text{ or 53 (cf. n.8) produces a progressively smaller mean daily motion and progressively greater disagreement.} \]
\[ ^{11} \text{It is interesting that this quotient lies almost exactly in the middle between the mean daily motion which Ptolemy gives explicitly (28 in the last sexagesimal place) and that underlying the sections for years and 18-year periods in the mean motion tables (27 in the last sexagesimal place, cf. p. 425 n.29). Is this an indication of incomplete revision?} \]
\[ ^{12} \text{Mars is still a problem here, since this method also produces 28857;41^d (cf. n.8).} \]
\[ ^{13} \text{For an interval } 1^h \text{ less (cf. n.4) one finds from the tables 338;25,30^\circ, in agreement with the result from the observations.} \]
change them on the basis of the observations he quotes (in this he was absolutely justified, since, as we saw above, an interval of 400 years is insufficient to guarantee more than 4 sexagesimal fractional places; he was not of course justified in concealing it from his readers).

This still leaves unexplained the basis of the actual mean motions. One might conjecture that they were derived from observations made over a shorter period (e.g. between Hipparchus and Ptolemy). It is easy to find, by Diophantine analysis, plausible intervals in time and longitude which produce the exact numbers, e.g. for Mars a motion in 274° 189; 16° of 128 revolutions plus 169; 32° leads to a mean daily motion of 0; 27, 41, 40, 19, 20, 57, 59°. But in the absence of any evidence for such observations by Hipparchus this remains mere arithmetical juggling, and we must admit that the origin of these numbers, at least for Jupiter and Mars, and probably for all the planets, remains unknown.14

14 An alternative conjecture is that the mean motions were indeed derived from the quoted observations, but by applying a 'correction' to an earlier (?Hipparchan) mean motion, in the same way as the mean motion in lunar anomaly was corrected in IV 7 (and in lunar latitude in the Canobic Inscription). But since no such mean motion is mentioned by Ptolemy, the details would be irrecoverable.
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